The Australian Journal of Mathematical Analysis and Applications

AJMAA

Volume 5, Issue 2, Article 12, pp. 1-11, 2009

LONG CORRELATIONS APPLIED TO THE STUDY OF AGRICULTURAL INDICES IN COMPARISON WITH THE S&P500 INDEX

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Received 8 August, 2007; accepted 24 November, 2008; published 11 March, 2009.

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ABSTRACT. Long-time correlations in agricultural indices are studied and their behavior is compared to the well-established S&P500 index. Hurst exponent and Detrended Fluctuation Analysis (DFA) techniques are used in this analysis. We detected long-correlations in the agricultural indices and briefly discussed some features specific in comparison to the S&P500 index.

Key words and phrases: Agricultural indices, Financial indices, Memory effects.

2000 *Mathematics Subject Classification.* 60H25, 60H30.

ISSN (electronic): 1449-5910

This work has been partially supported by the Economic Development Administration (U.S. Department of Commerce), ADVANCE - NSF, and Minigrant College of Arts & Sciences, NMSU.

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1. INTRODUCTION

A growing literature has recently analyzed the behavior of different economic time-series data, including the existence of long-term or short-term correlations in the behavior of financial markets. Highlights of this literature encompass analyze of major stock indices (established versus emerging markets [5], [7], [8], developed countries' market indices [2], [3], [17], [18], [28], Bombay stock exchange index [16], [26], Latin American indices [11]). This paper analyzes the presence of long-time correlations in some agricultural indices. We consider two NCREIF (National Council of Real Estate Investment Fiduciaries) indices, namely, the farmland index and timberland index, and compare their behavior to the S&P500 index of the NYSE.

We use Re-Scaled Range Analysis and Detrended Fluctuation Analysis methods to determine long-range correlations. Both methods characterize fractional behavior, but R/S analysis can yield more accurate results for small data sets [5]. As the time-series data sets for the agricultural indices are very small, and the exponents calculated could serve as verification and comparison of the results, both methods are used.

2. R / S ANALYSIS

Hurst developed the R/S analysis [9], [15]. He observed many natural phenomena that followed a biased random walk, i.e., every phenomenon showed a pattern. He measured the trend using an exponent now called the Hurst exponent.

The procedure used to calculate the Hurst exponent by using the R/S method is as follows:

1. Let N be the length of time series $(y_1, y_2, y_2, \cdot\cdot\cdot, y_m)$. First, the logarithmic ratio of the time series is obtained. The length of the new time series $M(t)$ will be $N-1$.

$$M(t) = \log(fracy_{t+1}y_t) \quad t = 1, 2, \cdot\cdot\cdot N-1$$

2. The time series is then divided into m sub-series of length n, where n represents the number of elements in the series and m represents the number of sub-series. Thus
$m \cdot n = N-1$. Each sub-series can be labeled as Q_a where $a = 1, 2, \cdot\cdot\cdot m$ and each element in Q_a can be labeled as $L_{k,a}$ for $k = 1, 2, \cdot\cdot\cdot n$.

3. For each Q_a, the average value is calculated:

$$Z_a = \frac{1}{n}\sum_{k=1}^{n} L_{k,a}$$

4. The cumulative deviation in each Q_a is calculated:

$$C_{k,a} = \sum_{j=1}^{k}(L_{k,a} - Z_a) \quad k = 1, 2, \cdot\cdot\cdot n$$

5. Thus the range of each sub-series Q_a is given as:

$$R(Q_a) = \max_k(C_{k,a}) - \min_k(C_{k,a})$$

6. The standard deviation of each sub-series Q_a is calculated:

$$S(Q_a) = \sqrt{\frac{1}{n}\sum_{j=1}^{n}(L_{j,a} - Z_a)^2}$$

7. Each sub-series is normalized by dividing the range, $R(Q_a)$, by the standard deviation, $S(Q_a)$. The average value of R/S for sub-series of length n is obtained by:

$$\frac{R}{S}_n = \frac{1}{m}\sum_{j=1}^{m}\frac{R(Q_a)}{S(Q_a)}$$

8. Steps 2 through 7 are repeated for all possible values of n, thus obtaining the corresponding R/S values for each n.

The relation between length of the sub-series, n and the rescaled range R/S is:

$$\frac{R}{S} = (c \cdot n)^H$$

where R/S is the rescaled range, n is the length of the sub-series of the time series and H is the Hurst exponent. Taking logarithms yields:

$$\log\frac{R}{S} = H \cdot \log n + H \cdot \log c$$

9. An ordinary least squares regression is performed using $\log(\frac{R}{S})$ as a dependent variable and $\log n$ as an independent variable. The slope of the equation provides the estimate of the Hurst exponent, H.

If the Hurst exponent, H, for the investigated time series is 0.5, then it implies that the time series follows a random walk which is an independent process. For data series with long memory effects, H would lie between 0.5 and 1. It suggests all the elements of the observation are dependent. This means that what happens now would have an impact on the future. This property of the time series is called persistent time series and this character enables prediction of any time series as it shows a trend. If H lies between 0 and 0.5, it implies that the time-series possess anti-persistent behavior (negative autocorrelation).

3. DETRENDED FLUCTUATION ANALYSIS

The Detrended Fluctuation Analysis *DFA* method is an important technique in revealing long range correlations in nonstationary time series. This method was developed by Peng [22], [23], and has been successfully applied to the study of cloud breaking [10], Latin-American market Indices [11], DNA [4], [24], cardiac dynamics [22], [25], climatic studies [13], [14], solid state physics [12], [29], and economic time series [1], [2], [6], [21].

In this work, we have done the following steps for the numerical implementation of the DFA analysis:

1. First the absolute value of $M(t)$, the logarithmic returns of the indices calculated in the R/S analysis is integrated:

$$y(t) = \sum_{i=1}^{t}|M(i)|$$

2. Then the integrated time series of length N is divided into m boxes of equal length n with no intersection between them. As the data is divided into equal lengths, there may be some left over at the end. To take account of these leftover values, the same procedure is repeated but beginning from the end, obtaining $\frac{2N}{n}$ boxes. Then, a least squares line is fitted to each box, representing the trend in each box, thus obtaining $y_n(t)$.

3. Finally the root mean square fluctuation is calculated by using the formula:

$$F(n) = \sqrt{\frac{1}{2N}\sum_{t=1}^{2N}[y(t) - y_n(t)]^2}$$

This computation is repeated over all box sizes to characterize a relationship between the box size n and $F(n)$. A linear relationship between $F(n)$ and n (i.e. box size) in a log-log plot reveals that the fluctuations can be characterized by a scaling exponent α, the slope of the line relating $\log F(n)$ to $\log n$.

For data series with no correlations or short-range correlation, α is expected to be 0.5. For data series with long-range power law correlations, α would lie between 0.5 and 1; and for power law anti-correlations, α would lie between 0 and 0.5. This method was used to measure correlations in financial series of high frequencies and in the daily evolution of some of the most relevant indices.

4. DATA

We studied the behavior of agricultural indices using data from the NCREIF farmland index and the NCREIF timberland index. As NCREIFï£¡s indices serve as one of the prominent indices in the field of agriculture and real estate, it has been considered for the study and there is no other specific reason to select these indices.

4.1. NCREIF Farmland Index.

The National Council of Real Estate Investment Fiduciaries (NCREIF) farmland index is a quarterly index which measures the performance of individual agricultural properties that is acquired in the private market for the purpose of investment [19]. The composition of the farmland index is not stable and changes over time. For example, when the data contributor for the index buys or sells the property, then the index changes. Also, when new investors are added as a data contributor, the composition of the index changes [19]. This index may not represent the agricultural investment as a whole [19]. Even though this index does completely represent the agricultural land investment market, it captures a reasonable reflection. Index data is available from the year 1992.

4.2. NCREIF Timberland Index.

The National Council of Real Estate Investment Fiduciaries (NCREIF) timberland index measures the level of return and variability on well-managed and industrial grade timberlands [19]. Like the farmland index, the timberland index is released quarterly, but dates back to 1987.

As the data is not the actual index, but the returns of the index, the returns were used to construct the indices:

$$Index_t = Index_{t-1} + [Index_{t-1} \times \frac{Returns_t}{100}], \quad t = 1, 2, \cdots n$$

The index value of $t = 1$ is assumed to be 1000 in both indices. Hence the index values for each quarter t are calculated for all n available periods.

Quarterly returns of the two indices were collected and the index value constructed: NCREIF Farmland Index, from 1992 to 2006 (60 data points); NCREIF Timberland Index, from 1987 to 2006 (80 data points). We compared these indices to the daily data of the S&P500, an index of the New York Stock Exchange, from 1987 to 2006 (5,045 data points) and from 1992 to 2006 (3,781 data points).

5. RESULTS AND DISCUSSIONS

Hurst as well as DFA analyses are performed to find the persistence of long correlations. The results obtained are summarized in Tables 1 and 2 and in Figures 1 through 8. The values obtained from the Hurst and DFA analyses are generally in the same range, thus verifying the values obtained.

The α and the Hurst exponent H for both indices are greater than 0.5, which confirm the existence of long range correlations in both farmland and timberland indices. We cannot conclude that the behavior of the farmland and timberland indices is similar to the S&P500 index based on α values and Hurst exponents. As can be seen from the figures, the S&P500 is a stable index compared to the other two. It is possible to conclude that even though the farmland and timberland indices do not behave similarly to the S&P500 index, the long memory effects are in the same range for these indices when compared to the S&P500, i.e. the degree to which the long term memory effects occur is similar.

The diagram suggests that the farmland index fluctuates more than the timberland. The farmland index is not a stable index due to its inconsistency in the mixture of components that represent the index, whereas the Timberland index consists only of two contributors since its beginning [27]. This is warranted by a study which concludes that the volatility of the timberland index is very low [20]. We can see from the figures obtained using both R/S analysis and DFA analysis, that the relationship between $\log n$ and $\log F(n)$ or $\log(R/S)$ does not represent a perfect linear trend. This effect can be due to the small number of available data.

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Index	α **(DFA Exponent)**	**Error**
Farmland	0.6108	0.15
Timberland	0.6527	0.07
S&P 500 (87-06)	0.6818	0.04
S&P 500 (92-06)	0.6135	0.03

Table 5.1: Values of the α *exponent for all indices calculated by using the DFA Method*

Index	H **(Hurst Exponent)**	**Error**
Farmland	0.6618	0.06
Timberland	0.6824	0.07
S&P 500 (87-06)	0.6043	0.02
S&P 500 (92-06)	0.6004	0.02

Table 5.2: Values of the H *exponent for all indices calculated by using the R/S Analysis*

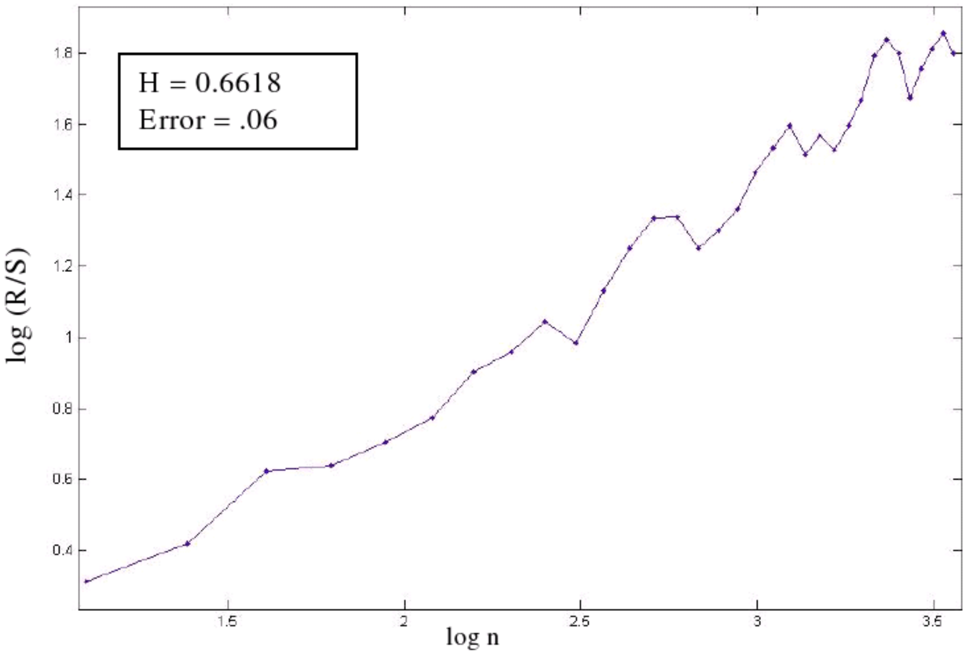

Figure 1: R/S Analysis applied to the data series (1992 - 2006) of the NCREIF Farmland index.

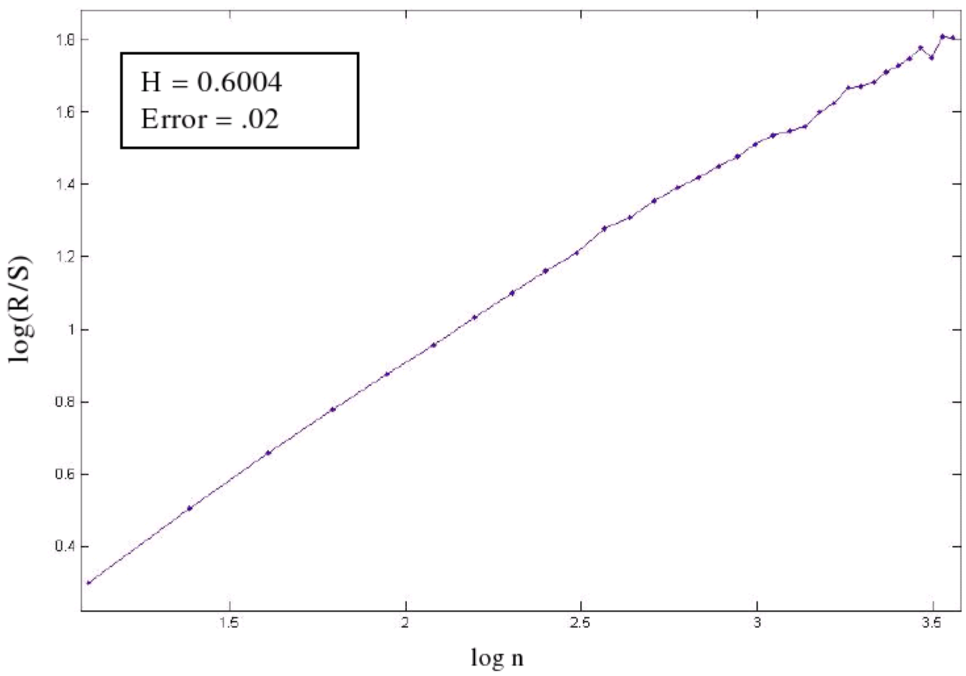

Figure 2: R/S Analysis applied to the data series (1992 - 2006) of the S & P 500 index.

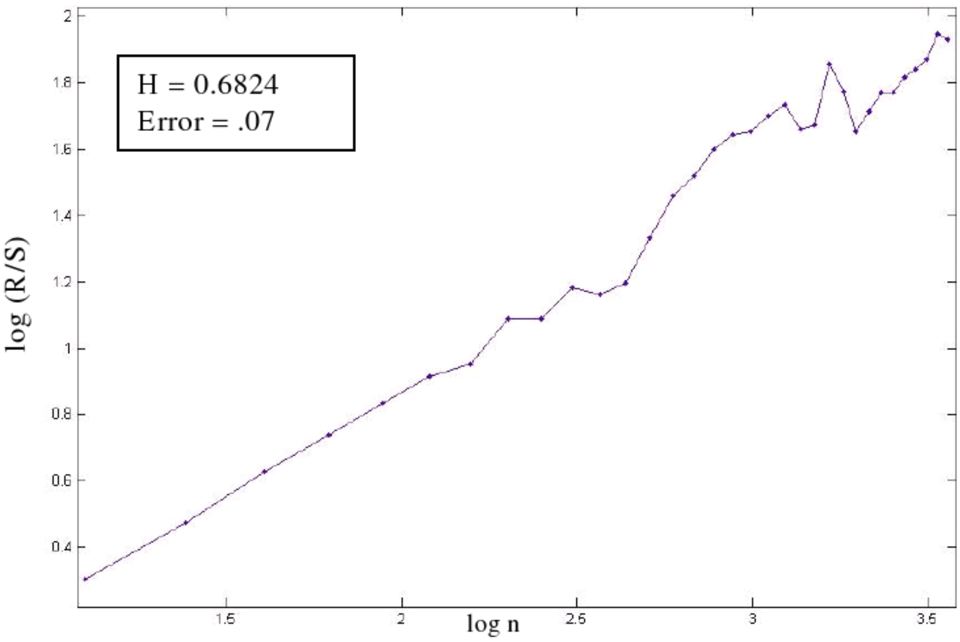

Figure 3: R/S Analysis applied to the data series (1987 - 2006) of the NCREIF Timberland index.

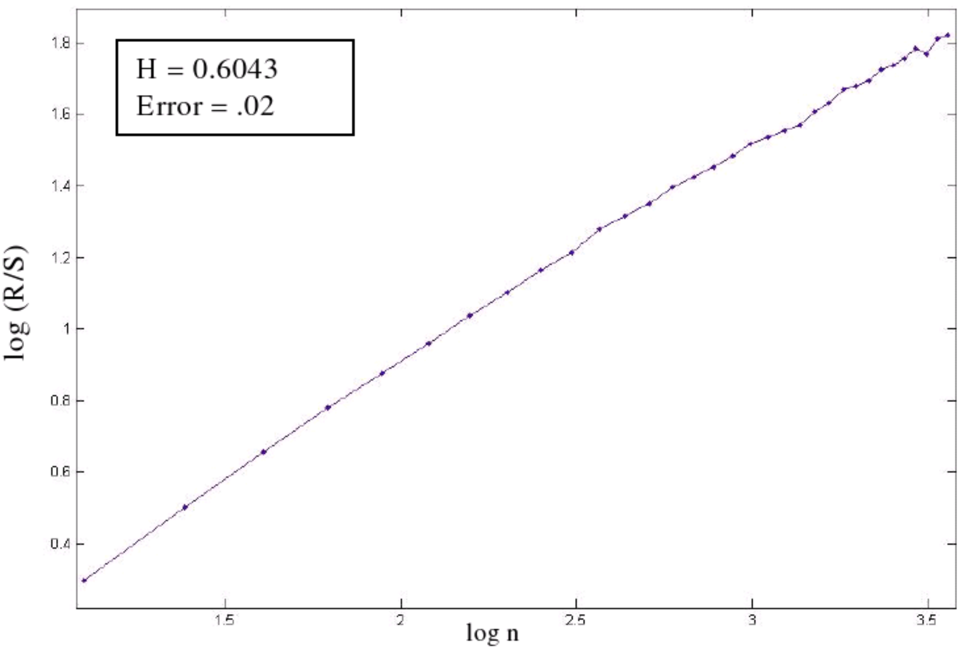

Figure 4: R/S Analysis applied to the data series (1987 - 2006) of the S & P 500 index.

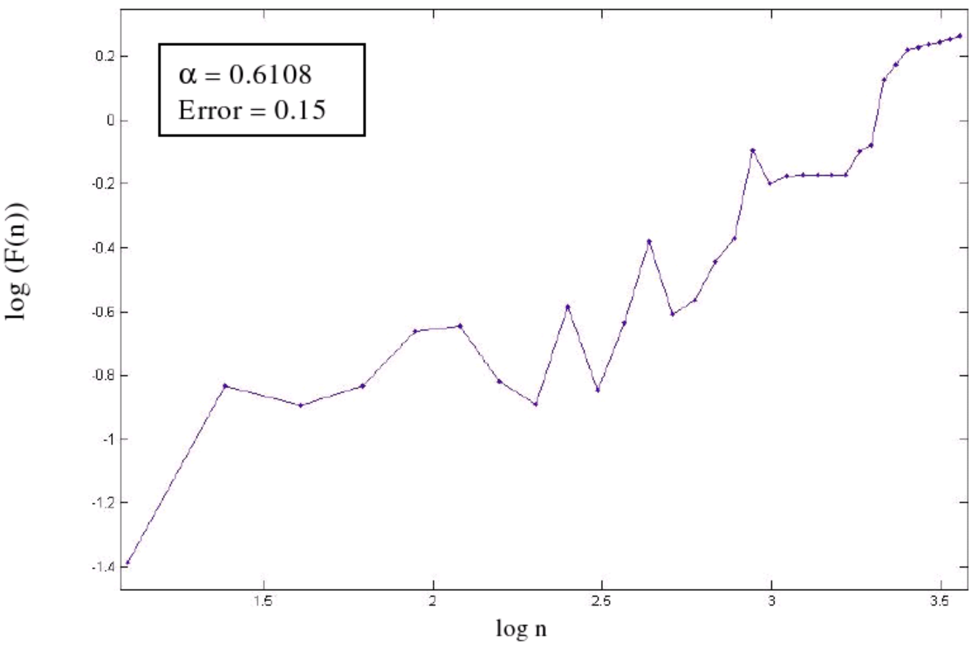

Figure 5: DFA method applied to the data series (1992 - 2006) of the CDREIF Farmland index.

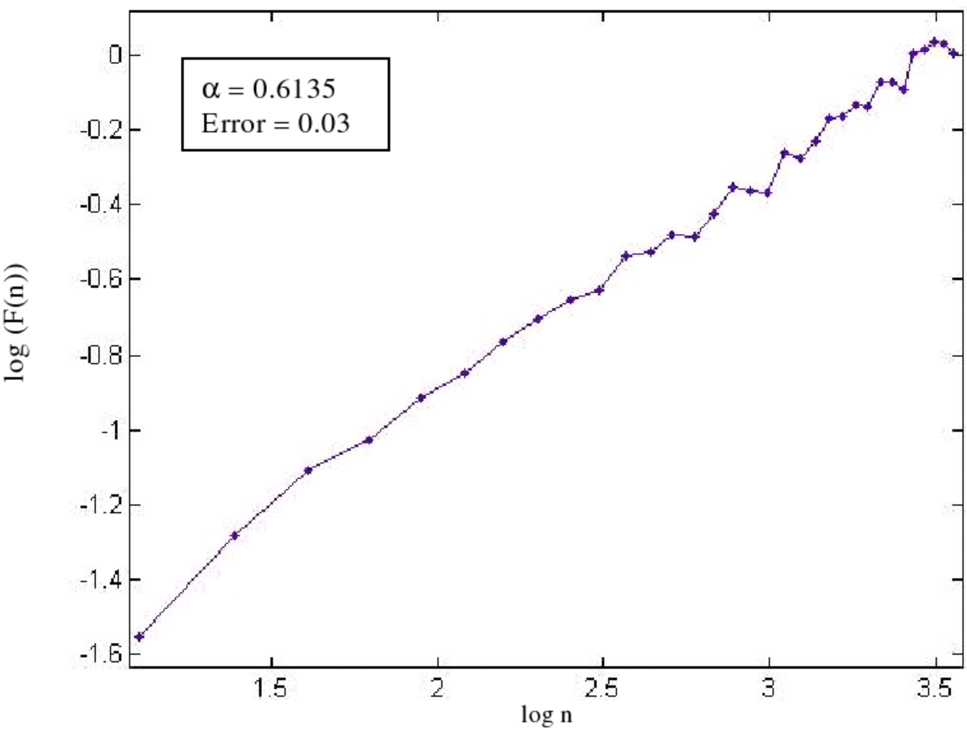

Figure 6: DFA method applied to the data series (1992 - 2006) of the S & P 500 index.

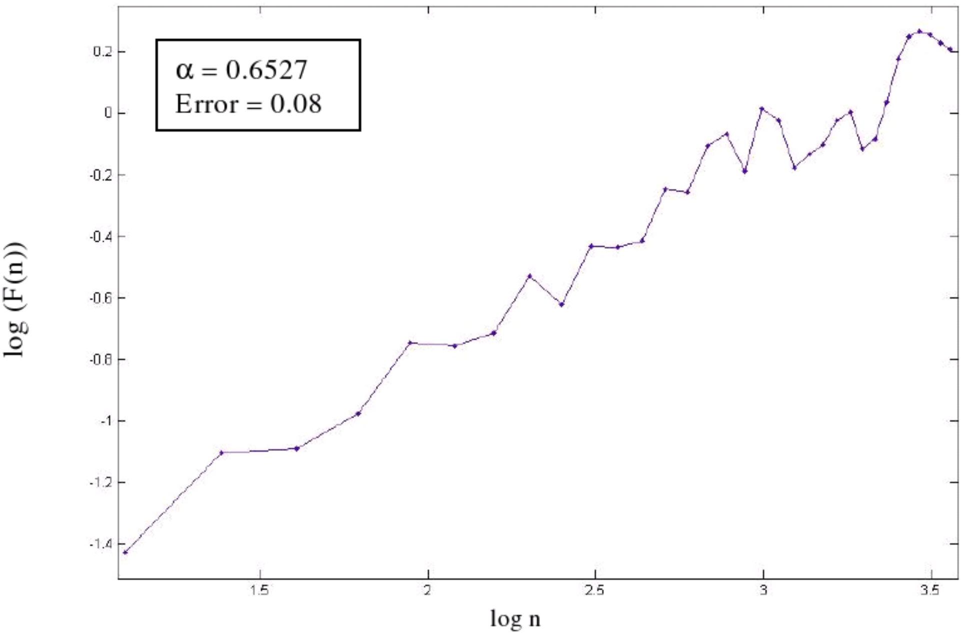

Figure 7: DFA method applied to the data series (1987 - 2006) of the NCREIF Timberland index.

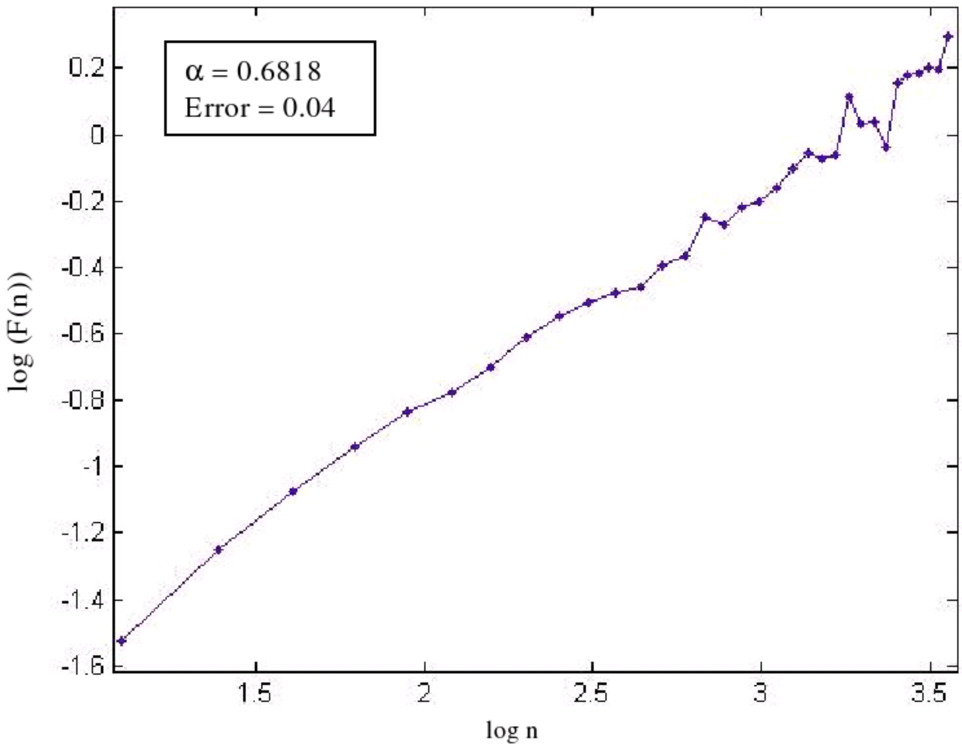

Figure 8: DFA method applied to the data series (1987 - 2006) of the S & P 500 index.