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A NEW STEP SIZE RULE IN NOOR'S METHOD FOR SOLVING GENERAL VARIATIONAL INEQUALITIES

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ABSTRACT. In this paper, we propose a new step size rule in Noor's method for solving general variational inequalities. Under suitable conditions, we prove that the new method is globally convergent. Preliminary numerical experiments are included to illustrate the advantage and efficiency of the proposed method.

Key words and phrases: General variational inequalities, Self-adaptive rules, Projection method, Pseudomonotone operators.

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1. INTRODUCTION

Let H be a real Hilbert space, whose inner product and norm are denoted by $\langle\cdot,\cdot\rangle$ and $\|\cdot\|$, let I be the identity mapping on H, and $T, g : H \to H$ be two operators. Let K be a nonempty closed convex subset of H. We consider the problem of finding $u^* \in H$ such that $g(u^*) \in K$ and

$$\langle T(u^*), g(v) - g(u^*)\rangle \geq 0, \qquad \forall g(v) \in K. \tag{1.1}$$

Problem (1.1) is called the general variational inequality, which first introduced and studied by Noor [11] in 1988. For the applications, formulation and numerical methods of general variational inequalities (1.1), see {[1], [5], [12]– [16], [19]}.

If $g \equiv I$, then the problem (1.1) is equivalent to finding $u^* \in K$ such that

$$\langle T(u^*), v - u^*\rangle \geq 0, \qquad \forall v \in K, \tag{1.2}$$

which is the classical variational inequality problem.

There are many theoretical results on the existence of solutions to variational inequalities. In essence, there are two general approaches to attack the existence problem. The first is a constructive approach in which, one assumes appropriate condition and proposes an algorithm that actually produces a solution. While, the second is an analytical approach in which one relies on an equivalent reformulation of the variational mathematical problem (such as a fixed-point problem, a constrained or unconstrained optimization problem).

We now have a variety of techniques to suggest and analyze various iterative algorithms for solving variational inequalities. The fixed-point theory has played an important role in the development of various algorithms for solving variational inequalities. The basic idea is very simple. Using the projection operator technique, one usually establishes an equivalence between the variational inequalities and the fixed-point problem. This alternative equivalent formulation was used by Lions and Stampacchia [10] to study the existence of a solution of the variational inequalities.

A well known projection method is the extragradient method of Korpelevich [9], which makes two projections on a closed convex set at each iteration, the distance between the iterative point and the solution set monotonically converges to zero. It is well known that the convergence of this method requires that the operator must be monotone and Lipschitz continuous. When the operator is not Lipschitz continuous or when the Lipschitz continuous is not known, the extragradient method and its variant forms require an Armijo-like line search procedure to compute the step size with a new projection need for each trial, which leads to expansive computation. To overcome these difficulties, several modified projection and extragradient-type methods have been suggested and developed for solving variational inequality problems, see {[3]–[8], [11]–[17]}. Recently, Noor [13, 14] proposed some new methods for solving general variational, the convergence of these methods only requires the pseudomonotonicity of the operator, which is weaker condition than monotonicity.

Inspired and motivated by the results of Noor [13], we propose a new method for solving general variational inequalities, by using a new step size.

Throughout this paper, we make following assumptions.

Assumptions:

- H is finite dimension space.
- g is homeomorphism on H i.e., g is bijective, continuous and g^{-1} is continuous.
- T is continuous and g-pseudomonotone operator on H i.e.,

$$\langle T(u), g(u') - g(u)\rangle \geq 0 \Rightarrow \langle T(u'), g(u') - g(u)\rangle \geq 0 \qquad \forall u', u \in H.$$

- The solution set of problem (1.1) denoted by S^*, is nonempty.

2. PRELIMINARIES

We summarize some preliminary results which are useful in the following analysis.

Lemma 2.1. *For a given $u \in K$, $z \in H$ satisfy the inequality*

$$\langle u - z, v - u \rangle \geq 0, \quad \forall v \in K$$

holds if and only if $u = P_K(z)$.

It follows from Lemma 2.1 that

$$\langle z - P_K(z), P_K(z) - v \rangle \geq 0, \qquad \forall z \in H, v \in K. \tag{2.1}$$

It follows that

$$\|P_K(z) - v\| \leq \|z - v\|, \quad \forall z \in H,\ v \in K, \tag{2.2}$$

It is well-known that the projection operator P_K is nonexpansive, that is,

$$\|P_K(u) - P_K(v)\| \leq \|u - v\|, \qquad \forall u, v \in H. \tag{2.3}$$

Lemma 2.2. *$u^* \in H$ is solution of Problem (1.1) if and only if $u^* \in H$ satisfies the relation:*

$$g(u^*) = P_K[g(u^*) - \rho T(u^*)], \tag{2.4}$$

From Lemma 2.2, it is clear that u is solution of (1.1) if and only if u is a zero point of the function

$$r(u, \rho) := g(u) - P_K[g(u) - \rho T(u)].$$

It has been shown [1] that $\|r(u, \rho)\|$ is a non-decreasing function with respect to ρ .

Lemma 2.3. *For all $u \in H$ and $\rho' > \rho > 0$, it holds that*

$$\|r(u, \rho')\| \geq \|r(u, \rho)\|. \tag{2.5}$$

The fixed-point formulation (2.4) has been used in [13] and [1] to suggest and analyze the following algorithms for solving problem (1.1).

Algorithm 2.1. [13]
For a given $u^0 \in H$, compute the approximate solution u^{k+1} by the iterative schemes.
Predictor step.

$$g(w^k) = P_K[g(u^k) - \rho_k T(u^k)],$$

where ρ_k satisfies

$$\rho_n \langle T(u^k) - T(g^{-1}(P_K[g(u^k) - \rho_k T(u^k)])), r(u^k, \rho_k) \rangle \leq \delta \|r(u^k, \rho_k)\|^2, \quad \delta \in (0, 1).$$

Corrector step.

$$g(u^{k+1}) = P_K[g(u^k) - \alpha_k^{'} d(u^k)],$$

where

$$d(u^k, \rho_k) = r(u^k, \rho_k) - \rho_k T(u^k) + \rho_k T(w^k),$$

and

$$\alpha_k^{'} = \frac{(1 - \delta)\|r(u^k, \rho_k)\|^2}{\|d(u^k, \rho_k)\|^2}$$

is the corrector step size.

Algorithm 2.2. [1]

Step 0. Given $\epsilon > 0, \gamma \in [1, 2), \mu \in (0, 1), \rho > 0, \delta \in (0, 1), \delta_0 \in (0, 1)$ *and* $u^0 \in H$*, set* $k = 0$.

Step 1. Set $\rho_k = \rho$. *If* $\|r(u^k, \rho)\| < \epsilon$*, then stop; otherwise, find the smallest no-negative integer* m_k*, such that* $\rho_k = \rho\mu^{m_k}$ *satisfying*

$$\|\rho_k(T(u^k) - T(w^k))\| \le \delta\|r(u^k, \rho_k)\|,$$

where

$$w^k = g^{-1}(P_K[g(u^k) - \rho_k T(u^k)]).$$

Step 2. Compute

$$d(u^k, \rho_k) := r(u^k, \rho_k) - \rho_k T(u^k) + \rho_k T(w^k),$$
$$\phi(u^k, \rho_k) := \|r(u^k, \rho_k)\|^2 - \rho_k \langle r(u^k, \rho_k), T(u^k) - T(w^k)\rangle$$

and the step size

$$\alpha_k^{''} = \frac{\phi(u^k, \rho_k)}{\|d(u^k, \rho_k)\|^2}.$$

Step 3. Get the next iterate

$$g(u^{k+1}) = P_K[g(u^k) - \gamma\alpha_k^{''} d(u^k, \rho_k)].$$

Step 4. If

$$\|\rho_k(T(u^k) - T(w^k))\| \le \delta_0\|r(u^k, \rho_k)\|,$$

then set $\rho = \frac{\rho_k}{\mu}$*, else set* $\rho = \rho_k$*. Set* $k := k + 1$*, and go to Step 1.*

3. BASIC RESULTS

In this Section, we describe our method and we prove some basic properties, which will be used to establish the sufficient and necessary conditions for the convergence of the proposed method. We propose the following algorithm for solving problem (1.1).

Algorithm 3.1. *Step 0. Given* $\epsilon > 0, \gamma \in [1, 2), \mu \in (0, 1), \rho > 0, \delta \in (0, 1), \delta_0 \in (0, 1)$ *and* $u^0 \in H$*, set* $k = 0$.

Step 1. Set $\rho_k = \rho$. *If* $\|r(u^k, \rho)\| < \epsilon$*, then then stop; otherwise, find the smallest no-negative integer* m_k*, such that* $\rho_k = \rho\mu^{m_k}$ *satisfying*

$$\|\rho_k(T(u^k) - T(w^k))\| \le \delta\|r(u^k, \rho_k)\|, \tag{3.1}$$

where

$$w^k = g^{-1}(P_K[u^k - \rho_k T(u^k)]). \tag{3.2}$$

Step 2. Set

$$\varepsilon^k = \rho_k(T(w^k) - T(u^k)), \tag{3.3}$$

$$d(u^k, \rho_k) := g(u^k) - g(w^k) + \varepsilon^k, \tag{3.4}$$

$$d_1(u^k, \rho_k) := g(u^k) - g(w^k) + \rho_k T(w^k), \tag{3.5}$$

$$\phi(u^k, \rho_k) := \langle g(u^k) - g(w^k), d(u^k, \rho_k)\rangle \tag{3.6}$$

and the step size

$$\alpha_k^{'''} := \frac{\phi(u^k, \rho_k)}{\|d_1(u^k, \rho_k)\|^2} \tag{3.7}$$

Step 3. Get the next iterate

$$g(u^{k+1}) = P_K[g(u^k) - \gamma\alpha_k^{'''} d_1(u^k, \rho_k))].$$

Step 4. If

$$\|\rho_k(T(u^k) - T(w^k))\| \leq \delta_0 \|r(u^k, \rho_k)\|,$$

then set $\rho = \frac{\rho_k}{\mu}$, else set $\rho = \rho_k$. Set $k := k+1$, and go to Step 1.

Remark 3.1. (3.1) *implies that*

$$|\langle g(u^k) - g(w^k), \varepsilon^k\rangle| \leq \delta\|r(u^k, \rho_k)\|^2, \qquad 0 < \delta < 1. \tag{3.8}$$

The next lemma shows that α_k and $\phi(u^k, \rho_k)$ are lower bounded away from zero.

Lemma 3.1. *For given $u^k \in H : g(u^k) \in K$ and $\rho_k > 0$, let w^k and ε^k satisfy to* (3.2) *and* (3.3)*, then*

$$\phi(u^k, \rho_k) \geq (1-\delta)\|r(u^k, \rho_k)\|^2 \tag{3.9}$$

and

$$\alpha_k^{'''} \geq c, \tag{3.10}$$

where $c > 0$.

Proof. It follows from (3.6) and (3.8) that

$$\begin{aligned} \phi(u^k, \rho_k) &= \|g(u^k) - g(w^k)\|^2 + \langle g(u^k) - g(w^k), \varepsilon^k\rangle \\ &\geq (1-\delta)\|r(u^k, \rho_k)\|^2. \end{aligned}$$

Since $\delta \in (0,1)$, then we can find a constant $c > 0$ such that

$$\alpha_k^{'''} := \frac{\varphi(u^k, \rho_k)}{\|d_1(u^k, \rho_k)\|^2} \geq c.$$

We can get the assertion of this lemma. ∎

Lemma 3.2. $\forall u^k \in H : g(u^k) \in K, u^* \in S^*$ *and* $\rho > 0$, *we have*

$$\langle g(u^k) - g(u^*), d_1(u^k, \rho_k)\rangle \geq \phi(u^k, \rho_k) \tag{3.11}$$

where $d_1(u^k, \rho_k)$ and $\phi(u^k, \rho_k)$ are defined in (3.5) *and* (3.6) *respectively.*

Proof. For any $u^* \in S^*$ solution of problem (1.1), we have

$$\langle \rho_k T(u^*), g(w^k) - g(u^*)\rangle \geq 0, \qquad \forall \rho_k > 0. \tag{3.12}$$

Using the g-pseudomonotonicity of T, we obtain

$$\langle \rho_k T(w^k), g(w^k) - g(u^*)\rangle \geq 0. \tag{3.13}$$

Substituting $z = g(u^k) - \rho_k T(u^k)$ and $v = g(u^*)$ into (2.1), we get

$$\langle g(u^k) - \rho_k T(u^k) - g(w^k), g(w^k) - g(u^*)\rangle \geq 0. \tag{3.14}$$

Adding (3.13) and (3.14), we have

$$\langle g(u^k) - g(w^k) - \rho_k[T(u^k) - T(w^k)], g(w^k) - g(u^*)\rangle \geq 0,$$

which can be rewritten as

$$\langle g(u^k) - g(w^k) - \rho_k[T(u^k) - T(w^k)], g(w^k) - g(u^k) + g(u^k) - g(u^*)\rangle \geq 0,$$

then

$$\begin{aligned} \langle g(u^k) - g(u^*), g(u^k) - g(w^k) + \rho_k T(w^k)\rangle &\geq \|g(u^k) - g(w^k)\|^2 - \rho_k\langle g(u^k) - g(w^k), T(u^k) - T(w^k)\rangle \\ &\quad + \langle g(u^k) - g(u^*), \rho_k T(u^k)\rangle. \end{aligned}$$

Using the g-pseudomonotonicity of T, the last term in the right side of the above inequality is positive, we obtain

$$\langle g(u^k) - g(u^*), d_1(u^k, \rho_k)\rangle \geq \|g(u^k) - g(w^k)\|^2 - \rho_k \langle g(u^k) - g(w^k), T(u^k) - T(w^k)\rangle,$$

and the conclusion of Lemma 3.2 is proved. ∎

4. GLOBAL CONVERGENCE

In this section, we prove the global convergence of the proposed method. The following theorem plays a crucial role in the convergence of the proposed method.

Theorem 4.1. *Let $u^* \in H$ be a solution of problem (1.1) and let $\{u^k\}$ be the sequence obtained from Algorithm 3.1. Then $\{u^k\}$ is bounded and*

$$\|g(u^{k+1}) - g(u^*)\|^2 \leq \|g(u^k) - g(u^*)\|^2 - \gamma(2-\gamma)c(1-\delta)\|r(u^k, \rho_k)\|^2. \tag{4.1}$$

Proof. Let $u^* \in H$ be a solution of problem (1.1), then

$$\begin{aligned}
\|g(u^{k+1}) - g(u^*)\|^2 &\leq \|g(u^k) - g(u^*) - \gamma\alpha_k^{'''} d_1(u^k, \rho_k)\|^2 \\
&= \|g(u^k) - g(u^*)\|^2 - 2\gamma\alpha_k^{'''}\langle g(u^k) - g(u^*), d_1(u^k, \rho_k)\rangle \\
&\quad + \gamma^2 \alpha_k^{'''2}\|d_1(u^k, \rho_k)\|^2 \\
&\leq \|g(u^k) - g(u^*)\|^2 - 2\gamma\alpha_k^{'''}\phi(u^k, \rho_k) + \gamma^2\alpha_k^{'''}\phi(u^k, \rho_k) \\
&\leq \|g(u^k) - g(u^*)\|^2 - \gamma(2-\gamma)c(1-\delta)\|r(u^k, \rho_k)\|^2,
\end{aligned}$$

where the first inequality follows from (2.2), the second inequality follows from (3.7) and (3.11), and the third inequality follows from Lemma 3.1. Since $\gamma \in [1, 2)$ and $\delta \in (0, 1)$ we have

$$\|g(u^{k+1}) - g(u^*)\| \leq \|g(u^k) - g(u^*)\| \leq \ldots \leq \|g(u^0) - g(u^*)\|.$$

Since g is homeomorphism and from the above inequality, it is easy to verify that the sequence u^k is bounded, we can get the assertion of this theorem. ∎

Theorem 4.2. *The sequence $\{u^k\}$ generated by the proposed method converges to a solution point of problem* (1.1).

Proof. It follows from (4.1) that

$$\sum_{k=0}^{\infty} \gamma(2-\gamma)c(1-\delta)\|r(u^k, \rho_k)\|^2 \leq \|g(u^0) - g(u^*)\|^2,$$

which means that

$$\lim_{k\to\infty} \|r(u^k, \rho_k)\| = 0.$$

By using the definition of $r(u^k, \rho_k)$, we obtain

$$\lim_{k\to\infty} \|g(u^k) - g(w^k)\| = 0. \tag{4.2}$$

Since g is homeomorphisme, we have

$$\lim_{k\to\infty} \|u^k - w^k\| = 0,$$

consequently $\{w^k\}$ is also bounded. Since $\|r(u^k,\rho)\|$ is a non-decreasing function of ρ, it follows from $\rho_k \ge \rho_{\text{min}}$ that

$$\begin{aligned}
\|r(w^k,\rho_{\text{min}})\| &\le \|r(w^k,\rho_k)\| \\
&= \|g(w^k) - P_K[g(w^k) - \rho_k T(w^k)]\| \\
\text{(using (3.2) and (3.3))} \quad &= \|P_K[g(u^k) - \rho_k T(w^k) + \varepsilon^k] - P_K[g(w^k) - \rho_k T(w^k)]\| \\
\text{(using (2.3))} \quad &\le \|g(u^k) - g(w^k) + \varepsilon^k\| \\
\text{(using (3.1))} \quad &\le (1+\delta)\|g(u^k) - g(w^k)\|
\end{aligned}$$

and from (4.2), we get

$$\lim_{k\to\infty} r(w^k,\rho_{\text{min}}) = 0. \tag{4.3}$$

Let $\bar{u}$ be a cluster point of $\{w^k\}$ and the subsequence $\{w^{k_j}\}$ converges to $\bar{u}$. Since $r(u,\rho)$ is a continuous function of u, it follows from (4.3) that

$$r(\bar{u},\rho_{\text{min}}) = \lim_{j\to\infty} r(w^{k_j},\rho_{\text{min}}) = 0.$$

According to Lemma 2.2, $\bar{u}$ is a solution point of problem (1.1). Note that inequality (4.1) is true for all solution point of problem (1.1), hence we have

$$\|g(u^{k+1}) - g(\bar{u})\| \le \|g(u^k) - g(\bar{u})\|, \quad \forall k \ge 0. \tag{4.4}$$

Since $\{g(w^{k_j})\} \to g(\bar{u})$ and $g(u^k) - g(w^k) \to 0$, for any given $\varepsilon > 0$, there is an $l > 0$, such that

$$\|g(w^{k_l}) - g(\bar{u})\| < \varepsilon/2 \qquad \text{and} \qquad \|g(u^{k_l}) - g(w^{k_l})\| < \varepsilon/2. \tag{4.5}$$

Therefore, for any $k \ge k_l$, it follows from (4.4) and (4.5) that

$$\|g(u^k) - g(\bar{u})\| \le \|g(u^{k_l}) - g(\bar{u})\| \le \|g(u^{k_l}) - g(w^{k_l})\| + \|g(w^{k_l}) - g(\bar{u})\| < \varepsilon$$

and thus the sequence $\{g(u^k)\}$ converges to $g(\bar{u})$. Using g is homeomorphism, we obtain $\{u^k\}$ converges to $\bar{u}$.

In the following, we prove that the sequence $\{u^k\}$ has exactly one cluster point. Assume that $\tilde{u}$ is another cluster point and satisfies

$$\delta := \|g(\tilde{u}) - g(\bar{u})\| > 0.$$

Since $\bar{u}$ is a cluster point of the sequence $\{u^k\}$ and g is homeomorphism, there is a $k_0 > 0$ such that

$$\|g(u^{k_0}) - g(\bar{u})\| \le \frac{\delta}{2}.$$

On the other hand, since $\bar{u} \in S^*$ and from (4.1), we have

$$\|g(u^k) - g(\bar{u})\| \le \|g(u^{k_0}) - g(\bar{u})\| \quad \text{for all} \quad k \ge k_0,$$

it follows that

$$\|g(u^k) - g(\tilde{u})\| \ge \|g(\tilde{u}) - g(\bar{u})\| - \|g(u^k) - g(\bar{u})\| \ge \frac{\delta}{2} \quad \forall k \ge k_0.$$

This contradicts the assumption that $\tilde{u}$ is cluster point of $\{u^k\}$, thus the sequence $\{u^k\}$ converges to $\bar{u} \in S^*$. ∎

5. PRELIMINARY COMPUTATIONAL RESULTS

In this section, we present some numerical results for the proposed Algorithm 3.1. In order to verify the theoretical assertions, we consider the following problems:

$$\text{(5.1a)} \qquad \min \quad h(u) = \sum_{j=1}^{n} u_j \log(u_j/p_j)$$

$$\text{(5.1b)} \qquad \text{s.t.} \quad Au \in \Pi$$

$$\text{(5.1c)} \qquad u \geq 0$$

where A is an $n \times n$ matrix, Π is a simple closed convex set in R^n, $0 < p \in R^n$ is a parameter vector. It has been shown [1] that solving problem (5.1) is equivalent to find a pair (u^*, y^*), such that

$$\text{(5.2)} \qquad \beta f(u^*) = A^T y^*$$

and

$$\text{(5.3)} \qquad g(u^*) \in \Pi, \quad (g(v) - g(u^*))^T y^* \geq 0, \quad \forall g(v) \in \Pi,$$

where

$$g(u) = Au.$$

In the test we let $v' \in R^n$ be a randomly generated vector, $v'_j \in (-0.5, 0.5)$, and let $A = I - 2\frac{v'v'^T}{v'^T v'}$ be an $n \times n$ Householder matrix. Let

$$u_j^* \in (0.1, 1.1) \qquad \text{and} \qquad y_i^* \in (-0.5, 0.5).$$

Note that

$$f_j(u^*) = (\nabla h(u^*))_j = \log(u_j^*) - \log(p_j) + 1.$$

Since

$$f(u^*) = A^T y^*,$$

we set

$$p_j = u_j^* \exp(1 - e_j^T A^T y^*),$$

and we take

$$\Pi = \{z \mid l_B \leq z \leq u_B\}$$

where

$$(l_B)_i = \begin{cases} (Au^*)_i & \text{if } y_i^* \geq 0, \\ (Au^*)_i + y_i^* & \text{otherwise,} \end{cases}$$

$$(u_B)_i = \begin{cases} (Au^*)_i & \text{if } y_i^* < 0, \\ (Au^*)_i + y_i^* & \text{otherwise.} \end{cases}$$

In this way, we have

$$Au^* \in \Pi \qquad \text{and} \qquad Au^* = P_\Pi[Au^* - y^*].$$

In all tests we take $\mu = 2/3$, $\delta = 0.95$, $\delta_0 = 0.5$ and $\gamma = 1.95$. The calculations are started with a vector u^0, whose elements are randomly chosen in (0,1), and stopped whenever $\|r(u^k, \rho_k)\|_\infty \leq 10^{-7}$. All codes are written in Matlab and run on a P4-2.00G note book computer. We test the problem with dimensions $n = 200$ and $n = 300$. The iteration numbers and the computational time for Algorithm 2.1, Algorithm 2.2 and Algorithm 3.1 with different dimensions and initial parameter ρ_0 are given in the Tables 5.1-5.2.

Table 5.1: The Numerical results for problem (5.3) with $n = 200$

ρ_0	Algorithm 2.1		Algorithm 2.2		Algorithm 3.1	
	No. It.	CPU(Sec.)	No. It.	CPU(Sec.)	No. It.	CPU(Sec.)
10^5	307	4.31	42	2.59	37	2.17
10^4	377	4.33	32	2.25	32	1.80
10^2	293	2.81	25	1.32	20	1.05
1	321	2.66	17	0.51	9	0.36
10^{-1}	280	2.25	8	0.92	3	0.75
10^{-3}	11484	51.23	18	2.06	6	1.62

Table 5.2: The Numerical results for problem (5.3) with $n = 300$

ρ_0	Algorithm 2.1		Algorithm 2.2		Algorithm 3.1	
	No. It.	CPU(Sec.)	No. It.	CPU(Sec.)	No. It.	CPU(Sec.)
10^5	539	8.39	48	3.25	39	2.47
10^4	645	9.53	42	2.43	34	1.95
10^2	527	6.81	31	1.53	22	1.37
1	572	6.43	16	0.65	11	0.66
10^{-1}	516	6.21	14	1.11	5	0.84
10^{-3}	11532	120.01	16	1.66	5	1.05

The numerical results show that the new method is attractive in practice. Moreover, it demonstrates computationally that the new method is more effective than the methods presented in [1] and [13] in the sense that the new method needs fewer iteration and less computational time.

6. CONCLUSIONS

The presented study deals with a new method for solving general variational inequalities. The main contribution of this paper, firstly we used a new step size α_k, secondly we proposed a self-adaptive strategy of adjusting the parameter ρ_k and thirdly the numerical results showed that our algorithm works well for problems tested. How to design other efficient methods for solving general variational inequalities and linear general variational inequalities is worthy of further investigations in the future.

REFERENCES

[1] A. BNOUHACHEM, A self-adaptive method for solving general mixed variational inequalities, *J. Math. Anl. Appl.*, **309** (2005), pp. 136–150.

[2] R. GLOWINSKI, J. L. LIONS and R. TREMOLIERES, *Numerical Analysis of Variational Inequalities,* North-Holland, Amsterdam, Holland, 1981.

[3] B. S. HE, A new method for a class of linear variational inequalities, *Math. Prog.*, **66** (1994), pp. 137–144.

[4] B. S. HE, A modified projection and contraction method for a class of linear complementarity problems, *J. Computational Math.*, **14** (1996), pp. 54–63.

[5] B. S. HE, Inexact implicit methods for monotone general variational inequalities, *Math. Prog.*, **86** (1999), pp. 199–216.

[6] B. S. HE, and L. Z. LIAO, Improvement of some projection methods for monotone variational inequalities, *J. Optim. Theory Appl.*, **112** (2002), pp. 111–128.

[7] A. N. IUSEM and B. F. SVAITER, A variant of Korplevich's method for variational inequalities with a new search strategy, *Optimization*, **42** (1997), pp. 309–321.

[8] E. N. KHOBOTOV, Modification of the extragradient method for solving variational inequalities and certain optimization problems, *USSR Computational Mathematics and Mathematical Physics*, **27** (1987), pp. 120–127.

[9] G. M. KORPELEVICH, The extragradient method for finding saddle points and other problems, *Ekonomika i Matematcheskie Metody*, **12** (1976), pp. 747–756.

[10] J. L. LIONS and G. STAMPACCHIA, Variational inequalities. *Comm. Pure Appl. Math.*, **20** (1967), pp. 493–512.

[11] M. A. NOOR, General variational inequalities, *Appl. Math. Letters*, **1** (1988), pp. 119–121.

[12] M. A. NOOR and T.M. RASSIAS, A class of projection methods for general variational inequalities, *J. Math Anal. Appl.*, **268** (2002), pp. 334–343.

[13] M. A. NOOR, Pseudomonotone general mixed variational inequalities, *Appl. Math. Computation*, **141** (2003), pp. 529–540.

[14] M. A. NOOR, New extragradient-type methods for general variational inequalities, *J. Math. Anal. Appl.*, **277** (2003), pp. 379–395.

[15] M. A. NOOR, Some developments in general variational inequalities, *Appl. Math. Computation*, **152** (2004), pp. 199-277.

[16] M. A. NOOR and K.I. NOOR, Self-adaptive projection algorithms for general variational inequalities, *Appl. Math. Computation*, **151** (2004), pp. 659–670.

[17] M. V. SOLODOV and P. TSENG, Modified projection-type methods for monotone variational inequalities, *SIAM J. Control and Optim.*, **34** (1996), pp. 1814–1830.

[18] M. V. SOLODOV and B. F. SVAITER, A new projection method for variational inequality problems, *SIAM J. Control and Optim.*, **37** (1999), pp. 765–776.

[19] N. XIU, J. ZHANG and M. A. NOOR, Tangent projection equations and general variational inequalities, *J. Math. Anal. Appl.*, **258** (2001), pp. 755–762.