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RESULTS CONCERNING FIXED POINT FOR SOFT WEAKLY CONTRACTION IN SOFT METRIC SPACES

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ABSTRACT. The basic objective of the proposed research work is to make people acquainted with the concept of soft metric space by generalizing the notions of soft $(\tilde{\psi}, \tilde{\phi})$-weakly contractive mappings in soft metric space, as well as to look at specific fundamental and topological parts of the underlying spaces. A compatible example is given to explain the idea of said space structure. The theory is very useful in decision making problems and secure transmission as fixed point provides exact output. The fixed-point theorems on subsets of $\mathbb{R}^m$ that are useful in game theoretic settings.

Key words and phrases: Self-soft mapping; Soft metric spaces; soft contractions; Soft computing; Secure Transmission.

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1. INTRODUCTION

In cases where the theory's parameterization tool proves inadequate, Molodtsov [19] presented "the soft set" theory, a novel mathematical tool designed to address intrinsic challenges and uncertainties in theories. The study of soft metric space based on soft-points, or soft sets, was first undertaken by Das and Samanta [6] in order to apply soft set theory to actual issues and different realms for improved outputs. They presented the ideas of a soft null, absolute soft sets, soft subsets, soft unions, intersections, and complements in [8] based on these ideas. Currently, it is a recognized field of study and has drawn interest from several computer scientists, mathematicians, and economists [7] [[9]-[10]] [[18]-[19]] [[29]-[31]]. Cagman et al. [3] established the idea of a soft- topology on a soft set and examined some fundamental characteristics of soft topological spaces. A soft metric space variant of the well-known FPT given by Meir-keeler was established by Chen and Lin [4]. Similar kind of ideology was used by S. Ghosh et al. [32] to establish the concept of neutrosophic fuzzy metric space. Also with an alike conception M. Shabir et al. [29] worked on soft topological spaces. The presence and characteristics of fixed points are the subject of FPTs. The goal of FPT is to identify self-correspondences with at least one constant element. A fixed point is the point where a function maps to itself inside its domain. $Tu = u$ if u is referred to as a fixed point of T. A study given by Poincare laid foundation for the idea of fixed points, which was further expanded upon by Brouwer [17], S. Banach [2], and Kakutani [20], to mention a few. To solve Tx = x, Brouwer [17] developed a fixed-point answer. Kakutani expanded on Brouwer's findings. S. Banach [2] established the groundwork for modern fixed-point results in 1922 by demonstrating FPTs of contractive transformation in a whole metric space. Von Nemann [14] then introduced his minimax theorem for games with two players and Nash equilibrium [12] for games with three or more players. In order to derive results for minimax theorem, intersection lemma given by Von Neumann, Kakutani [20] discovered a fixed-point theorem in 1941. John Nash [13] presented his widely accepted equilibrium theory. Having vast applications in various fields not just limited to Mathematics and Engineering but also to Game Theory, Artificial Intelligence, Image processing etc., to name a few, fixed point results are useful in finding coincidence points of the functions representing the supply and demand in economics, nash-equilibrium in game theory with finite strategic games. Fan [16] and Glicksberg [11] extended Kakutani's theorem for locally convex Hausdorff topological vector spaces .Again Fan expanded the von Neumann intersection lemma by using his own fixed-point theory. Another intersection theorem for a finite family of sets with convex sections was discovered by Fan [15] in 1964. Maji [18] expanded on this in 1969 to encompass infinite families by the use of Fan's expansion of the von Neumann intersection lemma. Maji used his finding to apply Nash's theorem for arbitrary families as well as an analytical formulation of Fan type. Please take note that our most recent works [1] [[21]-[28]] expand on all of the aforementioned results in a number of ways. In actuality, convex subsets of (Hausdorff) topological vector spaces are the primary focus of those results. This study introduces a novel technique to expand soft metric ideas, such as the soft $(\tilde{\phi}, \tilde{\psi})$-weakly contractive theorem, to such topologies. It also suggests a generalized SMS and introduces the idea of a fixed- point under soft $(\tilde{\phi}, \tilde{\psi})$-weakly contractive conditions. Several other works focusing on similar kind of metric spaces where different contractions of same category has been considered include [[30]-[34]] [[36]-[42]]. Even after establishing various fixed point results in such type of metric spaces, some mathematicians have also given different applications where the deduced results has been applied in different other fields [35, 43].

2. PRELIMINARIES

Definition 2.1. [8] A pair (F, E) is called a soft set over X, where F is a function given by $F : E \to P(X)$ and E is a set of parameters. In other words, a soft set over X is a parameterized family of subsets of the universe X. For any parameter $x \in E,\ F(x)$ is taken as the defined collection of x-approximate elements of the soft set denoted by (F, E).

Definition 2.2. [8] Let *(F,E)* and *(G,D)* be two soft set over *X*. We say that *(F,E)* is a sub-soft set of *(G,D)* and denote it by $(F, E) \subset (G, D)$, if:
1. $E \subseteq D$, and
2. $F(e) \subseteq G(e), \forall e \in E$.

Definition 2.3. [6] *(F,E)* is a soft set over *X*:
1. *(F,E)* is said to be a null soft set denoted by $\tilde{\phi}$ if for every $e \in E, F(e) = \phi$.
2. *(F,E)* is said to be an absolute soft set denoted by $\tilde{X}$, if for every $e \in E, F(e) = X$.

Definition 2.4. [4] Given parameter set A where $A \subseteq E$. A pair (a, r) : $r \in R$, $a \in A$, is known to be SPS. (a, r) is non-negative when ever $r \geq 0$.
If $(a, r)\,, (b, r')$ are two SPSs then (a, r) is said to be no less than (b, r') represented as $(a, r) \geq (b, r')$ if $r \geq r'$.

3. SOFT METRIC SPACE

This part deals with presence and uniqueness of FPT for soft $\left(\widetilde{\psi}, \widetilde{\varphi}\right)$-weakly contractive-transformation in soft metric space

$$\tilde{\Psi} = \left\{\tilde{\psi} : [0, +\infty) \to [0, +\infty) \text{ is increasing and continuous function.}\right\}$$

$$\tilde{\Phi} = \left\{\tilde{\phi} : [0, +\infty) \to [0, +\infty) \text{ is increasing and continuous and } \phi(t) = 0.\right\}$$

In order to get our main results, we introduce some definitions and give one example to support our results. Fixed point theorems of soft contractive mappings and related concepts can be viewed in [5]. The details about soft metric space and related terms can be seen in [9].

Definition 3.1. Let $\left(\tilde{X}, \tilde{d}\right)$ be a soft metric space over $\tilde{U}$. A soft sequence $\{(\tilde{u}_n, \tilde{v}_n)\}_{n=1}^{\infty}$ in $(\tilde{U}, \tilde{V})$ is called convergent to $\tilde{u}$ if $\lim_{n\to+\infty} d(\tilde{u}_n, \tilde{u}) = d(\tilde{u}, \tilde{u})$.

Definition 3.2. Let $\left(\tilde{X}, \tilde{d}\right)$ be a soft metric space over $\tilde{U}$. A soft sequence $\{(\tilde{u}_n, \tilde{v}_n)\}_{n=1}^{\infty}$ in $(\tilde{U}, \tilde{V})$ is called Cauchy if $\lim_{n,m\to+\infty} d(\tilde{u}_n, \tilde{u}_m) = 0$.

Definition 3.3. Let $(\tilde{X}, \tilde{d})$ is said to be a complete soft metric space over $\tilde{U}$. For a Cauchy soft sequence $\{(\tilde{u}_n, \tilde{v}_n)\}_{n=1}^{\infty}$ in $(\tilde{U}, \tilde{V})$, there exists a $\tilde{u} \in \tilde{U}$ such that $\lim_{n,m\to+\infty} d(\tilde{u}_n, \tilde{u}_m) = \lim_{n\to+\infty} d(\tilde{u}_n, \tilde{u}) = d(\tilde{u}, \tilde{u})$.

Definition 3.4. Let $\tilde{f}$ and $\tilde{g}$ be two self- soft mappings on set $\tilde{U}$. if $\tilde{w} = (\tilde{f}u) = (\tilde{g}u)$, for some $\tilde{u}$ in $\tilde{U}$, then $\tilde{u}$ is said to be the coincidence point of $\tilde{f}$ and $\tilde{g}$, where $\tilde{w}$ is called the point of coincidence of $\tilde{f}$ and $\tilde{g}$. Let $\tilde{C}(\tilde{f}, \tilde{g})$ denote the set of all soft coincidence points of $\tilde{f}$ and $\tilde{g}$.

Definition 3.5. Let $\tilde{f}$ and $\tilde{g}$ be two self- soft mappings on set $\tilde{U}$. If for some $\tilde{u} \in \tilde{U}$, then $\tilde{u}$ is said to be weakly soft compatible if they commute at every coincidence point, that is, $(\tilde{f}u) = (\tilde{g}u) => f\tilde{g}u = g\tilde{f}u$ for every $\tilde{u} \in \tilde{C}(\tilde{f}, \tilde{g})$.

Corollary 3.1. *Let $(\tilde{X}, \tilde{d})$ be a complete soft metric space with a constant $s \geq 1$ and the two soft mappings $\tilde{f}$ and $\tilde{g}$ have a unique point of coincidence in $\tilde{X}$. Moreover, if the two soft maps $\tilde{f}$ and $\tilde{g}$ are weakly compatible, then $\tilde{f}$ and $\tilde{g}$ have a unique common fixed point.*

Example 3.1. *Let $\tilde{X} = [0, +\infty)$ and a soft metric space $\tilde{d} : \tilde{X} \times \tilde{X} \to [0, +\infty)$ defined by, $\tilde{d}(\tilde{U}, \tilde{\eta}) = (\tilde{U} + \tilde{\eta})^2$.*
Then $(\tilde{X}, \tilde{d})$ is a complete soft metric space with s=2 a constant. Define $\tilde{F}\tilde{U} = \frac{\tilde{U}}{4}$ and $\tilde{G}\tilde{U} = 1 + \frac{\tilde{U}}{8}$ are soft mappings $\tilde{f}$ and $\tilde{g}$ on $\tilde{X}$. Since $\tilde{k} \geq (1 + \tilde{k})$ for each $\tilde{k} \in [0, +\infty), \forall\, \tilde{U}, \tilde{\eta} \in (\tilde{X}, \tilde{d})$ we have,

$$\tilde{d}(\tilde{F}\tilde{U}, \tilde{F}\tilde{\eta}) = \left(\frac{\tilde{U}}{4} + \frac{\tilde{\eta}}{4}\right)^2 = \left(\frac{2\tilde{U}}{8} + \frac{2\tilde{\eta}}{8}\right)^2 = 4\left(\frac{\tilde{U}}{2} + \frac{\tilde{\eta}}{2}\right)^2$$
$$\geq 4\left(\left(1 + \frac{\tilde{U}}{8}\right) + \left(1 + \frac{\tilde{\eta}}{8}\right)\right)^2 = 4\tilde{d}(\tilde{G}\tilde{U}, \tilde{G}\tilde{\eta})$$

Which means that $\tilde{d}(\tilde{F}\tilde{U}, \tilde{F}\tilde{\eta}) \geq \tilde{\alpha}\tilde{d}(\tilde{G}\tilde{U}, \tilde{G}\tilde{\eta})$, where $\alpha = 4$. Hence all the conditions of Corollary 3.1 are satisfied, hence the mappings $\tilde{F}$ and $\tilde{G}$ have a unique point of coincidence actually 0 is the unique point of coincidence. Further by $F\tilde{G}0 = G\tilde{F}0$, we observe that 0 is unique fixed point of $\tilde{F}$ and $\tilde{G}$.

4. MAIN RESULT

Theorem 4.1. *Let $(\tilde{X}, \tilde{d})$ be a complete soft metric space with parameter $s \geq 1$ and let $\tilde{f}, \tilde{g} : \tilde{X} \to \tilde{X}$ be given self- soft mappings satisfying $\tilde{f}(\tilde{X}) \subset \tilde{g}(\tilde{X})$ where $\tilde{g}(\tilde{X})$ is a closed soft subset of $\tilde{X}$, if there are functions $\tilde{\psi} \in \Psi$ and $\tilde{\phi} \in \Phi$ such that*

$$\tilde{\psi}\left(s^2\left[\left(\tilde{d}(f\tilde{u}, f\tilde{v})\right)\right]^2\right) \leq \tilde{\psi}\left(F_s\left(\tilde{u}, \tilde{v}\right)\right) - \tilde{\phi}\left(E_s\left(\tilde{u}, \tilde{v}\right)\right). \tag{4.1}$$

where,

$$F_s(\tilde{u}, \tilde{v}) = \max\left\{[d(f\tilde{u}, g\tilde{u})]^2, [d(g\tilde{u}, g\tilde{v})]^2, [d(f\tilde{v}, g\tilde{v})]^2, d(f\tilde{u}, g\tilde{u}), d(f\tilde{u}, f\tilde{v}), d(g\tilde{u}, g\tilde{v})\right\}$$

$$E_s(\tilde{u}, \tilde{v}) = \max\left\{\begin{array}{c} [d(f\tilde{v}, g\tilde{v})]^2, [d(f\tilde{u}, g\tilde{v})]^2, [d(g\tilde{u}, g\tilde{v})]^2, \frac{[d(f\tilde{u}, g\tilde{u})]^2[1 + [d(g\tilde{u}, g\tilde{v})]^2]}{1 + [d(f\tilde{u}, g\tilde{v})]^2}, \\ \frac{[d(f\tilde{v}, g\tilde{v})]^2[1 + [d(g\tilde{v}, g\tilde{u})]^2]}{1 + [d(f\tilde{v}, g\tilde{u})]^2} \end{array}\right\}$$

Then $\tilde{f}$ and $\widetilde{g}$ have a coincidence point in $\widetilde{X}$. Moreover, $\tilde{f}$ and $\tilde{g}$ have common fixed point provided that $\widetilde{f}$ and $\widetilde{g}$ are soft weakly compatible.

Proof. Let $\tilde{u}_0 \in \widetilde{X}$. As $f\left(\widetilde{X}\right) \subset g\left(\widetilde{X}\right)$. Now we define the sequence $\widetilde{\{u_n\}}$ and $\widetilde{\{v_n\}}$ in $\widetilde{X}$ by $\widetilde{v_n} = f\widetilde{u_n} = g\widetilde{u_{n+1}}\ \forall n \in N$. Applying 4.1 with $\widetilde{u} = \widetilde{u_n}$ and $\widetilde{v} = \widetilde{u_{n+1}}$, then we have

$$\widetilde{\psi}(s^2[d(\widetilde{v_n}, \widetilde{v_{n+1}})^2] = \tilde{\psi}(s^2\tilde{d}(f\tilde{u_n}, f\widetilde{u_{n+1}})]^2 \leq \tilde{\psi}F_s(\tilde{u_n}, \widetilde{u_{n+1}}) - \tilde{\varphi}(E_s(\tilde{u_n}, \widetilde{u_{n+1}}))$$

where,

$$F_s(\tilde{u_n}, \widetilde{u_{n+1}}) = \max\left\{\begin{array}{c} [d(\tilde{v_n}, \widetilde{v_{n-1}})]^2, [d(\widetilde{v_{n-1}}, \tilde{v_n})]^2, [d(\widetilde{v_{n+1}}, \tilde{v_n})]^2, \\ d(\tilde{v_n}, \widetilde{v_{n-1}}), d(\tilde{v_n}, \widetilde{v_{n+1}}), [d(\tilde{v_n}, \widetilde{v_{n-1}})]^2 \end{array}\right\}$$

$$E_s(\widetilde{u_n},\widetilde{u_{n+1}}) = \max\left\{ \begin{array}{c} [d(\widetilde{v_{n+1}},\widetilde{v_n})]^2, [d(\widetilde{v_n},\widetilde{v_n})]^2, [d(\widetilde{v_{n-1}},\widetilde{v_n})]^2, \\ \frac{[d(\widetilde{v_n},\widetilde{v_{n-1}})]^2[1+[d(\widetilde{v_n},\widetilde{v_{n-1}})]^2]}{1+[d(\widetilde{v_n},\widetilde{v_n})]^2}, \frac{[d(\widetilde{v_{n+1}},\widetilde{v_n})]^2[1+[d(\widetilde{v_n},\widetilde{v_n})]^2]}{1+[d(\widetilde{v_{n+1}},\widetilde{v_{n-1}})]^2} \end{array} \right\}$$

If $\widetilde{d}(\widetilde{v_n},\widetilde{v_{n+1}}) \geq \widetilde{d}(\widetilde{v_n},\widetilde{v_{n-1}}) > 0$, for some $n \in N$, then we have

$$F_s\left(\widetilde{u_n},\widetilde{u_{n+1}}\right) = \left[1 + \left[d\left(\widetilde{v_n},\widetilde{v_{n+1}}\right)\right]^2\right]$$

$$E_s\left(\widetilde{u_n},\widetilde{u_{n+1}}\right) \geq \left[1 + \left[d\left(\widetilde{v_n},\widetilde{v_{n+1}}\right)\right]^2\right]$$

The above inequalities that

$$\widetilde{\psi}(\left[d\left(\widetilde{v_n},\widetilde{v_{n-1}}\right)\right]^2) \leq \widetilde{\psi}(s^2 d(v_n,v_{n+1})]^2$$
$$\leq \widetilde{\psi}\left(F_s\left(\widetilde{u_n},\widetilde{u_{n+1}}\right)\right) - \widetilde{\varphi}\left(E_s\left(\widetilde{u_n},\widetilde{u_{n+1}}\right)\right)$$
$$\leq \widetilde{\psi}(\left[d\left(\widetilde{v_n},\widetilde{v_{n+1}}\right)\right]^2) - \widetilde{\varphi}\left(\left[d\left(\widetilde{v_n},\widetilde{v_{n+1}}\right)\right]^2\right)$$

Which implies, $\widetilde{\varphi}\left(\left[d\left(\widetilde{v_n},\widetilde{v_{n+1}}\right)\right]^2\right) = 0$, that is $\widetilde{v_n} = \widetilde{v_{n+1}}$ a contradiction. Which is non-increasing sequence and there exists $r \geq 0$ such that $\lim_{n\to\infty} d\left(\widetilde{v_n},\widetilde{v_{n+1}}\right) = r$. We have,

$$F_s\left(\widetilde{u_n},\widetilde{u_{n+1}}\right) = \left[1 + \left[d\left(\widetilde{v_n},\widetilde{v_{n-1}}\right)\right]^2\right]$$

$$E_s\left(\widetilde{u_n},\widetilde{u_{n+1}}\right) \geq \left[1 + \left[d\left(\widetilde{v_n},\widetilde{v_{n-1}}\right)\right]^2\right]$$

It follows that,

$$\widetilde{\psi}(\left[d\left(\widetilde{v_n},\widetilde{v_{n+1}}\right)\right]^2) \leq \widetilde{\psi}\left(F_s\left(\widetilde{u_n},\widetilde{u_{n+1}}\right)\right) - \widetilde{\varphi}\left(E_s\left(\widetilde{u_n},\widetilde{u_{n+1}}\right)\right)$$
$$\leq \widetilde{\psi}(\left[d\left(\widetilde{v_n},\widetilde{v_{n-1}}\right)\right]^2) - \widetilde{\varphi}\left(\left[d\left(\widetilde{v_n},\widetilde{v_{n-1}}\right)\right]^2\right)$$

Now suppose that $r > 0$. By taking the limit as $n \to \infty$ then we have $\lim_{n\to\infty} d\left(\widetilde{v_n},\widetilde{v_{n+1}}\right) = r = 0$ there exists $\varepsilon > 0$ for which one can find soft sequences $\widetilde{v_{m_k}}$ and $\widetilde{v_{n_k}}$ of $\{\widetilde{v_n}\}$ where $\widetilde{n_k}$ is the smallest soft index for which $\widetilde{n_k} > \widetilde{m_k} > \widetilde{k}$, $\varepsilon \leq d\left(\widetilde{v_{m_k}},\widetilde{v_{n_k}}\right)$ and $d\left(\widetilde{v_{m_k}},\widetilde{v_{n_{k-1}}}\right) < \varepsilon$.

In the triangle inequality in soft metric space, we get,

$$\varepsilon^2 \leq \left[d\left(\widetilde{v_{m_k}},\widetilde{v_{n_k}}\right)\right]^2 \leq \left[sd\left(\widetilde{v_{m_k}},\widetilde{v_{n_{k-1}}}\right) + sd\left(\widetilde{v_{n_{k-1}}},\widetilde{v_{n_k}}\right)\right]^2$$
$$= s^2\left[d\left(\widetilde{v_{m_k}},\widetilde{v_{n_{k-1}}}\right)\right]^2 + s^2\left[d\left(\widetilde{v_{n_{k-1}}},\widetilde{v_{n_k}}\right)\right]^2 + 2s^2 d\left(\widetilde{v_{m_k}},\widetilde{v_{n_{k-1}}}\right) d(\widetilde{v_{n_{k-1}}},\widetilde{v_{n_k}})$$
$$\leq s^2\varepsilon^2 + s^2\left[d\left(\widetilde{v_{n_{k-1}}},\widetilde{v_{n_k}}\right)\right]^2 + 2s^2 d\left(\widetilde{v_{m_k}},\widetilde{v_{n_{k-1}}}\right) d(\widetilde{v_{n_{k-1}}},\widetilde{v_{n_k}})$$

Using above inequality and taking upper limit as $k \to +\infty$, we obtain,

$$\varepsilon^2 \leq \lim_{k\to+\infty} \left[d\left(\widetilde{v_{m_k}},\widetilde{v_{n_k}}\right)\right]^2 \leq s^2\varepsilon^2.$$

Now we deduce the equation,

$$\varepsilon^2 \leq \left[d\left(\widetilde{v_{m_k}},\widetilde{v_{n_k}}\right)\right]^2 \leq \left[sd\left(\widetilde{v_{m_k}},\widetilde{v_{n_{k-1}}}\right) + sd\left(\widetilde{v_{n_{k-1}}},\widetilde{v_{n_k}}\right)\right]^2$$
$$= s^2\left[d\left(\widetilde{v_{m_k}},\widetilde{v_{n_{k-1}}}\right)\right]^2 + s^2\left[d\left(\widetilde{v_{n_{k-1}}},\widetilde{v_{n_k}}\right)\right]^2 + 2s^2 d\left(\widetilde{v_{m_k}},\widetilde{v_{n_{k-1}}}\right) d(\widetilde{v_{n_{k-1}}},\widetilde{v_{n_k}})$$
$$\left[d\left(\widetilde{v_{m_k}},\widetilde{v_{n_k}}\right)\right]^2 \leq \left[sd\left(\widetilde{v_{m_k}},\widetilde{v_{m_{k-1}}}\right) + sd\left(\widetilde{v_{m_{k-1}}},\widetilde{v_{n_k}}\right)\right]^2$$
$$= s^2\left[d\left(\widetilde{v_{m_k}},\widetilde{v_{m_{k-1}}}\right)\right]^2 + s^2\left[d\left(\widetilde{v_{m_{k-1}}},\widetilde{v_{n_k}}\right)\right]^2 + 2s^2 d\left(\widetilde{v_{m_k}},\widetilde{v_{m_{k-1}}}\right) d(\widetilde{v_{m_{k-1}}},\widetilde{v_{n_k}})$$
$$\left[d\left(\widetilde{v_{m_{k-1}}},\widetilde{v_{n_k}}\right)\right]^2 \leq \left[sd\left(\widetilde{v_{m_{k-1}}},\widetilde{v_{m_k}}\right) + sd\left(\widetilde{v_{m_k}},\widetilde{v_{n_k}}\right)\right]^2$$
$$= s^2\left[d\left(\widetilde{v_{m_{k-1}}},\widetilde{v_{m_k}}\right)\right]^2 + s^2\left[d\left(\widetilde{v_{m_k}},\widetilde{v_{n_k}}\right)\right]^2 + 2s^2 d\left(\widetilde{v_{m_{k-1}}},\widetilde{v_{m_k}}\right) d(\widetilde{v_{m_k}},\widetilde{v_{n_k}})$$

Then we have,

$$\frac{\varepsilon^2}{s^2} \leq \lim_{k\to+\infty} \left[d\left(\widetilde{v_{m_k}},\widetilde{v_{n_{k-1}}}\right)\right]^2 \leq \varepsilon^2$$

And

$$\frac{\varepsilon^2}{s^2} \leq \lim_{k\to+\infty} \left[d\left(\widetilde{v_{m_{k-1}}}, \widetilde{v_{n_k}}\right)\right]^2 \leq s^4\varepsilon^2$$

Similarly we deduce that,

$$\left[d\left(\widetilde{v_{m_{k-1}}}, \widetilde{v_{n_{k-1}}}\right)\right]^2 \leq \left[sd\left(\widetilde{v_{m_{k-1}}}, \widetilde{v_{m_k}}\right) + sd\left(\widetilde{v_{m_k}}, \widetilde{v_{n_{k-1}}}\right)\right]^2$$

$$= s^2\left[d\left(\widetilde{v_{m_{k-1}}}, \widetilde{v_{m_k}}\right)\right]^2 + s^2\left[d\left(\widetilde{v_{m_k}}, \widetilde{v_{n_{k-1}}}\right)\right]^2 + 2s^2 d\left(\widetilde{v_{m_{k-1}}}, \widetilde{v_{m_k}}\right) d(\widetilde{v_{m_k}}, \widetilde{v_{n_{k-1}}})$$

$$\left[d\left(\widetilde{v_{m_k}}, \widetilde{v_{n_k}}\right)\right]^2 \leq \left[sd\left(\widetilde{v_{m_k}}, \widetilde{v_{m_{k-1}}}\right) + sd\left(\widetilde{v_{m_{k-1}}}, \widetilde{v_{n_k}}\right)\right]^2$$

$$= s^2\left[d\left(\widetilde{v_{m_k}}, \widetilde{v_{m_{k-1}}}\right)\right]^2 + s^2\left[d\left(\widetilde{v_{m_{k-1}}}, \widetilde{v_{n_k}}\right)\right]^2 + 2s^2 d\left(\widetilde{v_{m_k}}, \widetilde{v_{m_{k-1}}}\right) d\left(\widetilde{v_{m_{k-1}}}, \widetilde{v_{n_k}}\right)$$

$$\leq s^2\left[d\left(\widetilde{v_{m_k}}, \widetilde{v_{m_{k-1}}}\right)\right]^2 + s^2\left[sd\left(\widetilde{v_{m_{k-1}}}, \widetilde{v_{n_{k-1}}}\right) + sd\left(\widetilde{v_{n_{k-1}}}, \widetilde{v_{n_k}}\right)\right]^2$$

$$+2s^2 d\left(\widetilde{v_{m_k}}, \widetilde{v_{m_{k-1}}}\right)\left[sd\left(\widetilde{v_{m_{k-1}}}, \widetilde{v_{n_{k-1}}}\right) + sd\left(\widetilde{v_{n_{k-1}}}, \widetilde{v_{n_k}}\right)\right]$$

It follows that,

$$\frac{\varepsilon^2}{s^4} \leq \lim_{k\to+\infty} \left[d\left(\widetilde{v_{m_{k-1}}}, \widetilde{v_{n_{k-1}}}\right)\right]^2 \leq s^2\varepsilon^2$$

Through the definition of $F_s\left(\widetilde{u},\widetilde{v}\right)$, we have,

$$F_s\left(\widetilde{u_{m_k}}, \widetilde{u_{n_k}}\right) = max\left\{\begin{array}{c}\left[d\left(\widetilde{v_{m_k}}, \widetilde{v_{m_{k-1}}}\right)\right]^2, \left[d\left(\widetilde{v_{m_{k-1}}}, \widetilde{v_{n_{k-1}}}\right)\right]^2, \left[d\left(\widetilde{v_{n_k}}, \widetilde{v_{n_{k-1}}}\right)\right]^2, \\ d\left(\widetilde{v_{m_{k-1}}}, \widetilde{v_{m_k}}\right) d\left(\widetilde{v_{m_k}}, \widetilde{v_{n_{k-1}}}\right) d\left(\widetilde{v_{m_k}}, \widetilde{v_{m_{k-1}}}\right) d\left(\widetilde{v_{m_{k-1}}}, \widetilde{v_{n_{k-1}}}\right)\end{array}\right\}$$

This yields that, $\lim_{k\to+\infty} \sup F_s\left(\widetilde{u_{m_k}}, \widetilde{u_{n_k}}\right) \leq \max\left\{0, s^2, \varepsilon^2, 0, 0, 0\right\} = \varepsilon^2 s^2$.

Also,

$$E_s(\widetilde{u_{m_k}}, \widetilde{u_{n_k}}) = \max\left\{\begin{array}{c}[d(\widetilde{v_{m_k}},\widetilde{v_{n_{k-1}}})]^2,[d(\widetilde{v_{m_k}},\widetilde{v_{n_{k-1}}})]^2,[d(\widetilde{v_{m_{k-1}}},\widetilde{v_{n_{k-1}}})]^2, \\ \frac{[d(\widetilde{v_{m_k}},\widetilde{v_{m_{k-1}}})]^2[1+[d(\widetilde{v_{m_{k-1}}},\widetilde{v_{n_{k-1}}})]^2]}{1+[d(\widetilde{v_{m_k}},\widetilde{v_{n_{k-1}}})]^2}, \frac{[d(\widetilde{v_{m_{k-1}}},\widetilde{v_{n_{k-1}}})]^2[1+[d(\widetilde{v_{n_{k-1}}},\widetilde{v_{n_k}})]^2]}{1+[d(\widetilde{v_{m_{k-1}}},\widetilde{v_{n_{k-1}}})]^2}\end{array}\right\}$$

Then we show that,

$$\lim_{k\to+\infty} \inf E_s\left(\widetilde{u_{m_k}}, \widetilde{u_{n_k}}\right) \geq \max\left\{0, \frac{\varepsilon^2}{s^2}, \frac{\varepsilon^2}{s^4}, 0\right\} \geq \frac{\varepsilon^2}{s^4},$$

Then we have,

$$\widetilde{\psi}\left(\left[d\left(\widetilde{v_{m_k}}, \widetilde{v_{n_k}}\right)\right]^2\right) \leq \widetilde{\psi}\left(s^2\left(\left[d\left(\widetilde{v_{m_k}}, \widetilde{v_{n_k}}\right)\right]^2\right)\right) \leq \widetilde{\psi}\left(F_s\left(\widetilde{u_{m_k}}, \widetilde{u_{n_k}}\right)\right) - \widetilde{\varphi}\left(E_s\left(\widetilde{u_{m_k}}, \widetilde{u_{n_k}}\right)\right)$$

In the above result it can be obtained that,

$$\widetilde{\psi}\left(s^2\varepsilon^2\right) \leq \widetilde{\psi}\left(s^2 \lim_{k\to+\infty} sup\left(\left[d\left(\widetilde{fu_{m_k}}, \widetilde{fu_{n_k}}\right)\right]^2\right)\right)$$

$$\leq \widetilde{\psi}\left(\lim_{k\to+\infty} \sup F_s\left(\widetilde{u_{m_k}}, \widetilde{u_{n_k}}\right)\right) - \widetilde{\varphi}\left(\lim_{k\to+\infty} \inf E_s\left(\widetilde{u_{m_k}}, \widetilde{u_{n_k}}\right)\right)$$

$$\leq \widetilde{\psi}\left(s^2\varepsilon^2\right) - \widetilde{\varphi}\left(\lim_{k\to+\infty} \inf E_s\left(\widetilde{u_{m_k}}, \widetilde{u_{n_k}}\right)\right)$$

Which implies that,

$$\left(\lim_{k\to+\infty} \inf E_s\left(\widetilde{u_{m_k}}, \widetilde{u_{n_k}}\right)\right) = 0$$

This is Contradiction to above result. It follows that $\widetilde{\{v_n\}}$ is a Cauchy sequence in $\widetilde{X}$ and $\lim_{n,m\to\infty} d\left(\widetilde{v_n}, \widetilde{v_m}\right) = 0$. since $\widetilde{X}$ is a complete soft metric space, there exists $\widetilde{w} \in \widetilde{X}$ such that,

$$\lim_{n,\to\infty} d\left(\widetilde{v_n}, \widetilde{w}\right) = \lim_{n\to+\infty} d\left(\widetilde{fu_n}, \widetilde{w}\right) = \lim_{n\to+\infty} d\left(\widetilde{gu_{n+1}}, \widetilde{w}\right) = \lim_{n,m\to+\infty} d\left(\widetilde{v_n}, \widetilde{v_m}\right) = d\left(\widetilde{w}, \widetilde{w}\right) = 0.$$

Furthermore, we have $\widetilde{w} \in \widetilde{g}(\widetilde{X})$ since $\widetilde{g}(\widetilde{X})$ is closed soft set. It follows that one can choose a $\widetilde{z} \in \widetilde{X}$ such that $\widetilde{w} = g\widetilde{z}$, and we can write above equation as,

$$\lim_{n\to+\infty} d\left(\widetilde{u_n}, g\widetilde{z}\right) = \lim_{n\to+\infty} d\left(\widetilde{fu_n}, \widetilde{gz}\right) = \lim_{n\to+\infty} d\left(\widetilde{gu_{n+1}}, g\widetilde{z}\right) = 0$$

If $\widetilde{fz} \neq \widetilde{gz}$, taking $\widetilde{u} = \widetilde{u}_{n_k}$ and $\widetilde{v} = \widetilde{z}$ in soft contractive condition in given equation, we get,

$$\widetilde{\psi}\left(s^2\left(\left[d\left(\widetilde{v_{n_k}}, \widetilde{fz}\right)\right]^2\right)\right) = \widetilde{\psi}\left(s^2\left(\left[d\left(\widetilde{fu_{n_k}}, \widetilde{fz}\right)\right]^2\right)\right)$$
$$\leq \widetilde{\psi}\left(F_s\left(\widetilde{u_{n_k}}, \widetilde{z}\right)\right) - \widetilde{\varphi}\left(\lim_{k\to+\infty} \inf E_s\left(\widetilde{u_{n_k}}, \widetilde{z}\right)\right)$$

Where,

$$F_s\left(\widetilde{u_{n_k}}, \widetilde{z}\right) = \max\left\{\begin{array}{c} [d(\widetilde{v_{n_k}}, \widetilde{v_{n_{k-1}}})]^2, [d(\widetilde{v_{n_{k-1}}}, \widetilde{gz})]^2, [d(\widetilde{fz}, \widetilde{gz})]^2, \\ d(\widetilde{v_{n_k}}, \widetilde{v_{n_{k-1}}})d(\widetilde{v_{n_k}}, \widetilde{fz}), d(\widetilde{v_{n_k}}, \widetilde{v_{n_{k-1}}}), d(\widetilde{v_{n_{k-1}}}, \widetilde{gz}) \end{array}\right\}$$

$$\lim_{k\to+\infty} \inf E_s\left(\widetilde{u_{n_k}}, \widetilde{z}\right) =$$

$$\max\left\{\begin{array}{c} [d(\widetilde{fz}, \widetilde{gz})]^2, [d(\widetilde{v_{n_k}}, \widetilde{gz})]^2, [d(\widetilde{v_{n_{k-1}}}, \widetilde{gz})]^2, \\ \dfrac{[d(\widetilde{v_{n_k}}, \widetilde{v_{n_{k-1}}})]^2[1 + [d(\widetilde{v_{n_{k-1}}}, \widetilde{gz})]^2]}{1 + [d(\widetilde{v_{n_k}}, \widetilde{gz})]^2}, \dfrac{[d(\widetilde{v_{n_{k-1}}}, \widetilde{n_k})]^2[1 + [d(\widetilde{v_{n_{k-1}}}, \widetilde{gz})]^2]}{1 + [d(\widetilde{fz}, \widetilde{gz})]^2} \end{array}\right\}$$

And then we obtain,

$$\lim_{k\to+\infty} \sup F_s\left(\widetilde{u_{n_k}}, \widetilde{z}\right) = max\left\{0, 0, \left[d\left(\widetilde{gz}, \widetilde{fz}\right)\right]^2, 0, 0\right\} = \left[d\left(\widetilde{gz}, \widetilde{fz}\right)\right]^2$$

$$\lim_{k\to+\infty} \inf \lim_{k\to+\infty} \inf E_s\left(\widetilde{u_{n_k}}, \widetilde{z}\right) = \max\left\{\left[d\left(\widetilde{gz}, \widetilde{fz}\right)\right]^2, 0, 0, 0, 0\right\} = \left[d\left(\widetilde{gz}, \widetilde{fz}\right)\right]^2$$

Now taking the upper limit as $k \to +\infty$, we have,

$$\left(\widetilde{\psi}\left[d\left(\widetilde{gz}, \widetilde{fz}\right)\right]^2\right) = \widetilde{\psi}\left(s^2.\frac{1}{s^2}\left[d\left(\widetilde{gz}, \widetilde{fz}\right)\right]^2\right) \leq \widetilde{\psi}\left(s^2\left[d\left(f\widetilde{u_{n_k}}, \widetilde{fz}\right)\right]^2\right)$$
$$\leq \widetilde{\psi}\left(\lim_{k\to+\infty} \sup F_s\left(\widetilde{u_{n_k}}, \widetilde{z}\right)\right) - \widetilde{\varphi}\left(\lim_{k\to+\infty} inf \lim_{k\to+\infty} \inf E_s\left(\widetilde{u_{n_k}}, \widetilde{z}\right)\right)$$
$$= \widetilde{\psi}\left(\left[d\left(\widetilde{gz}, \widetilde{fz}\right)\right]^2\right) - \widetilde{\varphi}\left(\left[d\left(\widetilde{gz}, \widetilde{fz}\right)\right]^2\right),$$

This implies that,

$$\phi(\left[d\left(\widetilde{fz}, \widetilde{gz}\right)\right]^2) = 0.$$

It follows that $d\left(\widetilde{fz}, \widetilde{gz}\right) = 0$ that is, $\widetilde{fz} = \widetilde{gz}$. therefore $\widetilde{w} = \widetilde{fz} = \widetilde{gz}$ is a point of coincidence for $\widetilde{f}$ and $\widetilde{g}$ we obtain that,

$$\widetilde{\psi}\left(\left[d\left(\widetilde{fz}, \widetilde{fz'}\right)\right]^2\right) \leq \widetilde{\psi}\left(s^2\left[d\left(\widetilde{fz}, \widetilde{fz'}\right)\right]^2\right)$$

$$\leq \widetilde{\psi}\left(F_s(\widetilde{z}, \widetilde{z'})\right) - \widetilde{\varphi}\left(\lim_{k\to+\infty} \inf E_s(\widetilde{z}, \widetilde{z'})\right)$$
$$\leq \widetilde{\psi}\left(\left[d\left(\widetilde{fz}, \widetilde{fz'}\right)\right]^2\right) - \widetilde{\varphi}\left(\left[d\left(\widetilde{fz}, \widetilde{fz'}\right)\right]^2\right)$$

Hence $\widetilde{fz} = \widetilde{fz'}$. that is, the point of coincidence is unique. Considering the soft weak of $\widetilde{f}$ and $\widetilde{g}$, it can be shown that $\widetilde{z}$ is a soft unique fixed point. This completes the proof. ∎

Corollary 4.2. *Let $(\widetilde{X}, \widetilde{d})$ be a complete soft metric space with parameter $s \geq 1$ and let $f, g : \widetilde{X} \to \widetilde{X}$ be given self- soft mappings satisfying $\widetilde{f}(\widetilde{X}) \subset \widetilde{g}(\widetilde{X})$ where $\widetilde{g}(\widetilde{X})$ a closed soft subset of $\widetilde{X}$. if the following condition is satisfied:*

$$s^2[\tilde{d}(f\tilde{u}, f\tilde{v}))]^2 \leq F_s(\widetilde{u}, \widetilde{v}) - L[\tilde{d}(f\tilde{u}, f\tilde{v})]^2$$

Where $L \in (0, 1)$ is a constant and then $\widetilde{f}$ and $\widetilde{g}$ have a unique coincidence point in $\widetilde{X}$. Moreover, $\widetilde{f}$ and $\widetilde{g}$ have a unique fixed point provided that $\widetilde{f}$ and $\widetilde{g}$ are soft weakly compatible.

Corollary 4.3. *Let $(\widetilde{X}, \widetilde{d})$ be a complete soft metric space with parameter $s \geq 1$ and let $f, g : \widetilde{X} \to \widetilde{X}$ be given self- soft mappings satisfying $\widetilde{f}(\widetilde{X}) \subset \widetilde{g}(\widetilde{X})$ where $\widetilde{g}(\widetilde{X})$ a closed soft subset of $\widetilde{X}$. If the following condition is satisfied:*

$$s^2[\tilde{d}(f\tilde{u}, f\tilde{v})]^2 \leq F_s(\widetilde{u}, \widetilde{v}) - (E_s(f\tilde{u}, f\tilde{v}))$$

Then $\widetilde{f}$ and $\widetilde{g}$ have a soft unique coincidence point in $\widetilde{X}$. Moreover, if $\widetilde{f}$ and $\widetilde{g}$ are soft weakly compatible, then $\widetilde{f}$ and $\widetilde{g}$ have a soft unique fixed point.

5. CONCLUSION

In this article, we have inserted new conceptions in a soft metric space. We have discussed a fixed point under generalized soft $(\tilde{\psi}, \tilde{\phi})$ -weakly contractive mappings in soft metric space without continuity of mappings. In future established results can be generalized for rough metric spaces soft fuzzy metric spaces also. In future the established result will be useful for application in game theory with the concept of J. Nash [13] and the concept of operation research

Conflicts of interest

The authors declare that they have no conflicts of interest regarding the publication of this paper.

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