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USING DIRECT AND FIXED POINT TECHNIQUE OF CUBIC FUNCTIONAL EQUATION AND ITS HYERS-ULAM STABILITY

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ABSTRACT. In this present work, we introduce a new type of finite dimensional cubic functional equation of the form

$$\Lambda\left(\sum_{\mathfrak{e}=1}^{\phi}\mathfrak{e}f_{\mathfrak{e}}\right)=\sum_{1\leq\mathfrak{e}<\mathfrak{f}<\mathfrak{g}\leq\phi}\Lambda\left(\mathfrak{e}f_{\mathfrak{e}}+\mathfrak{f}f_{\mathfrak{f}}+\mathfrak{g}f_{\mathfrak{g}}\right)+(3-\phi)\sum_{1\leq\mathfrak{e}<\mathfrak{f}\leq\phi}\Lambda\left(\mathfrak{e}f_{\mathfrak{e}}+\mathfrak{f}f_{\mathfrak{f}}\right)$$
$$+\left(\frac{\phi^2-5\phi+6}{2}\right)\sum_{\mathfrak{e}=0}^{\phi-1}(\mathfrak{e}+1)^3\left(\frac{\Lambda(f_{\mathfrak{e}+1})-\Lambda(-f_{\mathfrak{e}+1})}{2}\right),$$

where $\phi\geq 4$ is an integer, and derive its general solution. The main purpose of this work is to investigate the Hyers-Ulam stability results for the above mentioned functional equation in Fuzzy Banach spaces by means of direct and fixed point methods.

Key words and phrases: Cubic-functional equation ($\mathcal{FE}$); Hyers-Ulam stability ($\mathcal{HUS}$); Fuzzy normed space ($\mathcal{FNS}$).

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1. INTRODUCTION

A function $\mathcal{F}:\mathcal{X}\to\mathcal{Y}$ between real vector spaces is called a cubic function if

$$\mathcal{F}(2\mathfrak{p}+\mathfrak{q})+\mathcal{F}(2\mathfrak{p}-\mathfrak{q})=2\mathcal{F}(\mathfrak{p}+\mathfrak{q})+2\mathcal{F}(\mathfrak{p}-\mathfrak{q})+12\mathcal{F}(\mathfrak{p}),\ \mathfrak{p},\mathfrak{q}\in X. \tag{1.1}$$

The $\mathcal{FE}$ (1.1) is known as a cubic $\mathcal{FE}$. As of late, impressive consideration has been expanding to the issue of fuzzy stability of $\mathcal{FE}$s. A few different fuzzy stability results concerning Cauchy, basic quadratic, and cubic $\mathcal{FE}$s have been examined [4, 6, 9], [12]-[15].

In demonstrating applied issues just halfway data might be known (or) there might be a level of vulnerability in the boundaries utilized in the model or a few estimations might be uncertain. Because of such highlights, we are enticed to think about the investigation of $\mathcal{FE}$s in the fuzzy setting. Throughout the previous 40 years, the fuzzy hypothesis has become a particularly unique space of assessment and a lot of progress has been made in the theory of fuzzy sets. This branch finds a wide extent of employments in the field of science and planning.

A. K. Katsaras [10] introduced a thought about the $\mathcal{FN}$ on a linear space, around a similar time Cpmgxin Wu [21] introduced a thought of $\mathcal{FNS}$ to give a theory of the Kolmogoroff normalized speculation for fuzzy topological linear spaces. In 1991, R. Biswas [3] portrayed and considered fuzzy inner product spaces in linear space. In 1992, C. Felbin [7] introduced an elective importance of a $\mathcal{FN}$ on linear topological plans of a $\mathcal{FNS}$. In 1994, S. C. Cheng [5] introduced an importance of the $\mathcal{FNS}$ so that the relating started fuzzy metric is of I. Kramosil [11]. In 2003, T. Pack [1, 2] changed the importance of S. C. Cheng [5] by dispensing with a normal condition. Actually various results have been analyzed by different makers one can insinuate [16]-[20].

In this present work, we introduce a new type of finite dimensional cubic $\mathcal{FE}$ of the form

$$\begin{aligned}\Lambda\left(\sum_{\mathfrak{e}=1}^{\phi}\mathfrak{e}f_{\mathfrak{e}}\right)&=\sum_{1\le\mathfrak{e}<\mathfrak{f}<\mathfrak{g}\le\phi}\Lambda\left(\mathfrak{e}f_{\mathfrak{e}}+\mathfrak{f}f_{\mathfrak{f}}+\mathfrak{g}f_{\mathfrak{g}}\right)+(3-\phi)\sum_{1\le\mathfrak{e}<\mathfrak{f}\le\phi}\Lambda\left(\mathfrak{e}f_{\mathfrak{e}}+\mathfrak{f}f_{\mathfrak{f}}\right)\\&\quad+\left(\frac{\phi^2-5\phi+6}{2}\right)\sum_{\mathfrak{e}=0}^{\phi-1}(\mathfrak{e}+1)^3\left(\frac{\Lambda(f_{\mathfrak{e}+1})-\Lambda(-f_{\mathfrak{e}+1})}{2}\right),\end{aligned}\tag{1.2}$$

where $\phi\ge 4$ is an integer, and derive its general solution. The main purpose of this work is to investigate the $\mathcal{HUS}$ results for the above mentioned $\mathcal{FE}$ in Fuzzy Banach spaces by means of direct and fixed point approaches.

2. PRELIMINARIES

We recall some basic facts concerning $\mathcal{FNS}$s and some preliminary results.

Definition 2.1. A function $\mathcal{N}:\mathcal{X}\times\mathcal{R}\to[0,1]$ is called a fuzzy norm on $\mathcal{X}$ if

(FNS01) $\mathcal{N}(\mathfrak{p},t)=0$ for $t\le 0$.
(FNS02) $x=0\Leftrightarrow\mathcal{N}(\mathfrak{p},t)=1,\ \forall\, t>0$.
(FNS03) $\mathcal{N}(c\mathfrak{p},t)=\mathcal{N}\left(\mathfrak{p},\frac{t}{|\mathfrak{g}|}\right)$.
(FNS04) $\mathcal{N}(\mathfrak{q}+\mathfrak{p},s+t)\ge\min\{\mathcal{N}(\mathfrak{q},s),\mathcal{N}(\mathfrak{p},t)\}$.
(FNS05) $\lim_{t\to\inf}\mathcal{N}(\mathfrak{p},t)=0$ and $\mathcal{N}(\mathfrak{p},.)$ is non-decreasing on $\mathcal{R}$.
(FNS06) For $x\ne 0,\mathcal{N}(\mathfrak{p},.)$ is continuous on $\mathcal{R},\ \forall\,\mathfrak{p},\mathfrak{q}\in\mathcal{X},\ s,t\in\mathcal{R}$.

The pair $(\mathcal{X},\mathcal{N})$ is called a $\mathcal{FNS}$ or fuzzy normed linear space.

Example 2.1. *Let* $(\mathfrak{p}, \|.\|)$ *be a normed linear space and* $\alpha, \beta > 0$. *Define* $\mathcal{N} : \mathcal{X} \times \mathcal{R} \to [0,1]$ *by*

$$N(\mathfrak{p}, t) = \begin{cases} \frac{\alpha t}{\alpha t + \beta \|x\|}, & \textit{if } t > 0, \mathfrak{p} \in \mathcal{X} \\ 0, & \textit{if } t \leq 0, \mathfrak{p} \in \mathcal{X}. \end{cases}$$

It is easy to check that $\mathcal{N}$ *is fuzzy norm on* $\mathcal{X}$.

Theorem 2.1. *If* $\Lambda : \mathcal{X} \to \mathcal{X}$ *is strictly contractive and* $(\mathfrak{p}, d)$ *be a complete metric space. Suppose that* $d(\Lambda^{k+1}a, \Lambda^{ka}) < \infty,\ k \geq 0$, *then*

(1) $\left\{\Lambda^f \mathfrak{e}\right\}_{n=1}^{\infty}$ *converges to a fixed point* $b \in \mathcal{X}$ *of* Λ.
(2) b *is the unique fixed point of* Λ *in the set* $Y = \{\mathfrak{q} \in \mathcal{X} | d(\Lambda^k \mathfrak{e}, \mathfrak{q}) < \infty\}$.
(3) $d(y, b) \leq \frac{1}{1-\phi} d(\mathfrak{q}, \Lambda \mathfrak{q}),\ \forall\ \mathfrak{q} \in \mathcal{Y}$.

3. GENERAL SOLUTION FOR THE EQUATION (1.2)

In this part, we achieve the general solution of the cubic $\mathcal{FE}$ (1.2).

Theorem 3.1. *If* $\Lambda : \mathcal{P} \to \mathcal{Q}$ *satisfies the* $\mathcal{FE}$ (1.2), *then the function* $\Lambda : \mathcal{P} \to \mathcal{Q}$ *satisfies the* $\mathcal{FE}$ (1.1).

Proof. Assume that $\Lambda : \mathcal{P} \to \mathcal{Q}$ satisfies the $\mathcal{FE}$ (1.2), for all $f_1, f_2, f_3, ..., f_\phi \in \mathcal{X}$. Substituting $(f_1, f_2, f_3, ..., f_\phi)$ by $(0, 0, 0, ..., 0)$ in (1.2), we receive

$$\begin{aligned}
\Lambda(0) &= \left(\frac{\phi^3 - 3\phi^2 + 2\phi}{6}\right)\Lambda(0) + (3-\phi)\left(\frac{\phi^2 - \phi}{2}\right)\Lambda(0) \\
&\quad + \left(\frac{\phi^2 - 5\phi + 6}{2}\right)\left(\frac{\phi^4 - 2\phi^3 + \phi^2 + 1036\phi - 1680}{4}\right)\Lambda(0) \\
&= \left(\frac{4\phi^3 - 12\phi^2 + 8\phi}{24}\right)\Lambda(0) + \left(\frac{36\phi^2 - 36\phi - 12\phi^3 + 12\phi^2}{24}\right)\Lambda(0) \\
&\quad + 3\left(\frac{\phi^2 - 5\phi + 6}{6}\right)\left(\frac{\phi^4 - 2\phi^3 + \phi^2 + 1036\phi - 1680}{4}\right)\Lambda(0) \\
\Lambda(0) &= \left(\frac{\phi^6 - 2\phi^5 + 51\phi^4 + 3064\phi^3 - 10446\phi^2 + 43848\phi - 10108}{24}\right)\Lambda(0) \\
0 &= \left(\frac{\phi^6 - 2\phi^5 + 51\phi^4 + 3064\phi^3 - 10446\phi^2 + 43848\phi - 10108}{24}\right)\Lambda(0) \\
\Lambda(0) &= 0.
\end{aligned}$$

Replacing $(f_1, f_2, f_3, ..., f_\phi)$ by $(f, 0, 0, ..., 0)$ in (1.2), we get

$$\begin{aligned}
\Lambda(f) &= \left(\frac{\phi^2-3\phi+2}{2}\right)\Lambda(f) + (3-\phi)(\phi-1)\Lambda(f) + \left(\frac{\phi^2-5\phi+6}{4}\right) \\
&\quad [\Lambda(f) - \Lambda(-f)] \\
&= \left(\frac{-\phi^2+5\phi-4}{2}\right)\Lambda(f) + \left(\frac{\phi^2-5\phi+6}{4}\right)\Lambda(f) - \left(\frac{\phi^2-5\phi+6}{4}\right)\Lambda(-f) \\
&= \left(\frac{-2\phi^2+10\phi-8+\phi^2-5\phi+6}{4}\right)\Lambda(f) - \left(\frac{\phi^2-5\phi+6}{4}\right)\Lambda(-f) \\
&= \left(\frac{-\phi^2+5\phi-2}{4}\right)\Lambda(f) - \left(\frac{\phi^2-5\phi+6}{4}\right)\Lambda(-f) \\
&= -\left(\frac{\phi^2-5\phi+6}{4}\right)\Lambda(f) - \left(\frac{\phi^2-5\phi+6}{4}\right)\Lambda(-f) \\
\Lambda(-f) &= -\Lambda(f), \ \forall\, f \in \mathcal{P}.
\end{aligned}$$

Hence Λ is odd function. Again replacing $(f_1, f_2, f_3, ..., f_\phi)$ by $(0, f, 0, ..., 0)$ in (1.2), we have

$$\begin{aligned}
\Lambda(2f) &= \left(\frac{\phi^2-3l+2}{2}\right)\Lambda(2f) + (3-\phi)(\phi-1)\Lambda(2f) \\
&\quad + 8\left(\frac{\phi^2-5\phi+6}{2}\right)\Lambda(f) \\
&= \left(\frac{-\phi^2+5\phi-4}{2}\right)\Lambda(2f) + 8\left(\frac{\phi^2-5\phi+6}{2}\right)\Lambda(f) \\
0 &= \left(\frac{-\phi^2+5\phi-6}{2}\right)\Lambda(2f) + 8\left(\frac{\phi^2-5\phi+6}{2}\right)\Lambda(f) \\
0 &= -\left(\frac{\phi^2-5\phi+6}{2}\right)\Lambda(2f) + 8\left(\frac{\phi^2-5\phi+6}{2}\right)\Lambda(f) \\
\left(\frac{\phi^2-5\phi+6}{2}\right)\Lambda(2f) &= 8\left(\frac{\phi^2-5\phi+6}{2}\right)\Lambda(f)
\end{aligned}$$

$$\Lambda(2f) = 8\Lambda(f), \ \forall\, f \in \mathcal{P}. \tag{3.1}$$

Now, letting f by $2f$ in (3.1), we get

$$\Lambda(2^2 f) = 2^3\Lambda(2f) = 2^6\Lambda(f), \ \forall\, f \in \mathcal{P}. \tag{3.2}$$

Now, letting f by $2f$ in (3.2), we get

$$\Lambda(2^3 f) = 2^6\Lambda(2f) = 2^9\Lambda(f), \ \forall\, f \in \mathcal{P}. \tag{3.3}$$

In general,

$$\Lambda(2^{\mathfrak{e}} f) = 2^{\mathfrak{e}}\Lambda(f), \ \forall\, f \in \mathcal{P}. \tag{3.4}$$

Setting $(f_1, f_2, f_3, ..., f_\phi)$ by $\left(\mathfrak{p}, \frac{-\mathfrak{p}}{2}, \frac{\mathfrak{p}}{3}, \frac{y}{4}, 0, ..., 0\right)$ in (1.2), and using (3.1), we receive

$$\begin{aligned}
\Lambda(\mathfrak{p}+\mathfrak{q}) &= \Lambda(2\mathfrak{p}+\mathfrak{q}) + (2\phi-8)\Lambda(\mathfrak{p}+\mathfrak{q}) - (\phi-4)\Lambda(\mathfrak{p}-\mathfrak{q}) + (\phi-4)\Lambda(2\mathfrak{p}) \\
&\quad + \left(\frac{\phi^2-9l+22}{2}\right)\Lambda(\mathfrak{p}) + \left(\frac{\phi^2-9l+24}{2}\right)\Lambda(\mathfrak{q}) + 2(3-\phi)\Lambda(\mathfrak{p}+\mathfrak{q}) \\
&\quad - (3-\phi)\Lambda(\mathfrak{p}-\mathfrak{q}) + (3-\phi)\Lambda(2\mathfrak{p}) + \left(\frac{\phi^2-5\phi+6}{2}\right)\Lambda(\mathfrak{p}) \\
&\quad + \left(\frac{\phi^2-5\phi+6}{2}\right)\Lambda(\mathfrak{q}) \\
&= \Lambda(2\mathfrak{p}+\mathfrak{q}) + (2\phi-8+6-2\phi)\Lambda(\mathfrak{p}+\mathfrak{q}) - (\phi-4+3-\phi)\Lambda(\mathfrak{p}-\mathfrak{q}) \\
&\quad + (\phi-4+3-\phi)\Lambda(2\mathfrak{p}) \\
&\quad + \left(\frac{\phi^2-9\phi+22+6\phi-24-2\phi^2+8\phi+\phi^2-5\phi+6}{2}\right)\Lambda(\mathfrak{p}) \\
&\quad + \left(\frac{\phi^2-9\phi+22+6\phi-24-2\phi^2+8\phi+\phi^2-5\phi+6}{2}\right)\Lambda(\mathfrak{q}) \\
&= \Lambda(2\mathfrak{p}+\mathfrak{q}) - 2\Lambda(\mathfrak{p}+\mathfrak{q}) + \Lambda(\mathfrak{p}-\mathfrak{q}) - \Lambda(2\mathfrak{p}) + 2\Lambda(\mathfrak{p}) + 3\Lambda(\mathfrak{q})
\end{aligned}$$

$$3\Lambda(\mathfrak{p}+\mathfrak{q}) = \Lambda(2\mathfrak{p}+\mathfrak{q}) + \Lambda(\mathfrak{p}-\mathfrak{q}) - 6\Lambda(\mathfrak{p}) + 3\Lambda(\mathfrak{q}),\ \forall\ \mathfrak{p},\mathfrak{q} \in \mathcal{X}. \tag{3.5}$$

Replacing $\mathfrak{q}$ by $-\mathfrak{q}$ in (3.5), we obtain

$$3\Lambda(\mathfrak{p}-\mathfrak{q}) = \Lambda(2\mathfrak{p}-\mathfrak{q}) + \Lambda(\mathfrak{p}+\mathfrak{q}) - 6\Lambda(\mathfrak{p}) + 3\Lambda(-\mathfrak{q})$$

$$3\Lambda(\mathfrak{p}-\mathfrak{q}) = \Lambda(2\mathfrak{p}-\mathfrak{q}) + \Lambda(\mathfrak{p}+\mathfrak{q}) - 6\Lambda(\mathfrak{p}) - 3\Lambda(\mathfrak{q}),\ \forall\ \mathfrak{p},\mathfrak{q} \in \mathcal{X}. \tag{3.6}$$

Adding (3.5) and (3.6), We obtain our result (1.1). ∎

4. STABILITY OF THE $\mathcal{FE}$ (1.2): DIRECT METHOD

In the rest of this section, we take $A, (B, P)$ and (Z, Q) are linear space, fuzzy Banach space and $\mathcal{FNS}$, respectively. For notational convenience, we use $\Lambda : \mathcal{P} \to \mathcal{Q}$ by

$$\begin{aligned}
D\Lambda(f_1, f_2, f_3, ..., f_\phi) = \Lambda\left(\sum_{\mathfrak{e}=1}^{\phi} \mathfrak{e} f_\mathfrak{e}\right) - \sum_{1\le \mathfrak{e}<\mathfrak{f}<c\le\phi} \Lambda(\mathfrak{e} f_\mathfrak{e} + \mathfrak{f} f_\mathfrak{f} + \mathfrak{g} f_\mathfrak{g}) - (3-\phi) \\
\sum_{1\le\mathfrak{e}<\mathfrak{f}\le\phi} \Lambda(\mathfrak{e} f_\mathfrak{e} + \mathfrak{f} f_\mathfrak{f}) \\
- \left(\frac{\phi^2-5\phi+6}{2}\right)\sum_{\mathfrak{e}=0}^{\phi-1}(\mathfrak{e}+1)^3\left(\frac{\Lambda(f_{\mathfrak{e}+1}) - \Lambda(-f_{\mathfrak{e}+1})}{2}\right),
\end{aligned}$$

for every $f_1, f_2, f_3, ..., f_\phi \in \mathcal{P}$. In this section, we examine a fuzzy version of the $\mathcal{HUS}$ for the $\mathcal{FE}$ (1.2) in $\mathcal{FNS}$s by means of direct method.

Theorem 4.1. *Let $u \in \{-1, 1\}$ and $\chi : A^\phi \to \mathcal{Z}$ is defined by*

$$Q(\chi(0, 2^u f, 0, ..., 0), \varepsilon) \ge Q(\varsigma^u \chi(0, f, 0, ..., 0), \varepsilon),\ \varsigma > 0,\ \left(\frac{\varsigma}{2^3}\right)^u < 1, \tag{4.1}$$

including

$$\lim_{m\to\infty} Q\left(\chi(2^{um}f_1, 2^{um}f_2, 2^{um}f_3, ..., 2^{um}f_\phi), 2^{3um}\varepsilon\right) = 1,$$

for all $f_1, f_2, f_3, ..., f_\phi \in \mathcal{P}$ *and* $\varepsilon > 0$. *Then an odd mapping* $\Lambda : \mathcal{P} \to \mathcal{Q}$ *with* $\Lambda(0) = 0$ *fulfils*

(4.2) $$P(D_\Lambda(f_1, f_2, f_3, ..., f_\phi), \varepsilon) \geq Q(\chi(f_1, f_2, f_3, ..., f_\phi), \varepsilon),$$

for all $f_1, f_2, f_3, ..., f_\phi \in \mathcal{P}$ *and* $\varepsilon > 0$. *Then the limit*

$$C(f) = P - \lim_{m\to\infty} \frac{\Lambda(2^{um}n)}{2^{3um}} \; exists, \; \forall f \in \mathcal{P}$$

and $C : \mathcal{P} \to \mathcal{Q}$ *is a unique cubic mapping such that*

(4.3) $$P(\Lambda(f) - C(f), \varepsilon) \geq Q(\chi(f_1, f_2, f_3, ..., f_\phi), \left(\frac{\phi^2 - 5\phi + 6}{2}\right) |2^3 - \varsigma| \varepsilon),$$

for all $f \in \mathcal{P}$ *and* $\varepsilon > 0$.

Proof. Initially, we consider $\mathfrak{e} = 1$. Substituting $(f_1, f_2, f_3, ..., f_\phi)$ through $(0, f, 0, ..., 0)$ in (4.2), we reach

$$P\left(\left(\frac{\phi^2 - 5\phi + 6}{2}\right) \Lambda(2f) - 8\left(\frac{\phi^2 - 5\phi + 6}{2}\right) \Lambda(f), \varepsilon\right) \geq Q(\chi(0, f, 0, ..., 0), \varepsilon)$$

(4.4) $$P\left(\left(\frac{\Lambda(2f)}{2^3}\right) - \left(\frac{2\varepsilon}{8(\phi^2 - 5\phi + 6)}\right), \varepsilon\right) \geq Q(\chi(0, f, 0, ..., 0), \varepsilon), \quad f \in \mathcal{P}, \varepsilon > 0.$$

Exchanging f through $2^m f$ in (4.4), we acquire

$$P\left(\left(\frac{\Lambda(2^{(m+1)}f)}{2^3}\right) - \Lambda(2^m f), \left(\frac{\varepsilon}{8(\phi^2 - 5\phi + 6)}\right)\right) \geq Q(\chi(2^m f, 2^m f, 0, ..., 0), \varepsilon)$$

$$P\left(\left(\frac{\Lambda(2^{(m+1)}f)}{2^{3(m+1)}}\right) - \frac{\Lambda(2^m f)}{2^{3m}}, \left(\frac{2\varepsilon}{2^{3(m+1)}(\phi^2 - 5\phi + 6)}\right)\right) \geq Q(\chi(0, 2^m f, 0, ..., 0), \varepsilon), \quad f \in \mathcal{P}, \varepsilon > 0.$$

Utilizing (4.1) and (3) in the above inequality, we reach

$$P\left(\left(\frac{\Lambda(2^{(m+1)}f)}{2^{3(m+1)}}\right) - \frac{\Lambda(2^m f)}{2^{3m}}, \left(\frac{2\varepsilon}{2^{3(m+1)}(\phi^2 - 5\phi + 6)}\right)\right) \geq Q(\chi(0, 2^m f, 0, ..., 0), \frac{\varepsilon}{\varsigma^m}), \quad f \in \mathcal{P}, \varepsilon > 0.$$

Switching ε through ς^m in the last inequality, we acquire

(4.5)
$$P\left(\left(\frac{\Lambda(2^{(m+1)}f)}{2^{3(m+1)}}\right) - \frac{\Lambda(2^m f)}{2^{3m}}, \left(\frac{2\varsigma^m\varepsilon}{2^{3(m+1)}(\phi^2 - 5\phi + 6)}\right)\right) \geq Q(\chi(0, 2^m f, 0, ..., 0), \varepsilon), \quad f \in \mathcal{P}, \varepsilon > 0.$$

From (4.5), we obtain

$$
\begin{aligned}
& P\left(\left(\frac{\Lambda(2^{(m)}f)}{2^{3(m)}}\right)-\Lambda(f),\sum_{\mathfrak{e}=0}^{m-1}\left(\frac{2\varsigma^{\mathfrak{e}}\varepsilon}{2^{3(\mathfrak{e}+1)}(\phi^2-5\phi+6)}\right)\right)\\
&= P\left(\sum_{\mathfrak{e}=0}^{m-1}\left(\frac{\Lambda(2^{\mathfrak{e}+1}f)}{2^{3(\mathfrak{e}+1)}}-\frac{\Lambda(2^{a}f)}{2^{3(a)}}\right),\sum_{\mathfrak{e}=0}^{m-1}\left(\frac{2\varsigma^{\mathfrak{e}}\varepsilon}{2^{3(\mathfrak{e}+1)}(\phi^2-5\phi+6)}\right)\right)\\
&\geq \min_{0\leq a\leq m-1} P\left(\left(\frac{\Lambda(2^{\mathfrak{e}+1}f)}{2^{3(\mathfrak{e}+1)}}-\frac{\Lambda(2^{a}f)}{2^{3(a)}}\right),\left(\frac{2\varsigma^{\mathfrak{e}}\varepsilon}{2^{3(\mathfrak{e}+1)}(\phi^2-5\phi+6)}\right)\right)\\
&\geq Q(\chi(0,f,0,...,0),\varepsilon),\ \forall\, f\in\mathcal{P},\varepsilon>0,m\in\mathcal{N}. \qquad (4.6)
\end{aligned}
$$

Substituting f by $2^s f$ in (4.6) and utilizing (4.1) with (3), we acquire

$$
\begin{aligned}
P\left(\left(\frac{\Lambda(2^{m+s}f)}{2^{3(m+s)}}-\frac{\Lambda(2^{s}f)}{2^{3(s)}}\right),\sum_{\mathfrak{e}=0}^{m-1}\left(\frac{2\varsigma^{\mathfrak{e}}\varepsilon}{2^{3(\mathfrak{e}+s+1)}(\phi^2-5\phi+6)}\right)\right) &\geq Q(\chi(0,2^sf,0,...,0),\varepsilon)\\
&\geq Q(\chi(0,f,0,...,0),\frac{\varepsilon}{\varsigma^s}).
\end{aligned}
$$

Also

$$
P\left(\left(\frac{\Lambda(2^{m+s}f)}{2^{3(m+s)}}-\frac{\Lambda(2^{s}f)}{2^{3(s)}}\right),\sum_{\mathfrak{e}=s}^{m+s-1}\left(\frac{2\varsigma^{\mathfrak{e}}\varepsilon}{2^{3(\mathfrak{e}+s+1)}(\phi^2-5\phi+6)}\right)\right)\geq Q(\chi(0,f,0,...,0),\varepsilon),
$$

$\forall\, f\in\mathcal{P},\varepsilon>0,s,m\geq 0$. Exchanging ε through $\frac{\varepsilon}{\sum_{\mathfrak{e}=s}^{m+s-1}\left(\frac{2\varsigma^{\mathfrak{e}}}{2^{3(\mathfrak{e}+1)}(\phi^2-5\phi+6)}\right)}$ in the last inequality, we obtain

$$
P\left(\left(\frac{\Lambda(2^{m+s}f)}{2^{3(m+s)}}-\frac{\Lambda(2^{s}f)}{2^{3(s)}}\right),\varepsilon\right)\geq Q\left(\chi(0,f,0,...,0),\frac{\varepsilon}{\sum_{\mathfrak{e}=s}^{m+s-1}\left(\frac{2\varsigma^{\mathfrak{e}}}{2^{3(\mathfrak{e}+1)}(\phi^2-5\phi+6)}\right)}\right), \qquad (4.7)
$$

$\forall\, f\in\mathcal{P},\varepsilon>0,s,m\geq 0$. Since

$$
\sum_{\mathfrak{e}=0}^{\infty}\left(\frac{2\varsigma}{8(\phi^2-5\phi+6)}\right)^{\mathfrak{e}}<\infty,
$$

it follows from (4.7) and (5) that $\left\{\frac{\Lambda(2^m f)}{2^{3m}}\right\}_{m=1}^{\infty}$ is Cauchy in $(\mathcal{Q},\mathcal{P})$ for each $f\in\mathcal{P}$. Since $(\mathcal{Q},\mathcal{P})$ is a fuzzy Banach space, this sequence converges to some point $C(f)\in\mathcal{Q}$ for each $f\in\mathcal{P}$. Define $C:\mathcal{P}\to\mathcal{Q}$ by

$$
C(f)=\mathcal{P}-\lim_{m\to\infty}\frac{\Lambda(2^m f)}{2^{3m}},\ f\in\mathcal{P}.
$$

Since Λ is odd, C is odd. Letting $s=0$ in (4.7), we obtain

$$
P\left(\left(\frac{\Lambda(2^{m}f)}{2^{3(m)}}-\Lambda(f)\right),\varepsilon\right)\geq Q\left(\chi(0,f,0,...,0),\frac{\varepsilon}{\sum_{\mathfrak{e}=0}^{m-1}\left(\frac{2\varsigma^{\mathfrak{e}}}{2^{3(\mathfrak{e}+1)}(\phi^2-5\phi+6)}\right)}\right), \qquad (4.8)
$$

$\forall\ f \in \mathcal{P}, \varepsilon > 0, m \geq 1$. Then

$$P(\Lambda(f) - C(f), \varepsilon + \alpha) \geq \min\left\{P\left(\left(\frac{\Lambda(2^m f)}{2^{3(m)}} - \Lambda(f)\right), \varepsilon\right), P\left(\left(\frac{\Lambda(2^m f)}{2^{3(m)}} - C(f)\right), \alpha\right)\right\}$$
$$\geq \{Q\left(\chi(0, f, 0, ..., 0), \frac{\varepsilon}{\sum_{\mathfrak{e}=0}^{m-1}\left(\frac{2\varsigma^{\mathfrak{e}}}{2^{3(\mathfrak{e}+1)}(\phi^2 - 5\phi + 6)}\right)}\right), P\left(\left(\frac{\Lambda(2^m f)}{2^{3(m)}} - C(f)\right), \alpha\right)\},$$

$\forall\ f \in \mathcal{P}, \varepsilon > 0, m \geq 1$. Taking $m \to \infty$ in the last inequality and using (3.4), we have

$$P(\Lambda(f) - C(f), \varepsilon + \alpha) \geq Q\left(\chi(0, f, 0, ..., 0), \left(\frac{(\phi^2 - 5\phi + 6)}{2}\right)(2^s - \varsigma)\varepsilon\right),$$

$f \in \mathcal{P}, \varepsilon, \alpha > 0$. Taking the limit as $\alpha \to 0$, we get (4.3). Now, we assert that C is cubic. It is clear that

$$P(DC(f_1, f_2, f_3, ..., f_\phi), 2\varepsilon) \geq \min\left\{P\left(DC(f_1, f_2, f_3, ..., f_\phi) - \frac{1}{2^{3m}} D_\Lambda(2^m f_1, 2^m f_2, 2^m f_3, ..., 2^m f_\phi), \varepsilon\right), P\left(\frac{1}{2^{3m}} D_\Lambda(2^m f_1, 2^m f_2, 2^m f_3, ..., 2^m f_\phi), \varepsilon\right)\right\}$$
$$\geq \min\left\{P\left(DC(f_1, f_2, f_3, ..., f_\phi) - \frac{1}{2^{3m}} D_\Lambda(2^m f_1, 2^m f_2, 2^m f_3, ..., 2^m f_\phi), \varepsilon\right), Q\left(\chi(2^m f_1, 2^m f_2, 2^m f_3, ..., 2^m f_\phi), 2^{3m}\varepsilon\right)\right\},\ f \in \mathcal{P}, \varepsilon > 0.$$

Since

$$\lim_{m\to\infty} P\left(DC(f_1, f_2, f_3, ..., f_\phi) - \frac{1}{2^{3m}} D_\Lambda(2^m f_1, 2^m f_2, 2^m f_3, ..., 2^m f_\phi), \varepsilon\right) = 1$$
$$\lim_{m\to\infty} C\left(\chi(2^m f_1, 2^m f_2, 2^m f_3, ..., 2^m f_\phi), 2^{3m}\varepsilon\right) = 1.$$

We infer $P\left(DC(f_1, f_2, f_3, ..., f_\phi), 2\varepsilon\right) = 1$ for all $f_1, f_2, f_3, ..., f_\phi \in \mathcal{P}$ and all $\varepsilon > 0$. Then (2) implies $DC(f_1, f_2, f_3, ..., f_\phi) = 0$ for all $f_1, f_2, f_3, ..., f_\phi \in \mathcal{P}$. Therefore $C : \mathcal{P} \to \mathcal{Q}$ is cubic by Theorem 3.1. To show the uniqueness of C, let $D : \mathcal{P} \to \mathcal{Q}$ be another cubic mapping fulfilling (4.3). Since $C(2^m f) = 2^{3m}C(f)$ and $D(2^m f) = 2^{3m}D(f), \forall f \in \mathcal{P},\ m \in \mathcal{N}$. From

(4.3) that

$$\begin{aligned}P(C(f)-D(f),\varepsilon) &= P\left(\left(\frac{C(2^m f)}{2^{3(m)}}-\frac{D(2^m f)}{2^{3(m)}}\right),\varepsilon\right)\\ &\geq \min\left\{P\left(\left(\frac{C(2^m f)}{2^{3(m)}}-\frac{\Lambda(2^m f)}{2^{3(m)}}\right),\frac{\varepsilon}{2}\right),\right.\\ &\left.P\left(\left(\frac{\Lambda(2^m f)}{2^{3(m)}}-\frac{D(2^m f)}{2^{3(m)}}\right),\frac{\varepsilon}{2}\right)\right\}\\ &\geq \min Q\left(\chi(0,2^m f,0,...,0),\left(\frac{(\phi^2-5\phi+6)(2^3-\varsigma)}{4}\right)\varepsilon\right)\\ &\geq \min Q\left(\chi(0,f,0,...,0),\left(\frac{(\phi^2-5\phi+6)(2^3-\varsigma)}{4\varsigma^m}\right)\varepsilon\right),\end{aligned}$$

for all $f\in\mathcal{P},\varepsilon>0$ and all $m\in\mathcal{N}$. Since

$$\lim_{m\to\infty}\frac{(\phi^2-5\phi+6)(2^3-\varsigma)}{4\varsigma^m}=\infty,$$

we have

$$\lim_{m\to\infty}Q\left(\chi(0,f,0,...,0),\left(\frac{(\phi^2-5\phi+6)(2^3-\varsigma)}{4\varsigma^m}\right)\varepsilon\right)=1.$$

Consequently, $P(C(f)-D(f),\varepsilon)=1$ for all $f\in\mathcal{P}$ and all $\varepsilon>0$. So $C(f)=D(f)$ for all $f\in\mathcal{P}$. For $u=-1$, we can demonstrate the consequence through homogeneous procedure. The proof of the theorem is now complete. ∎

5. STABILITY RESULTS FOR THE $\mathcal{FE}$ (1.2): FIXED POINT METHOD

In this segment, we scrutinize the generalized $\mathcal{HUS}$ of the $\mathcal{FE}$ (1.2) in $\mathcal{FNS}$s through the fixed point method. First, we define $\psi_{\mathfrak{e}}$ a as a constant such that

$$\psi_{\mathfrak{e}}=\begin{cases}2, & \text{if } \mathfrak{e}=0\\ \frac{1}{2}, & \text{if } \mathfrak{e}=1\end{cases}$$

and we consider $\Upsilon=\{v:\mathcal{P}\to\mathcal{Q}|v(0)=0\}$.

Theorem 5.1. *Let $\Lambda:\mathcal{P}\to\mathcal{Q}$ be a mapping with $\Lambda(0)=0$ and $\chi:\mathcal{P}^\phi\to\mathcal{Z}$ with condition*

$$\lim_{m\to\infty}Q\left(\chi\left(\psi_1^m f_1,\psi_1^m f_2,\psi_1^m f_3,...,\psi_1^m f_\phi\right),\psi_1^{3m}\varepsilon\right)=1, \tag{5.1}$$

for all $f_1,f_2,f_3,...,f_\phi\in\mathcal{P},\varepsilon>0$ and satisfying the inequality

$$P(D_\Lambda(f_1,f_2,f_3,...,f_\phi),\varepsilon)\geq Q(\chi(f_1,f_2,f_3,...,f_\phi),\varepsilon), \tag{5.2}$$

for all $f_1,f_2,f_3,...,f_\phi\in\mathcal{P},\varepsilon>0$. Let $\sigma(f)=\frac{2}{(\phi^2-5\phi+6)}\chi\left(\frac{f}{2},\frac{f}{2},0,0,...,0\right)$, for all $f\in\mathcal{P}$. If there exist $L=L_{\mathfrak{e}}\in(0,1)$ such that

$$Q\left(\frac{1}{\psi_{\mathfrak{e}}^3}\sigma(\psi_{\mathfrak{e}}n),\varepsilon\right)\geq Q(L\sigma(f),\varepsilon),\ f\in\mathcal{P},\varepsilon>0. \tag{5.3}$$

Then there exist $C:\mathcal{P}\to\mathcal{Q}$ fulfilling

$$P(\Lambda(f)-C(f),\varepsilon)\geq Q\left(\frac{\phi^{1-a}}{1-\phi}\sigma(f),\varepsilon\right),\ f\in\mathcal{P},\varepsilon>0. \tag{5.4}$$

Proof. Let ς be the generalized metric on Υ

$$\varsigma(v,w) = \inf\{r \in (0,\infty) | P(v(f) - w(f), \varepsilon) \geq Q(r\sigma(f), \varepsilon), f \in \mathcal{P}, \varepsilon > 0\}$$

and we take, as usual, $\inf \emptyset = +\infty$. Define $\Lambda_{\mathfrak{e}} : \Upsilon \to \Upsilon$ by $\Lambda_{\mathfrak{e}} v(f) = \frac{1}{\psi_{\mathfrak{e}}^3} v(\psi_{\mathfrak{e}} f)$ for all $f \in \mathcal{P}$. Let $v, w \in \Upsilon$ be given such that $\varsigma(v,w) \leq \alpha$. Then

$$P(v(f) - w(f), \varepsilon) \geq Q(\alpha\sigma(f), \varepsilon), f \in \mathcal{P}, \varepsilon > 0,$$

whence

$$P(\Lambda_{\mathfrak{e}} v(f) - \Lambda_{\mathfrak{e}} w(f), \varepsilon) \geq Q(\frac{\alpha}{\psi_{\mathfrak{e}}^3} \sigma(\psi_{\mathfrak{e}} f), \varepsilon),\ \varepsilon > 0,\ f \in \mathcal{P}.$$

From (5.3) that

$$P(\Lambda_{\mathfrak{e}} v(f) - \Lambda_{\mathfrak{e}} w(f), \varepsilon) \geq Q(\alpha L \sigma(f), \varepsilon), f \in \mathcal{P}, \varepsilon > 0.$$

Hence, we have $\varsigma(\Lambda_{\mathfrak{e}}, \Lambda_{\mathfrak{e}} w) \leq \alpha L$. This shows $\varsigma(\Lambda_{\mathfrak{e}}, \Lambda_{\mathfrak{e}} w) \leq \phi\varsigma(v,w)$, i.e., $\Lambda_{\mathfrak{e}}$ is strictly contractive on Υ. Substituting $(f_1, f_2, f_3, ..., f_\phi)$ by $(0, f, 0, ..., 0)$ in (5.2) and utilizing (f_3), we get

$$P\left(\frac{\Lambda(2f)}{2^3} - \Lambda(f), \varepsilon\right) \geq Q\left(\frac{2\chi(0,f,0,...,0)}{2^3(\phi^2 - 5\phi + 6)}, \varepsilon\right),\ f \in \mathcal{P}, \varepsilon > 0. \tag{5.5}$$

Using (5.3) when $a = 0$, it follows from (5.5) that

$$P\left(\frac{\Lambda(2f)}{2^3} - \Lambda(f), \varepsilon\right) \geq Q\left(L\sigma(f), \varepsilon\right),\ f \in \mathcal{P}, \varepsilon > 0.$$

Therefore

$$\varsigma(\Lambda_0 \Lambda, \Lambda) \leq \phi = \phi^{1-a}. \tag{5.6}$$

Exchanging n through $\frac{f}{2}$ in (5.5), we obtain

$$\begin{aligned} P\left(\Lambda(f) - 2^3\Lambda\left(\frac{f}{2}\right), 2^3\varepsilon\right) &\geq Q\left(\chi\left(\frac{f}{2}, \frac{f}{2}, 0, ..., 0\right), 2^3\left(\frac{\phi^2 - 5\phi + 6}{2}\right)\varepsilon\right) \\ &= Q\left(\sigma(f), 2^3\left(\frac{\phi^2 - 5\phi + 6}{2}\right)\varepsilon\right),\ f \in \mathcal{P}, \varepsilon > 0. \end{aligned}$$

Therefore

$$\varsigma(\Lambda_1 \Lambda, \Lambda) \leq \phi = \phi^{1-a}. \tag{5.7}$$

Then from (5.6) and (5.7), we conclude $\varsigma(\Lambda_{\mathfrak{e}}\Lambda, \Lambda) \leq \phi^{1-a} < \infty$. Now from the fixed point alternative Theorem 2.1, it follows that there exists a fixed point C of $\Lambda_{\mathfrak{e}}$ in Υ such that

(1) $\Lambda_{\mathfrak{e}} C = C$ and $\lim_{m\to\infty} \varsigma(\Lambda_{\mathfrak{e}}^m \Lambda, C)$
(2) $E = \{v \in \Upsilon | d(\Lambda, v) < \infty\}$
(3) $\varsigma(\Lambda, C) \leq \frac{1}{1-\phi}\varsigma(\Lambda, \Lambda_{\mathfrak{e}}\Lambda)$.

Letting $\lim_{m\to\infty} \varsigma(\Lambda_{\mathfrak{e}}^m \Lambda, C) = \alpha_m$, we get

$$P(\Lambda_{\mathfrak{e}}^m v(f) - C(f), \varepsilon) \geq Q(\alpha_m \sigma(f), \varepsilon), f \in \mathcal{P}, \varepsilon > 0.$$

Since $\lim_{m\to\infty} \alpha_m = 0$, we infer

$$C(f) = P - \lim_{m\to\infty} \frac{\Lambda(\psi_{\mathfrak{e}}^m f)}{\psi_{\mathfrak{e}}^{2m}},\ \ f \in \mathcal{P}.$$

Switching $(f_1, f_2, f_3, ..., f_\phi)$ by $(\psi_{\mathfrak{e}}^m f_1, \psi_{\mathfrak{e}}^m f_2, \psi_{\mathfrak{e}}^m f_3, ..., \psi_{\mathfrak{e}}^m f_\phi)$ in (5.2), we obtain

$$P\left(\frac{1}{\psi_{\mathfrak{e}}^{3m}} D\Lambda(\psi_{\mathfrak{e}}^m f_1, \psi_{\mathfrak{e}}^m f_2, \psi_{\mathfrak{e}}^m f_3, ..., \psi_{\mathfrak{e}}^m f_\phi), \varepsilon\right) \geq Q(\chi(\psi_{\mathfrak{e}}^m f_1, \psi_{\mathfrak{e}}^m f_2, \psi_{\mathfrak{e}}^m f_3, ..., \psi_{\mathfrak{e}}^m f_\phi), \psi_{\mathfrak{e}}^{3m}\varepsilon),$$

for all $\varepsilon > 0, f_1, f_2, f_3, ..., f_\phi \in \mathcal{P}$. Using the same argument as in the proof of Theorem 4.1, we can prove the function $C : \mathcal{P} \to \mathcal{Q}$ is cubic. Since $\varsigma(\Lambda_{\mathfrak{e}}\Lambda, \Lambda) \leq \phi^{1-a}$, it follows from (3) that $\varsigma(\Lambda, C) \leq \frac{\phi^{1-a}}{1-\phi}$ which means (5.4). To prove the uniqueness of C, let $D : \mathcal{P} \to \mathcal{Q}$ be another cubic mapping fulfilling (5.4). Since $C(2^m f) = 2^{3m}C(f)$ and $D(2^m f) = 2^{3m}D(f)$ for all $f \in P, m \in N$, we have

$$\begin{aligned} P(C(f) - D(f), \varepsilon) &= P\left(\frac{C(2^m f)}{2^{3m}} - \frac{D(2^m f)}{2^{3m}}, \varepsilon\right) \\ &\geq \min\left\{P\left(\frac{C(2^m f)}{2^{3m}} - \frac{\Lambda(2^m f)}{2^{3m}}, \frac{\varepsilon}{2}\right), \right. \\ &\quad \left. P\left(\frac{\Lambda(2^m f)}{2^{3m}} - \frac{D(2^m f)}{2^{3m}}, \frac{\varepsilon}{2}\right)\right\} \\ &\geq Q\left(\frac{\phi^{1-a}}{1-\phi}\sigma(2^m f), \frac{2^{3m}\varepsilon}{4}\right). \end{aligned}$$

By (5.1), we have

$$\lim_{m\to\infty} Q\left(\frac{\phi^{1-a}}{1-\phi}\sigma(2^m f), \frac{2^{3m}\varepsilon}{4}\right) = 1.$$

Consequently, $P(C(f) - D(f), \varepsilon) = 1$ for all $f \in \mathcal{P}$ and all $\varepsilon > 0$. So $C(f) = D(f)$ for all $f \in \mathcal{P}$, which ends the proof. ∎

The upcoming corollaries are instantaneous outcome of Theorems 4.1 and 5.1, regarding the stability for the equation (1.2). Assume that $\mathcal{P}, (\mathcal{Q}, \mathcal{P})$ and $(\mathcal{R}, Q)$ be a linear space, a fuzzy Banach space and a $\mathcal{FNS}$, respectively.

Corollary 5.2. *Suppose a function $\Lambda : \mathcal{P} \to \mathcal{Q}$ fulfils $\Lambda(0) = 0$ and the inequality*

$$P(D\Lambda(f_1, f_2, f_3, ..., f_\phi), \varepsilon) \geq Q\left(\tau + \theta\prod_{\mathfrak{e}=1}^{\phi}\|f_{\mathfrak{e}}\|^q, \varepsilon\right),$$

for all $f_1, f_2, f_3, ..., f_\phi \in \mathcal{P}$ and all $\varepsilon > 0$, where τ, θ, q are real constants with $lq \in (0, 3)$. Then there exists a unique cubic mapping $C : \mathcal{P} \to \mathcal{Q}$ such that

$$P(C(f) - D(f), \varepsilon) \geq Q(\tau, 7\varepsilon), \quad f \in \mathcal{P}, \varepsilon > 0.$$

Corollary 5.3. *Suppose a function $\Lambda : \mathcal{P} \to \mathcal{Q}$ fulfils $\Lambda(0) = 0$ and*

$$P(D\Lambda(f_1, f_2, f_3, ..., f_\phi), \varepsilon) \geq Q\left(\alpha\sum_{\mathfrak{e}=1}^{\phi}\|f_{\mathfrak{e}}\|^p + \theta\prod_{\mathfrak{e}=1}^{\phi}\|f_{\mathfrak{e}}\|^q, \varepsilon\right),$$

for all $f_1, f_2, f_3, ..., f_\phi \in \mathcal{P}$ and all $\varepsilon > 0$, where α, θ, p and q are real constants with $p, lq \in (0, 3) \cup (3, +\infty)$. Then there exists a unique cubic mapping $C : \mathcal{P} \to \mathcal{Q}$ such that

$$P(C(f) - D(f), \varepsilon) \geq Q\left(\alpha\|f_{\mathfrak{e}}\|^p, \left(\frac{\phi^2 - 5\phi + 6}{2}\right)|2^3 - 2^p|\varepsilon\right), \quad f \in \mathcal{P}, \varepsilon > 0.$$

Corollary 5.4. *Suppose a function* $\Lambda : \mathcal{P} \to \mathcal{Q}$ *fulfils* $\Lambda(0) = 0$ *and the inequality*

$$P(D\Lambda(f_1, f_2, f_3, ..., f_\phi), \varepsilon) \geq Q\left(\theta \prod_{\mathfrak{e}=1}^{\phi} \|f_{\mathfrak{e}}\|^q, \varepsilon\right),$$

for all $f_1, f_2, f_3, ..., f_\phi \in \mathcal{P}$ *and all* $\varepsilon > 0$, *where* θ *and* q *are real constants with* $0 < lq \neq 3$. *Then* Λ *is cubic.*

6. CONCLUSION

The method of combining general solution of cubic functional was obtained in (1.2). Furthermore the generalized Hyers-Ulam stability of Cubic functional of the form (1.2) in fuzzy normed space using direct and fixed point methods.

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