Aust. J. Math. Anal. Appl.
Vol. **19** (2022), No. 2, Art. 4, 8 pp.
AJMAA

TOEPLITZ DETERMINANT FOR SAKAGUCHI TYPE FUNCTIONS UNDER PETAL SHAPED DOMAIN

B. NANDHINI AND B. SRUTHA KEERTHI

Received 22 April, 2022; accepted 2 August, 2022; published 23 September, 2022.

DIVISION OF MATHEMATICS, SCHOOL OF ADVANCED SCIENCES, VELLORE INSTITUTE OF TECHNOLOGY
CHENNAI CAMPUS, CHENNAI - 600 048, INDIA.
nandhinibaskar1996@gmail.com

DIVISION OF MATHEMATICS, SCHOOL OF ADVANCED SCIENCES, VELLORE INSTITUTE OF TECHNOLOGY
CHENNAI CAMPUS, CHENNAI - 600 048, INDIA
keerthivitmaths@gmail.com

ABSTRACT. We introduce a new general subclass $\mathcal{GP}^{t,\rho}$ of Sakaguchi kind function on a Petal shaped domain. We obtain coefficients bounds and upper bounds for the Fekete-Szegö functional over the class. From these functions we obtain the bounds of first four coefficients, and then we have derived the Toeplitz determinant $\mathcal{T}_2(2)$ and $\mathcal{T}_3(1)$ whose diagonal entries are the coefficients of functions.

Key words and phrases: Analytic function; subordination; Logarithmic coefficient; Fekete-Szegö inequality.

2010 *Mathematics Subject Classification.* 30C45, 30C50.

ISSN (electronic): 1449-5910
© 2022 Austral Internet Publishing. All rights reserved.

1. INTRODUCTION AND PRELIMINARIES

Let $\mathcal{A}$ be the family of all analytic functions , which is normalized under the condition $f(0) = f'(0) - 1 = 0$ in $\mathbb{D} = \{z : z \in \mathbb{C}$ and $|z| < 1\}$ and given by the following Taylor-Maclaurin series:

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n \tag{1.1}$$

Further, $\mathcal{S}$ indicates the the class of all functions in $\mathcal{A}$ which are univalent in $\mathbb{D}$. By recalling the subordination principle of two analytic functions, let the functions f and g be analytic in $\mathbb{D}$. Then we say that the function f is subordinate to g if there exists a Schwarz function $\omega(z)$, analytic in $\mathbb{D}$ with

$$\omega(0) = 0,\ |\omega(z)| < 1, (z \in \mathbb{D})$$

such that $f(z) = g(\omega(z))$
We denote this subordination by,

$$f \prec g\ (or)\ f(z) \prec g(z)$$

In particular, if the function g is univalent in $\mathbb{D}$, the above subordination is equivalent to $f(0) = g(0)$, $f(\mathbb{D}) \subset g(\mathbb{D})$

In geometric function theory, Ma and Minda[12] gave a unified treatment of distortion, growth and covering theorems for the functions $f \in \mathcal{R}^*_\eta$ and $f \in \mathcal{N}$ for which either of the quantity $\frac{zf'(z)}{f(z)}$ or $1 + \frac{zf''(z)}{f'(z)}$ is subordinate to a more general subordi- nate function. The most basic and important subfamilies of the set $\mathcal{S}$ are the family $\mathcal{R}^*$ of starlike functions and the family $\mathcal{N}$ of convex functions which are defined as follows:

$$\mathcal{N} = \left\{ f \in \mathcal{A} : 1 + \frac{zf''(z)}{f'(z)} \prec \Psi(z),\ z \in \mathbb{D} \right\}, \tag{1.2}$$

$$\mathcal{R}^* = \left\{ f \in \mathcal{A} : \frac{zf'(z)}{f(z)} \prec \Psi(z),\ z \in \mathbb{D} \right\}, \tag{1.3}$$

with

$$\Psi(z) = 1 + 2\sum_{n=2}^{\infty} z^n := \frac{1+z}{1-z}, (z \in \mathbb{D}) \tag{1.4}$$

By varying the function $\Psi(z)$ in (1.2),(1.3), we get some subfamilies of the sets $\mathcal{N}$ and $\mathcal{R}^*$ which have significant geometric sense.
For example, if we consider $\Psi(z) = 1 + sinh^{-1}z$, then the class $\mathcal{R}^*_\eta := \mathcal{R}^*(1 + sinh^{-1}z)$ was provided by Kumar and Arora[11] and is defined as a function $f \in \mathcal{A}$ which is in the family $\mathcal{R}^*_\eta$ if (1.3) holds for the function $\Psi(z) = \eta(z)$, where

$$\eta(z) = 1 + sinh^{-1}z \tag{1.5}$$

Clearly, the function $\eta(z)$ is a multivalued function and has the branch cuts about the line segments $(-i\infty, -i) \cup (i, i\infty)$, on imaginary axis and hence, it is holomorphic in $\mathbb{D}$. In a geometric point of view, the function $\eta(z)$ maps the unit disc $\mathbb{D}$ onto a petal-shaped region Ω_p,

$$\Omega_p = \{\omega \in \mathcal{C} : |sinh(\omega - 1)| < 1\} \tag{1.6}$$

Using this idea, we now consider a subfamily $\mathcal{GP}^{t,\rho}$ of analytic functions as

Definition 1.1. The function $f \in \mathcal{A}$ is in the class $\mathcal{GP}^{t,\rho}$ if

$$\frac{(1-t)[\rho z^2 f''(z) + zf'(z)]}{f(z) - f(tz)} \prec \tilde{\Psi}(z), \tag{1.7}$$

where $\tilde{\Psi}(z)$ is given by (1.6), with $|t| \leq 1$, $t \neq 1$, and $0 \leq \rho \leq 1$.

Remark 1.1. (i) For $\rho = 0$, we get the following new class $\mathcal{GP}$,

$$GP_p^{t,\rho} = \left\{ f \in \mathcal{A} : \frac{(1-t)[zf'(z)]}{f(z) - f(tz)} \prec \tilde{\Psi}(z), z \in \mathbb{D} \right\}$$

Now, for the function of the form (1.1), we define the Toeplitz Determinant $\mathcal{T}_q(n)$ with $q \geq 1$ and $n \geq 1$ by,

$$\mathcal{T}_q(n) = \begin{vmatrix} a_n & a_{n+1} & \dots & a_{n+q-1} \\ a_{n+1} & a_n & \dots & a_{n+q} \\ \vdots & \vdots & \dots & \vdots \\ a_{n+q-1} & a_{n+q} & \dots & a_n \end{vmatrix}$$

In particular, we can find some of the Toeplitz determinants as,

$$\mathcal{T}_2(2) = \begin{vmatrix} a_2 & a_3 \\ a_3 & a_2 \end{vmatrix}, \mathcal{T}_3(1) = \begin{vmatrix} a_1 & a_2 & a_3 \\ a_2 & a_1 & a_2 \\ a_3 & a_2 & a_1 \end{vmatrix}$$

2. A SET OF LEMMAS

Let $\mathcal{H}$ be the family of functions q that are holomorphic in $\mathbb{D}$ with $\Re(q(z)) > 0$ and the power series form as follows:

$$q(z) = 1 + \sum_{k=1}^{\infty} c_k z^k \qquad (z \in \mathbb{D}). \tag{2.1}$$

Lemma 2.1. [3, 14] *If $q \in \mathcal{H}$ be expressed in series expansion (2.1), then*

$$|c_k| \leq 2 \qquad \textit{for} \quad k \geq 1, \tag{2.2}$$

and for complex number γ, we have

$$\left|c_2 - \gamma c_1^2\right| \leq 2 \max\{1, |2\gamma - 1|\}, \tag{2.3}$$

Lemma 2.2. [9] *Let $q \in \mathcal{H}$ has power series expansion (2.1), then*

$$\left|Jc_1^3 - Kc_1c_2 + Lc_3\right| \leq 2|J| + 2|K - 2J| + 2|J - K + L|.$$

3. COEFFICIENTS ESTIMATES AND FEKETE-SZEGÖ INEQUALITY

Theorem 3.1. *If the function f of the form (1.1) belongs to $\mathcal{GP}^{t,\rho}$, then*

$$|a_2| \leq \frac{U_2}{u_2},$$

$$|a_3| \leq \frac{U_3}{U_2 u_3} max[1, U_2],$$

$$|a_4| \leq \frac{U_4}{U_3 u_4} \left\{ \left| \frac{5}{24} - \frac{U_2}{4} - \frac{U_3}{4U_2} + \frac{U_3}{4} \right| + \left| \frac{7}{12} - \frac{U_3}{2} \right| + \left| \frac{5}{24} + \frac{U_2}{4} + \frac{U_3}{4U_2} + \frac{U_3}{4} \right| \right\}$$

where

$$u_n = \frac{1-t^n}{1-t},\; U_n = \prod \frac{u_n}{n[1+(n-1)\rho]-u_n}\;, n = 1, 2, \ldots$$

Proof. Let $f \in \mathcal{GP}^{t,\rho}$. Then, (1.7) can be written in the form of the Schwarz function as

$$\frac{(1-t)[\rho z^2 f''(z) + zf'(z)]}{f(z)-f(tz)} = 1 + sinh^{-1}(w(z))\; (z \in \mathbb{D}) \tag{3.1}$$

Now, if $q \in \mathcal{H}$, then it may be written in terms of the Schwarz function w by

$$p(z) = \frac{1+w(z)}{1-w(z)} = 1 + c_1 z + c_2 z^2 + c_3 z^3 + \ldots,$$

equivalently,

$$w(z) = \frac{p(z)-1}{p(z)+1} = \frac{c_1 z + c_2 z^2 + c_3 z^3 + c_4 z^4 \ldots}{2 + c_1 z + c_2 z^2 + c_3 z^3 + c_4 z^4 \ldots} \tag{3.2}$$

Using (1.1) in (1.7), we get

$$\frac{(1-t)[\rho z^2 f''(z) + zf'(z)]}{f(z)-f(tz)}$$

$$= \frac{z + 2a_2(1+\rho)z^2 + 3a_3(1+2\rho)z^3 + 4a_4(1+3\rho)z^4 + ...}{z + a_2 u_2 z^2 + a_3 u_3 z^3 + a_4 u_4 z^4 + ...} \tag{3.3}$$

By simplification and using the series expansion (3.2), we obtain

$$\begin{aligned} 1 + sinh^{-1}(w(z)) &= 1 + \frac{1}{2}c_1 z + \left(-\frac{c_1^2}{4} + \frac{c_2}{2}\right) z^2 + \left(\frac{5c_1^3}{48} - \frac{c_1 c_2}{2} + \frac{c_3}{2}\right) z^3 \\ &\quad + \left(-\frac{1}{32}c_1^4 + \frac{5}{16}c_1^2 c_2 - \frac{1}{2}c_1 c_3 - \frac{1}{4}c_2^2 + \frac{1}{2}c_4\right) z^4 + \cdots . \end{aligned} \tag{3.4}$$

Using (3.3) and (3.4), we get

$$a_2 = \frac{U_2}{2u_2} c_1 \tag{3.5}$$

$$a_3 = \frac{U_3}{U_2 u_3}\left[\frac{c_2}{2} - \frac{c_1^2}{4} + \frac{c_1^2 U_2}{4}\right] \tag{3.6}$$

$$a_4 = \frac{U_4}{U_3 u_4}\left[\frac{c_3}{2} + c_1^3\left(\frac{5}{48} - \frac{U_2}{8} - \frac{U_3}{8U_2} + \frac{U_3}{8}\right) - c_1 c_2\left(\frac{1}{2} - \frac{U_2}{4} - \frac{U_3}{4U_2}\right)\right] \tag{3.7}$$

Now implementing (2.2) in (3.5), we obtain

$$|a_2| \leq \frac{U_2}{u_2}. \tag{3.8}$$

Now implementing (2.3) in (3.6), we obtain

$$|a_3| \leq \frac{U_3}{U_2 u_3}\, max(1, U_2). \tag{3.9}$$

Implementation of triangle inequality and Lemma 2.2 in (3.7), leads us to

$$|a_4| \leq \frac{U_4}{u_4 U_3}\left[\frac{c_3}{2} + c_1^3\left(\frac{5}{48} - \frac{U_2}{8} - \frac{U_3}{8U_2} + \frac{U_3}{8}\right) - c_1 c_2\left(\frac{1}{2} - \frac{U_2}{4} - \frac{U_3}{4U_2}\right)\right]. \tag{3.10}$$

This completes the proof ∎

Letting $\rho = 0$, in Theorem 3.1, we get the following result.

Corollary 3.2. *If the function f of the form (1.1) belongs to $\mathcal{GP}^{t,0}$, then*

$$|a_2| \leq \frac{1}{2-u_2},$$

$$|a_3| \leq \frac{1}{3-u_3} max\left[1, \frac{u_2}{2-u_2}\right],$$

$$|a_4| \leq \frac{1}{4[4-u_4]}\left[\left|\frac{5}{6} - \frac{u_2}{2-u_2} - \frac{u_3}{3-u_3} + \frac{u_2u_3}{[2-u_2][3-u_3]}\right| + \left|\frac{7}{3} - \frac{2u_2u_3}{[2-u_2][3-u_3]}\right| + \left|\frac{5}{6} + \frac{u_2}{2-u_2} + \frac{u_3}{3-u_3} + \frac{u_2u_3}{[2-u_2][3-u_3]}\right|\right]$$

4. FEKETE-SZEGO INEQUALITY FOR THE CLASS $\mathcal{GP}(t,\rho)$

Fekete-Szego inequality is one of the famous problem related to coefficient of univalent analytic functions. It was first given by [10] , the classical Fekete-Szego inequality for the coefficients of $f \in \mathcal{S}$ is

$$|a_3 - \mu a_2^2| \leq 1 + 2e^{\frac{-2\mu}{1-\mu}} \ for \ \mu \in [0,1)$$

Further we are deriving this result to the above class.

Theorem 4.1. *If the function f of the form (1.1) belonging to the class $\mathcal{GP}^{t,\rho}$, then*

$$\left|a_3 - \gamma a_2^2\right| \leq \frac{U_3}{u_3U_2} max\left[1, \left| -U_2 + \gamma\frac{U_2^3u_3}{u_2^2U_3}\right|\right] \ for \ \gamma \in \mathbb{C} \tag{4.1}$$

Proof. By applying the values of a_2 and a_3, we obtain

$$|a_3 - \gamma a_2^2| = \left|\left[\frac{1}{2}c_2 - \frac{1}{4}c_1^2(1-U_2)\right]\frac{U_3}{u_3U_2} - \gamma\frac{c_1^2U_2^2}{4u_2^2}\right| \tag{4.2}$$

By rearranging , it yields

$$|a_3 - \gamma a_2^2| = \frac{U_3}{2u_3U_2}\left|c_2 - \frac{c_1^2}{2}\left[1 - U_2 + \gamma\frac{U_2^3u_3}{u_2^2U_3}\right]\right. \tag{4.3}$$

Application of (2.3) leads us to

$$|a_3 - \gamma a_2^2| \leq \frac{U_3}{2u_3U_2} 2max\left[1, \left|\left(1 - U_2 + \gamma\frac{U_2^3u_3}{u_2^2U_3}\right) - 1\right|\right] \tag{4.4}$$

After the simplification, we obtain

$$|a_3 - \gamma a_2^2| \leq \frac{U_3}{u_3U_2} max\left[1, \left| -U_2 + \gamma\frac{U_2^3u_3}{u_2^2U_3}\right|\right] \tag{4.5}$$

∎

Letting $\rho = 0$, in Theorem 4.1, we get the following consequence.

Corollary 4.2. *If the function f of the form (1.1) belongs to $\mathcal{GP}^{t,\rho}$,
then for any complex number ρ*

$$|a_3 - \gamma a_2^2| \leq \frac{1}{3-u_3} max\left[1, \left|\frac{-u_2}{2-u_2} + \gamma\frac{3-u_3}{[2-u_2]^2}\right|\right]$$

5. TOEPLITZ DETERMINANT $\mathcal{T}_2(2)$ AND $\mathcal{T}_3(1)$ FOR THE CLASS $\mathcal{GP}(t,\rho)$

Toeplitz matrices and their determinants play an important role in several branches of mathematics and have many applications Toeplitz [1]. For information on applications of Toeplitz matrices to several areas of pure and applied mathematics, we refer to the survey article by Ye and Lim [2]. We recall that Toeplitz symmetric matrices have constant entries along the diagonal. For the function $f(z) = z + \sum_{k=2}^{\infty} a_k z^k \quad (z \in \mathbb{D})$, we associate a determinant $T_q(n)$ defined by

$$\mathcal{T}_q(n) = \begin{vmatrix} a_n & a_{n+1} & \cdots & a_{n+q-1} \\ a_{n+1} & a_n & \cdots & a_{n+q} \\ \vdots & \vdots & \cdots & \vdots \\ a_{n+q-1} & a_{n+q} & \cdots & a_n \end{vmatrix}$$

Ali et al [4] convex and close to convex. Motivated by Ali et al.[4], some researchers in the last three years studied $\mathcal{T}_q(n)$ for low values of n and q, where entries are the coefficients of functions in several subclasses of analytic functions. Some recent work on coefficient problems includes Cho et al [7]; Cudna et al[5]; Lecko et al [6]; O. P. Ahuja et al.[13] In this paper, we obtain estimates for Toeplitz determinants $\mathcal{T}_2(2)$ and $\mathcal{T}_3(1)$ for functions belonging to the classes $\mathcal{GP}^{t,\rho}$

Theorem 5.1. *If $f \in \mathcal{GP}^{t,\rho}$, then the Toeplitz determinant $\mathcal{T}_2(2)$ is given by,*

$$|\mathcal{T}_2(2)| \le \left[\frac{U_3}{u_3 U_2}\, max(1, U_2)\right]^2 + \frac{U_2^2}{u_2^2}.$$

Proof. Since $\mathcal{T}_2(2) = \begin{vmatrix} a_2 & a_3 \\ a_3 & a_2 \end{vmatrix} = a_3^2 - a_2^2,$
we can write the bound of $\mathcal{T}_2(2)$ as,

$$|\mathcal{T}_2(2)| = |a_3^2 - a_2^2| \tag{5.1}$$

Applying triangle inequality to the above equation, we arrive at

$$|\mathcal{T}_2(2)| \le |a_3^2| + |a_2^2| \tag{5.2}$$

Now, substituting equations (3.8) and (3.9) to (5.2), we obtain,

$$|\mathcal{T}_2(2)| \le \left[\frac{U_3}{u_3 U_2}\, max(1, U_2)\right]^2 + \frac{U_2^2}{u_2^2}. \tag{5.3}$$

∎

Letting $\rho = 0$, in Theorem 5.1, we get the following consequence.

Corollary 5.2. *If the function f of the form (1.1) belongs to $\mathcal{GP}^{t,\rho}$, then*

$$|\mathcal{T}_2(2)| \le \left[\frac{1}{[3-u_3]}\, max\left(1, \frac{u_2}{2-u_2}\right)\right]^2 + \frac{1}{[2-u_2]^2}$$

Theorem 5.3. *The Toeplitz determinant $\mathcal{T}_3(1)$ is given by,*

$$|\mathcal{T}_3(1)| \le 1 + \frac{2U_2^2}{u_2^2} + \frac{U_3^2}{u_3^2 U_2^2} max\,(1, U_2)\; max\,\left[1, \left|\frac{2U_2^3 u_3}{u_2^2 U_3} - U_2\right|\right]$$

Proof. If $f \in\ \mathcal{GP}^{t,\rho}$, then the Toeplitz determinant $\mathcal{T}_3(1)$ is given by,

$$\mathcal{T}_3(1) = \begin{vmatrix} a_1 & a_2 & a_3 \\ a_2 & a_1 & a_2 \\ a_3 & a_2 & a_1 \end{vmatrix} = 1 - 2a_2^2 - a_3(a_3 - 2a_2^2)$$

We can write the bound of $\mathcal{T}_3(1)$ as,

$$|\mathcal{T}_3(1)| = |1 - 2a_2^2 - a_3(a_3 - 2a_2^2)| \tag{5.4}$$

Applying triangle inequality to the above equation, we obtain

$$|\mathcal{T}_3(1)| \leq 1 + 2|a_2^2| + |a_3||a_3 - 2a_2^2| \tag{5.5}$$

Substituting equation (3.8) to (5.5), we get

$$|\mathcal{T}_3(1)| \leq 1 + \frac{2U_2^2}{u_2^2} + \frac{U_3^2}{u_3^2 U_2^2} max\ (1, U_2)\ max\ \left[1, \left|\frac{2U_2^3 u_3}{u_2^2 U_3} - U_2\right|\right]$$

∎

Letting $\rho = 0$, in Theorem 5.3, we get the following consequence.

Corollary 5.4. *If the function f of the form (1.1) belongs to $\mathcal{GP}^{t,\rho}$, then*

$$|\mathcal{T}_3(1)| \leq 1 + \frac{2}{[2-u_2]^2} + \frac{1}{[3-u_3]^2} max\ \left[1, \frac{u_2}{2-u_2}\right]\ max\ \left[1, \left|\frac{2[3-u_3]}{[2-u_2]^2} - \frac{u_2}{2-u_2}\right|\right]$$

6. CONCLUSION

The purpose of this article is to obtain some applications of Quantum-Calculus in Geometric function theory, which is the recent concept for many researchers. In this method we have generated a new sub-class of Sakaguchi kind function in a petal shaped domain with the help of subordination. By working on these, resulted in coefficients for the Feketo-Szego functional over the class and also the Toeplitz determinant $\mathcal{T}_2(2)$ and $\mathcal{T}_3(1)$ for the class have been obtained.

REFERENCES

[1] O. TOEPLITZ, Zur Transformation der Scharen bilinearer Formen von unendlichvielen Veränderlichen, *Mathematischphysikalische, Klasse, Nachr. der Kgl. Gessellschaft derWissenschaften zu Göttingen* (1907), pp. 110–115.

[2] K. YE and L-H. LIM, Every matrix is a product of Toeplitz matrices, *Found. Comput. Math.*, **16** (2016), No. 3, pp. 577–598. MR3494505.

[3] F. R. KEOGH and E. P. MERKES, A coefficient inequality for certain classes of analytic functions, *Proc. Amer. Math. Soc.*, **20** (1969), pp. 8–12. MR0232926.

[4] M. F. ALI, D. K. THOMAS and A. VASUDEVARAO, Toeplitz determinants whose elements are the coefficients of analytic and univalent functions, *Bull. Aust. Math. Soc.*, **97** (2018), No. 2, pp. 253–264. MR3772655.

[5] K. CUDNA, O. S. KWON, A. LECKO, Y. J. SIM and B. ŚMIAROWSKA, The second and third-order Hermitian Toeplitz determinants for starlike and convex functions of order α, *Bol. Soc. Mat. Mex.*, **(3)26** (2020), No. 2, pp. 361–375. MR4110457.

[6] A. LECKO, Y. J. SIM and B. ŚMIAROWSKA, The fourth-order Hermitian Toeplitz determinant for convex functions, *Anal. Math. Phys.*, **10** (2020), No. 3, Paper No. 39, 11 pp. MR4137139.

[7] N. E. CHO, S. KUMAR, V. KUMAR, V. RAVICHANDRAN and H. M. SRIVASTAVA, Starlike Functions Related to the Bell Numbers, *Symmetry* (2019), 11, 21. [Online: https://doi.org/10.3390/sym11020219].

[8] O. M. BARUKAB, M. ARIF, M. ABBAS and S. A. KHAN, Sharp bounds of the coefficient results for the family of bounded turning functions associated with a petal-shaped domain, *J. Funct. Spaces* (2021), Art. ID 5535629, 9 pp. MR4221370.

[9] M. ARIF, M. RAZA, H. TANG, S. HUSSAIN and H. KHAN, Hankel determinant of order three for familiar subsets of analytic functions related with sine function, *Open Math.*, **17** (2019), No. 1, pp. 1615–1630. MR4048485.

[10] M. FEKETE and G. SZEGÖ, Eine Bemerkung Uber Ungerade Schlichte Funktionen, *J. London Math. Soc.*, **8** (1933), No. 2, 85–89. MR1574865.

[11] S. S. KUMAR and K. ARORA, Starlike functions associated with a petal shaped domain (2020). [Online: https://arxiv.org/abs/2010.10072].

[12] W. C. MA and D. MINDA, (1992) A unified treatment of some special classes of univalent functions, in *Proceedings of the Conference on Complex Analysis*, Tianjin, pp. 157–169. Lecture Notes Anal., I, Int. Press, Cambridge, MA, Conf, Proc.

[13] O. P. AHUJA, K. KHATTER V. RAVICHANDRAN, Toeplitz determinants associated with Ma-Minda classes of starlike and convex functions, *Iran. J. Sci. Technol. Trans. A Sci.*, **45** (2021), No. 6, pp. 2021–2027. MR4329220.

[14] C. POMMERENKE, *Univalent functions*, Studia Mathematica/Mathematische Lehrbücher, Band XXV, Vandenhoeck & Ruprecht, Göttingen, 1975. MR0507768.