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ANALYSIS OF THE DYNAMIC RESPONSE OF THE SOIL-PILE BEHAVIORAL MODEL UNDER LATERAL LOAD

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ABSTRACT. This work aims to extend and improve our previous study on mathematical and numerical analysis of stationary Pasternak model. In this paper a dynamic response of Pasternak model is considered. On the one hand we establish the existence and uniqueness of the solution by using the Lax-Milgram theorem and the spectral theory thus the existence of a Hilbert basis is shown and the spectral decomposition of any solution of the problem can be established and on the other hand the finite element method is used to determinate the numerical results. Furthermore, the influence of soil parameters G_p and K_p on the displacement of the pile is studied numerically at any time t_n.

Key words and phrases: Soil-pile interaction; Hilbert space; Eigenvalues and eigenfunctions; Variational formulation; Sobolev space; Lax-Milgram; Finite element method; Newmark method.

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1. INTRODUCTION

Deep foundations on piles, widely used in the construction of structures, are experiencing increasing development. The progress made in dimensioning methods, technological innovations in the construction of piles, the increasingly mediocre quality of the land left to builders and the large dimensions of the structures are at the origin of this development. In practice, these structures are dimensioned in order to take both axial and lateral forces and moments [1].

The behavior of piles, under vertical and lateral loads has been studied for several years with full-scale tests [2], tests on centrifuge models [3], theoretical analyzes [4] as well as numerical simulations.

Today, although complex, the study of the mechanical behavior of piles has already been the subject of several research works [5, 6, 7, 8, 9, 10, 11, 12]. These have resulted in modeling and calculation methods used for the design of such structures. Among these calculation methods we can cite that of finite differences and that of finite elements. Numerical methods by finite elements or by finite difference make it possible to solve soil-pile interaction problems with more rigor while including the effects of loadings on the interface, of the inclination of the piles and of the stiffness of the soil. It also turns out that the analytical approach remains more complex and has limits. It is in this context that we were interested in the numerical calculation of piles under lateral loads taking into account the soil-pile interaction. In our previous studies [10, 11], we worked on stationary behavioral models of soil-pile interaction. However, this study, on a dynamic model of soil-pile interaction, aims primarily to establish first the existence and uniqueness of the solution of the problem posed from the Lax-Milgram theorem and the spectral theory and then to present a rigorous numerical method based on the finite element method and the Newmark method in order to determine the responses of the pile at each instant by taking into account a large number of parameters relative to soils and piles.

2. PRESENTATION OF THE MODEL

The dynamic response of Pasternak model is defined as follows.
Find: $u : \Omega =]0, l[\times\mathbb{R}_+^* \to \mathbb{R}$ such that:
(2.1)

$$\begin{cases} \mathbf{m}\dfrac{\partial^2 u(z,t)}{\partial t^2} + E_p I_p \dfrac{\partial^4 u(z,t)}{\partial z^4} - G_p \dfrac{\partial^2 u(z,t)}{\partial z^2} + K_p u(z,t) = P(z,t),\ \forall t > 0, \forall z \in]0,l[\\ u(z,0) = u_0(z),\ \forall\ z \in]0,l[\\ \dfrac{\partial u(z,0)}{\partial t} = u_1(z),\ \forall\ z \in]0,l[\\ u(0,t) = \dfrac{\partial u(0,t)}{\partial z} = 0,\ \forall t > 0 \\ \dfrac{\partial u(l,t)}{\partial z} = 0,\ \forall t > 0 \\ \dfrac{\partial^3 u(l,t)}{\partial z^3} = \frac{H}{E_p I_p},\ \forall t > 0. \end{cases}$$

Where, $u(x,t)$ is the longitudinal deflection of the beam in terms of m; z is the space coordinate measured along the length of the beam in m; t is the time in (s); E_pI_p is the flexural rigidity of the beam in $(N.m^2)$; **m** is the mass per unit length of the beam in (kg/m); $P(z,t)$ is the applied external load per unit length in (N/m); K_p is the spring constant (the first parameter) of the soil per unit beam length in terms of (N/m^2), and G_p is the shear modulus (the second parameter) of the soil in (N/m^2) [12]; H the head trenchant effort of the free pile in (N). We see the description of the soil-pile interaction in the following figure 1.

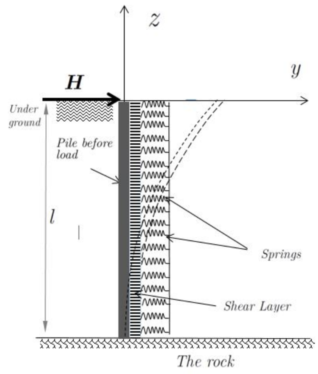

Figure 1: Pile under lateral loading.

First, we are interested in the existence and uniqueness of the solutions to the problem (2.1).

2.1. **Existence of a Hilbert basis of $\mathbb{L}^2(\Omega)$.** Since the (2.1) problem can be associated with an eigenvalue problem, we will solve it using a Hilbert basis of $\mathbb{L}^2(\Omega)$. Thus demonstrating the existence of a Hilbert basis of $\mathbb{L}^2(\Omega)$ amounts to verifying the hypotheses of the **[Theorem 7.2.8]** [14].
Consider the following problem:

$$(2.2) \qquad \begin{cases} E_pI_p\dfrac{d^4u(z)}{dz^4} - G_p\dfrac{d^2u(z)}{dz^2} + K_pu(z) = P(z) \ ,\forall z \in]0,l[, \\ u(0) = \dfrac{du(0)}{dz} = 0, \\ \dfrac{du(l)}{dz} = 0, \\ \dfrac{d^3u(l)}{dz^3} = \frac{H}{E_pI_p}. \end{cases}$$

We pose $w(z) = u(z) - \frac{H}{6E_pI_p}z^3 + \frac{Hl}{4E_pI_p}z^2$ and (2.2) becomes the following boundary problem:

$$(2.3) \qquad \begin{cases} E_pI_p\dfrac{d^4w(z)}{dz^4} - G_p\dfrac{d^2w(z)}{dz^2} + K_pw(z) = G(z) \ ,\forall z \in]0,l[, \\ w(0) = \dfrac{dw(0)}{dz} = 0, \\ \dfrac{dw(l)}{dz} = 0, \\ \dfrac{d^3w(l)}{dz^3} = 0. \end{cases}$$

with $G(z) = P(z) - K_p(\frac{H}{6E_pI_p}z^3 - \frac{Hl}{4E_pI_p}z^2) + G_p(\frac{H}{E_pI_p}z - \frac{Hl}{2E_pI_p})$
First we prove that (2.3) admits a unique solution.

Lemma 2.1. *According to the Lax-Milgram theorem the problem* (2.3) *admits a unique solution* $w \in V = \{v \in \mathbf{H^2}(\Omega); v(0) = v'(0) = v'(l) = 0\}$ *verifying the following variational* *formulation:*

$$a(w,v) = L(v)\ \forall\, v \in\ V \tag{2.4}$$

with

$$a(w,v) = E_pI_p\int_0^l \frac{d^2w(z)}{dz^2}\frac{d^2v(z)}{dz^2}dz\ + G_p\int_0^l \frac{dw(z)}{dz}\frac{dv(z)}{dz}dz\ + K_p\int_0^l w(z)v(z)dz$$

and

$$L(v) = \int_0^l G(z)v(z)dz$$

we define the space $\mathbb{L}^2(\Omega)$ provided with the scalar product:

$$\langle f, v\rangle = \int_0^l f(z)v(z)dz \text{ for all } f \text{ and } v \in \mathbb{L}^2(\Omega).$$

and the space V with the reduced norm

$$\|w\|_V = \|w^{(2)}\|_{\mathbb{L}^2(\Omega)} \text{ for all } w \in V.$$

Proof. V is a closed subspace of $\mathbf{H^2}(\Omega)$ therefore it is a Sobolev space in addition

$$|a(u,v)| \leq (E_pI_p + G_pl^2 + K_pl^4)\|u\|_V\|v\|_V$$

implies a is continuous [11], we also have

$$a(u,u) \geq E_pI_p\|u\|_V^2$$

then a is coercive and L is linear by definition and the inequality

$$|L(v)| \leq l^2\|G\|_{\mathbb{L}^2(\Omega)}\|v\|_V$$

shows it is continuous therefore the problem admits a unique solution according to Lax-Milgram theorem.

□

So we can define our operator as follows:

$$\begin{array}{rccc} \mathcal{L}: & \mathbb{L}^2(\Omega) & \rightarrow & V \\ & g & \mapsto & \mathcal{L}g \end{array} \tag{2.5}$$

with $\mathcal{L}g$ the solution of the equation (2.3). In other words, the operator $\mathcal{L}$ is defined by:

$$\mathcal{L}g \in V \text{ such that } a(\mathcal{L}g, v) = \langle g, v\rangle_{\mathbb{L}^2(\Omega)} \text{ for all } v \in V \tag{2.6}$$

Now, the objective is to show that the operator $\mathcal{L}$ thus defined is linear continuous, self-adjoint, compact and definite-positive.

Lemma 2.2. *The operator $\mathcal{L}$ defined in* (2.5) *is continuous linear, self-adjoint, compact and definite-positive.*

Proof. i) **Linearity of $\mathcal{L}$**

The linearity of $\mathcal{L}$ defined in (2.5) is a consequence of Lemma 2.1.

ii) **Continuity of $\mathcal{L}$**
By taking $v = \mathcal{L}g$ in (2.6), we obtain thanks to the coercivity of a and the continuous injection of V into $\mathbb{L}^2(\Omega)$ and from $\mathbb{L}^2(\Omega)$ to $\mathbb{L}^1(\Omega)$:

$$E_pI_p\|\mathcal{L}g\|_V^2 \leq a(\mathcal{L}g, \mathcal{L}g) = \langle g, \mathcal{L}g\rangle_{\mathbb{L}^2(\Omega)} \tag{2.7}$$

now

$$\langle g, \mathcal{L}g\rangle_{\mathbb{L}^2(\Omega)} \leq \|g\|_{\mathbb{L}^2(\Omega)}\|\mathcal{L}g\|_{\mathbb{L}^2(\Omega)} \leq l^2\|g\|_{\mathbb{L}^2(\Omega)}\|\mathcal{L}g\|_V$$

and so we get

$$E_pI_p\|\mathcal{L}g\|_V^2 \leq l^2\|g\|_{\mathbb{L}^2(\Omega)}\|\mathcal{L}g\|_V$$
$$\Longrightarrow \|\mathcal{L}g\|_V \leq \frac{l^2}{E_pI_p}\|g\|_{\mathbb{L}^2(\Omega)}$$

hence the continuity of$\mathcal{L}$.

iii) **Self-adjoint of $\mathcal{L}$**
By taking $v = \mathcal{L}h$ with $g, h \in \mathbb{L}^2(\Omega)$ in (2.6), we obtain thanks to the symmetry of a:

$$\begin{array}{rcl} \langle g, \mathcal{L}h\rangle_{\mathbb{L}^2(\Omega)} & = & a(\mathcal{L}g, \mathcal{L}h) \\ & = & a(\mathcal{L}h, \mathcal{L}g) \quad = \quad \langle h, \mathcal{L}g\rangle_{\mathbb{L}^2(\Omega)} \end{array}$$

iv) **Compactness of $\mathcal{L}$**
Let $\mathcal{I} : V \to \mathbb{L}^2(\Omega), g \mapsto \mathcal{I}g = g$ the injection operator and $\mathcal{L}g$ the operator defined in (2.5). So we have: $\mathcal{I} \circ \mathcal{L}$ defined from $\mathbb{L}^2(\Omega)$ to value in $\mathbb{L}^2(\Omega)$.

$$\mathcal{L}g \in V,\ \forall g \in \mathbb{L}^2(\Omega) \Rightarrow \mathcal{L}g = (\mathcal{I} \circ \mathcal{L})g,\ \forall g \in \mathbb{L}^2(\Omega)$$

and since $\mathcal{I}$ is compact then $\mathcal{L}$ is compact as a compound of compact and continuous operator.

v) **$\mathcal{L}$ is definite-positive**

it comes from the coercivity of a, indeed:

$$\langle g, \mathcal{L}g\rangle_{\mathbb{L}^2(\Omega)} = a(\mathcal{L}g, \mathcal{L}g) \geq E_pI_p\|\mathcal{L}g\|_V^2 > 0, \forall 0 \neq g \in \mathbb{L}^2(\Omega) \tag{2.8}$$

□

The hypotheses of the **[Theorem 7.2.8]** [14] are verified therefore the eigenvalues of $\mathcal{L}$ form a sequence $(\lambda_k)_{k\geq 1}$ of real numbers strictly positive which tend to 0, and there exists a Hilbertian basis $(u_k)_{k\geq 1}$ of V formed by eigenvectors of $\mathcal{L}$. Therefore, we get the spectral decomposition of any element v of V.

3. VARIATIONAL FORMULATION OF THE PROBLEM

We obtain the following variational formulation of the problem (2.1), find $u(t)\ :]0, T[\to V$ such that:

$$\left\{ \begin{array}{l} \frac{d^2}{dt^2}\langle u(t), v\rangle_{\mathbb{L}^2(\Omega)} + a(u(t), v) = \langle P(t), v\rangle_{\mathbb{L}^2(\Omega)}, \forall v \in V,\ 0 < t < T \\ u(t=0) = u_0; \frac{du}{dt}(t=0) = u_1 \end{array} \right. \tag{3.1}$$

with

$$V = \{v \in H^2(\Omega); v(0) = v'(0) = v'(l) = 0\};$$
$$P(t)\ :]0, T[\to \mathbb{L}^2(\Omega);$$
$$a(u(t), v) = E_pI_p \int_\Omega u''(z,t)v''(z)\,dz + G_p \int_\Omega u'(z,t)v'(z)\,dz + K_p \int_\Omega u(z,t)v(z)\,dz;$$

$$L(v) = \langle P(t), v\rangle_{\mathbb{L}^2(\Omega)} = \int_\Omega P(z,t)v(z)\,dz - \frac{H}{E_p I_p}v(l)$$

Remark 3.1. We denote by $u(z,t)$ the value $u(t)(z)$, $P(z,t)$ the value $P(t)(z)$.

3.1. **Semi-discretization in space.** Let N_h be the number of interior points of the discretization and $h = \frac{l}{N_h+1}$ the discretization step. We construct an internal variational approximation by introducing a subspace V_h of V of finite dimension. V_h will be a finite element subspace $\mathbb{P}_3$ on the discretization. The semi-discretization of (3.1) is therefore the following variational approximation: We look for $u_h(t)$ function of $]0,T[$ with values in V_h such that:

$$\text{(3.2)} \qquad \begin{cases} \frac{d^2}{dt^2}\langle u_h(t), v_h\rangle_{\mathbb{L}^2(\Omega)} + a(u_h(t), v_h) = \langle f_h(t), v_h\rangle_{\mathbb{L}^2(\Omega)}, \forall v_h \in V_h,\ 0<t<T \\ u_h(t=0) = u_{0,h};\ \frac{du_h}{dt}(t=0) = u_{1,h} \end{cases}$$

where $u_{0,h} \in V_h$ is an approximation of the initial data u_0 and $u_{1,h} \in V_h$ is also an approximation of the initial data u_1.
We introduce the basis $(w^{(i)}, z^{(j)})$ of V_h ([11]) for all $1 \le i \le N_h+1$ and $1 \le j \le N_h$.We are looking for $u_h(t)$ in the form

$$u_h(t) = \sum_{i=1}^{N_h+1} U_i^h w^{(i)}(z) + \sum_{j=1}^{N_h} (U_j^h)' z^{(j)}(z).$$

We denote by U^h the vector of coordinates of u_h in the same way we have:

$$u_{0,h}(t) = \sum_{i=1}^{N_h+1} U_i^{0,h} w^{(i)}(z) + \sum_{j=1}^{N_h} (U_j^{0,h})' z^{(j)}(z);$$

$$u_{1,h}(t) = \sum_{i=1}^{N_h+1} U_i^{1,h} w^{(i)}(z) + \sum_{j=1}^{N_h} (U_j^{1,h})' z^{(j)}(z)$$

where, $U^{0,h}$ denotes the vector of coordinates of $u_{0,h}$ and $U^{1,h}$ denotes the vector of coordinates of $u_{1,h}$. and (3.2) becomes for all $1 \le i \le N_h+1$ and $1 \le j \le N_h$

$$\text{(3.3)} \qquad \begin{cases} \frac{d^2}{dt^2}\langle u_h(t), w^{(i)}\rangle_{\mathbb{L}^2(\Omega)} + a(u_h(t), w^{(i)}) = \langle f_h(t), w^{(i)}\rangle_{\mathbb{L}^2(\Omega)}, \forall 0<t<T \\ \frac{d^2}{dt^2}\langle u_h(t), z^{(j)}\rangle_{\mathbb{L}^2(\Omega)} + a(u_h(t), z^{(j)}) = \langle f_h(t), z^{(j)}\rangle_{\mathbb{L}^2(\Omega)}, \forall 0<t<T \end{cases}$$

hence, the variational approximation (3.2) is equivalent to the following linear system of ordinary differential equations with constant coefficients:

$$\text{(3.4)} \qquad \begin{cases} \mathcal{M}_h \frac{d^2U^h}{dt^2} + \mathcal{K}_h U^h = B^h \\ U^h(0) = U^{0,h};\ \frac{dU^h}{dt}(0) = U^{1,h} \end{cases}$$

the mass matrix is defined by:

$$\mathcal{M}_h = \begin{pmatrix} M^{11} & M^{12} \\ M^{21} & M^{22} \end{pmatrix}$$

with

$$M^{11} = (\langle w^{(i)}, w^{(j)}\rangle)_{1\le i,j\le N_h+1};\ M^{12} = (\langle z^{(i)}, w^{(j)}\rangle)_{1\le i\le N_h; 1\le j\le N_h+1};$$
$$M^{21} = (\langle w^{(i)}, z^{(j)}\rangle)_{1\le i\le N_h+1; 1\le j\le N_h};\ M^{22} = (\langle z^{(i)}, z^{(j)}\rangle)_{1\le i,j\le N_h}.$$

The stiffness matrix is defined by:

$$\mathcal{K}_h = \begin{pmatrix} K^{11} & K^{12} \\ K^{21} & K^{22} \end{pmatrix}$$

with

$$K^{11} = (a(w^{(i)}, w^{(j)}))_{1\leq i,j\leq N_h+1}; K^{12} = (a(z^{(i)}, w^{(j)}))_{1\leq i\leq N_h;1\leq j\leq N_h+1};$$

$$K^{21} = (a(w^{(i)}, z^{(j)}))_{1\leq i\leq N_h+1;1\leq j\leq N_h}; K^{22} = (a(z^{(i)}, z^{(j)}))_{1\leq i,j\leq N_h}.$$

and the matrix of the second member is defined by:

$$B^h = \begin{pmatrix} B^1 \\ B^2 \end{pmatrix}$$

with $B^1 = (L(w^{(i)}))_{1\leq i\leq N_h+1}$ and $B^2 = (L(z^{(j)}))_{1\leq j\leq N_h}$

3.2. **Total discretization in space-time.** We decompose the time interval $[0, T]$ into N time step $\Delta t = \frac{T}{N}$, we set $t_n = n\Delta t$ for $n \in \{0, 1 \ldots, N\}$ and we denote by U_n^h the approximation of $U^h(t_n)$. To calculate numerically approximate solutions of ((3.4)) we use the following Newmark time-stepping method:

$$\text{(3.5)} \qquad \begin{cases} \mathcal{M}_h\ddot{U}_{n+1}^h + \mathcal{K}_hU_{n+1}^h = B_{n+1}^h \\ \dot{U}_{n+1}^h = \dot{U}_n^h + \Delta t\left((1-\delta)\ddot{U}_n^h + \delta\ddot{U}_{n+1}^h\right) \\ U_{n+1}^h = U_n^h + \Delta t\dot{U}_n^h + \frac{(\Delta t)^2}{2}\left((1-2\theta)\ddot{U}_n^h + 2\theta\ddot{U}_{n+1}^h\right) \end{cases}$$

Where, the real parameters δ and θ will be fixed as follows $0 \leq \delta \leq 1$; $0 \leq \theta \leq \frac{1}{2}$ [14], Δt is the time-step fixed later. So inserting the formula for U_{n+1}^h into $\mathcal{M}_h\ddot{U}_{n+1}^h + \mathcal{K}_hU_{n+1}^h = B_{n+1}^h$ at time t_{n+1} we obtain from (3.5) the following schema:

$$\text{(3.6)} \qquad \begin{cases} \ddot{U}_{n+1}^h = (\mathcal{M}_h + \theta(\Delta t)^2K_h)^{-1}\left(B_{n+1}^h - \mathcal{K}_h[U_n^h + \Delta t\dot{U}_n^h + \frac{(\Delta t)^2}{2}(1-2\theta)\ddot{U}_n^h]\right) \\ \dot{U}_{n+1}^h = \dot{U}_n^h + \Delta t\left((1-\delta)\ddot{U}_n^h + \delta\ddot{U}_{n+1}^h\right) \\ U_{n+1}^h = U_n^h + \Delta t\dot{U}_n^h + \frac{(\Delta t)^2}{2}\left((1-2\theta)\ddot{U}_n^h + 2\theta\ddot{U}_{n+1}^h\right) \end{cases}$$

and the acceleration

$$\text{(3.7)} \qquad \ddot{U}_n^h = \mathcal{M}_h^{-1}(B_n^h - \mathcal{K}_hU_n^h)$$

follows from the equation $\mathcal{M}_h\ddot{U}_n^h + \mathcal{K}_hU_n^h = B_n^h$. Knoving $U_n^h, \dot{U}_n^h, \ddot{U}_n^h$ we find $U_{n+1}^h, \dot{U}_{n+1}^h, \ddot{U}_{n+1}^h$.

3.3. **Parameters of the simulation.** the calculation of the coefficients of the matrices $\mathcal{M}_h$, $\mathcal{K}_h$ and B_h is carried out in the same way as in ([11]). The parameters of simulation [13, 12] are as follows:

Table 3.1: Parameters of the soil-pile interaction

l(m)	$E_pI_p(MN.m^2)$	$K_p(kN/m^2)$	$G_p(kN/m^2)$	**m**(kg/m)	$P(kN/m)$
20	3000	100	37393	48.2	200
20	3000	552	186966	48.2	200
20	3000	1103	373933	48.2	200
40	3000	87	63059	48.2	200
40	3000	437	315296	48.2	200
40	3000	874	630591	48.2	200

And we choose $T = 1s$, $dt = 0.01s$, $\delta = 0.6$ and $\theta = 0.4$ for the Newmark Scheme.

4. NUMERICAL RESULTS

The following pictures display differents shapes of deformation of the pile with respect to several parameters G_p and K_p of the soil at any time t_n.

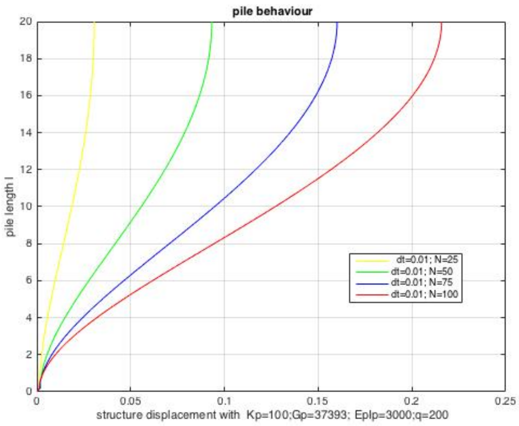

Figure 2: Behaviour of the Pile of length $l = 20m$ at t_{25},t_{50}, t_{75}, t_{100} for $G_p = 37393$ and $K_p = 100$

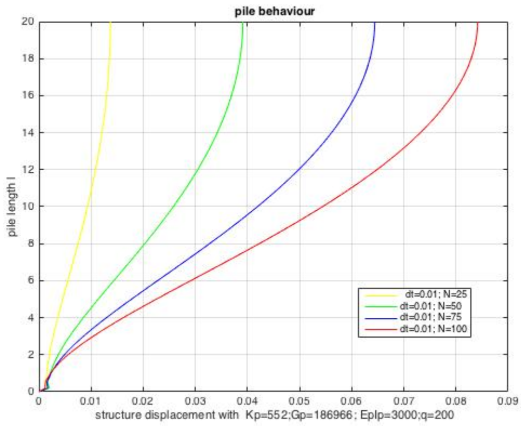

Figure 3: Behaviour of the Pile of length $l = 20m$ at t_{25},t_{50}, t_{75}, t_{100} for $G_p = 186966$ and $K_p = 552$

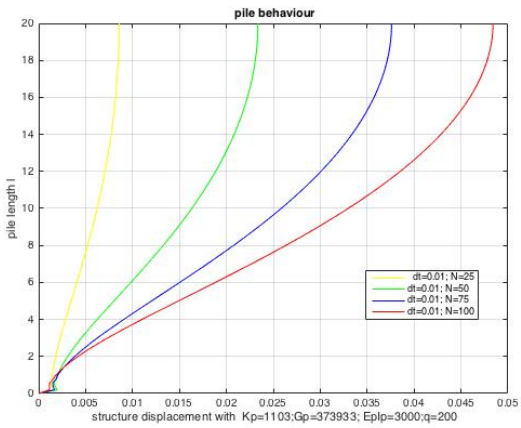

Figure 4: Behaviour of the Pile of length $l = 20m$ at t_{25},t_{50}, t_{75}, t_{100} for $G_p = 373933$ and $K_p = 1103$

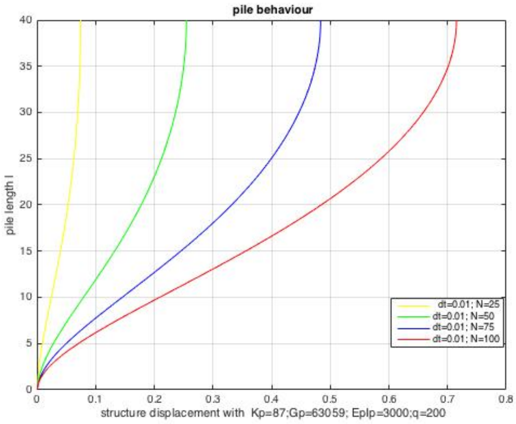

Figure 5: Behaviour of the Pile
of length $l = 40m$ at t_{25},t_{50}, t_{75}, t_{100}
for $G_p = 63059$ and $K_p = 87$

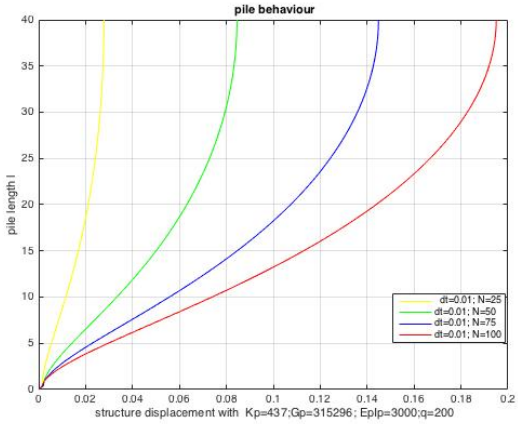

Figure 6: Behaviour of the Pile
of length $l = 40m$ at t_{25},t_{50}, t_{75}, t_{100}
for $G_p = 315296$ and $K_p = 437$

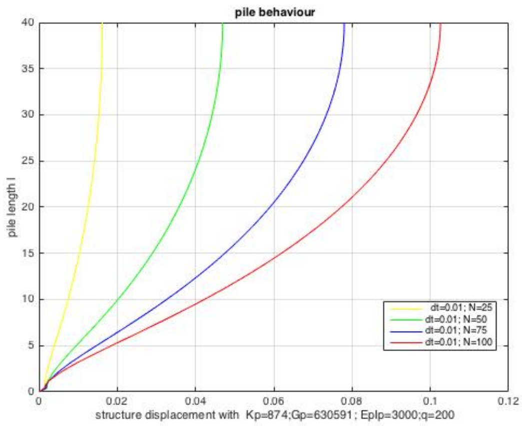

Figure 7: Behaviour of the Pile
of length $l = 40m$ at t_{25},t_{50}, t_{75}, t_{100}
for $G_p = 630591$ and $K_p = 874$

5. DISCUSSION OF THE RESULTS

As we know the soil-structure interaction (SSI) of the Pasternak model is essentially based on the two mechanical parameters which are: Pasternak shear modulus G_p and the Pasternak reaction coefficient K_p. The parametric study of our (SSI) model was emphasized on the variability of these parameters and the length of the sheet l. From figures 2, 3 and 4, it can be seen that the horizontal deflection of the pile depends on the parameters K_p and G_p at any time t_n. Indeed, the deflections vary in a decreasing way according to the values of Kp and Gp. These displacements are more influenced by the shear modulus G_p. We deduce from this that for the behavioral model, more the shear layer is incompressible (the values of the parameters K_p and G_p very large) less the pile moves in the soil mass. From figures 5, 6 and 7, for a more flexible

pile (l very large) and for incompressible shear layers (G_p very large), we obtain for smaller values of K_p, the same deflection forms but with greater amplitudes at any time t_n. In the previous studies [10, 11] we have established in the case of the stationary model of pile under lateral load that to reduce the deformations of the pile it is necessary to increase both the parameters G_p and K_p of the soil. In this work, we also establish that at each moment by increasing the parameters of the soil the deformation of the pile decreases.
Finally, we can keep that in order to reduce the deformation of the pile under latteral load and a trenchant effort on the free head, we must take into account soil parameters.

6. CONCLUSION

In this work, we use on the one hand mathematical analysis results to prove the existence and uniqueness of the solution and on the other hand we use finite element method to determine an approximate solution to partial differential equation. Moreover, numerical simulations show us the pious deformation and the influence of soil parameters on the structure in relation to time. We also observe that when soil parameters K_p, G_p increase then the displacement of the pious decreases even if the number of iterations N in time increases. It turned out from our study that a variability in the parameters of the shear layer (in particular the shear modulus) has a considerable influence on the displacements over time. This work confirms and reinforces the previous results obtained in the analysis of the deflection of the stationary Pasternak model. Finally, the authors believe that the present investigations could help engineers and researchers in studying and designing shell structures and a more suitable foundation model for obtaining the optimal dynamic response.

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