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ON SUBSPACE-SUPERCYCLIC OPERATORS

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ABSTRACT. In this paper, we prove that supercyclic operators are subspace-supercyclic and by this we give a positive answer to a question posed in (L. Zhang, Z. H. Zhou, Notes about subspace-supercyclic operators, *Ann. Funct. Anal.*, **6** (2015), pp. 60–68). We give examples of subspace-supercyclic operators that are not subspace-hypercyclic. We state that if T is an invertible supercyclic operator then T^n and T^{-n} is subspace-supercyclic for any positive integer n. We give two subspace-supercyclicity criteria. Surprisingly, we show that subspace-supercyclic operators exist on finite-dimensional spaces.

Key words and phrases: Supercyclic operators; Subspace-supercyclic operators; Subspace-hypercyclic operators.

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1. INTRODUCTION AND PRELIMINARIES

Let X be a Banach space. In this paper, we use the symbol $B(X)$ for the bounded linear operator on X and briefly we call its elements as operators. We say an operator $T \in B(X)$ is hypercyclic, if there exists $x \in X$ such that $orb(T, x)$ is dense in X, where $orb(T, x) = \{x, Tx, ..., T^n x, ...\}$. Hypercyclic operators are interesting for mathematicians because they are related to wellknown invariant closed subspace problem. You can see [3] and [6] for more information. Another interesting matter in dynamical systems is supercyclicity. The concept of supercyclic operators was introduced by Hilden and Wallen in [8]. We say an operator $T \in B(X)$ is supercyclic if there exists $x \in X$ such that $\{\lambda x, \lambda Tx, ..., \lambda T^n x, ... : \lambda \in \mathbb{C}\}$ is dense in X. In other words

$$\overline{\{\lambda T^n x : n \geq 0, \lambda \in \mathbb{C}\}} = \overline{\mathbb{C}.orb(T, x)} = X.$$

It is clear by definition that hypercyclic operators are supercyclic.

We say an operator T is $\mathbb{R}^+$-supercyclic if there is $x \in X$ such that $\{tT^n x : n \geq 0, t > 0\}$ is dense in X. It is obvious that $\mathbb{R}^+$-supercyclic operators are supercyclic but there are supercyclic operators that are not $\mathbb{R}^+$-supercyclic([4]). Leon-Saavedra and Muller proved in [10] that supercyclicity and $\mathbb{R}^+$-supercyclicity are equivalent where $\sigma_p(T^*) = \phi$, where $\sigma_p(T^*)$ is the point spectrum of T^*.

Theorem 1.1. *(*[10]*) Let $T \in B(X)$ be such that $\sigma_p(T^*) = \phi$ and let $x \in X$. Then x is supercyclic for T if and only if the set $\{tT^n x : n \geq 0, t > 0\}$ is dense in X.*

In 2011, Madore and Martinez-Avendano defined the concept of subspace-hypercyclicity. We say an operator is subspace-hypercyclic with respect to a closed and non-zero subspace M of X, if there is $x \in X$ such that $orb(T, x) \cap M$ is dense in M ([11]). They make examples of subspace-hypercyclic operators that are not hypercyclic. Also, they proved in [11] that subspace-hypercyclic operators do not exist on finite-dimensional spaces.

Zhao, Shu and Zhou in [17] defined subspace-supercyclic operators. We say an operator is subspace-supercyclic with respect to a closed and non-zero subspace M of X if there is $x \in X$ such that

$$\overline{\mathbb{C}.orb(T, x) \cap M} = M.$$

It is clear that subspace-hypercyclic operators are subspace-supercyclic. Authors in [17] mentioned some sufficient conditions for subspace-supercyclicity and a subspace-supercyclicity criteria. The following theorem is one of their theorems.

Theorem 1.2. *(*[17]*) Let $T \in B(X)$ and let M be a non-zero and closed subspace of X. If for any pair of non-empty and open sets $U \subseteq M$ and $V \subseteq M$, there exists $n \in \mathbb{N}$ and $\lambda \in \mathbb{C}$ such that $(\lambda T^n)^{-1}(U) \cap V \neq \phi$ and $T^n(M) \subseteq M$, then T is subspace-supercyclic with respect to M.*

Authors in [17] presented examples of subspace-supercyclic operators. For example, if $T : X \to X$ is a supercyclic operator, then $T \oplus I : X \oplus X \to X \oplus X$ is a subspace-supercyclic but not a supercyclic operator. They also, showed that an operator may be subspace-supercyclic with respect to a finite-dimensional subspace M.

One can see [15] for more information about subspace-supercyclic operators.

Zhang and Zhou in [16] ask question(Question 2.14) that if T is a supercyclic operator, is there a non-trivial subspace M such that T is M-supercyclic? In Section 2 of this paper, we prove that the answer to their question is positive and we show that supercyclic operators are subspace-supercyclic. We give examples of subspace-supercyclic operators that are not subspace-hypercyclic. We present conditions that under them both T and T^{-1} are subspace-supercyclic. We prove that if T is an invertible supercyclic operator then T^n and T^{-n} are subspace-supercyclic for any positive integer n and by this we give partial answer to Question 1.2 of [16]. In Section 3, we give some subspace-supercyclicity criteria and make some examples of subspace-supercyclic operators by using these criteria. Surprisingly, we show in Section 4, that subspace-supercyclic operators exist on finite-dimensional spaces.

2. SUPERCYCLIC OPERATORS ARE SUBSPACE-SUPERCYCLIC

Zhang and Zhou asked this question in [16, Question 2.14] that if T is a supercyclic operator, is there a closed and non-trivial subspace M of X such that T is subspace-supercyclic with respect to it? In the following, we prove that the answer to their question is positive. First, we recall a theorem from [2].

Theorem 2.1. *If A is a dense subset of a Banach space X, then there is a non-trivial closed subspace M of X such that $A \cap M$ is dense in M.*

Now we state our main theorem.

Theorem 2.2. *Let $T \in B(X)$ be a supercyclic operator. Then T is subspace-supercyclic with respect to a closed and non-trivial subspace M of X.*

Proof. By hypothesis, T is supercyclic. So there exists $x \in X$ such that $\overline{\{\lambda T^n x : n \geq 0, \lambda \in \mathbb{C}\}} = X$. If we consider $A := \{\lambda T^n x : n \geq 0, \lambda \in \mathbb{C}\}$ then by Theorem 2.1, there exists a closed and non-trivial subspace M of X such that $\overline{A \cap M} = M$. In the other words, T is subspace-supercyclic with respect to M. ∎

Now, we give an example of a subspace-supercyclic operator with a dense set of subspace-supercyclic vectors that is not a subspace-hypercyclic operator.

Example 2.1. *Let B be the unilateral backward shift on l^p, $1 \leq p < \infty$, that defined by $B(e_n) = e_{n-1}$ and $B(e_0) = 0$, where $(e_n)_{n \geq 0}$ is the canonical basis of l^p. Rolewiecz proved in* [13] *that λB is hypercyclic for any λ with $|\lambda| > 1$. Hence, B is supercyclic and by Theorem 2.2, is subspace-supercyclic. Also, B has a dense set of subspace-supercyclic vectors in l^p. Since for an arbitrary λ with $|\lambda| > 1$, λB is hypercyclic and hence has a dense set of hypercyclic vectors in l^p. It is not hard to see that any hypercyclic vector for λB is a subspace-supercyclic vector for B.*

But B is not hypercyclic nor subspace-hypercyclic since it is a contradiction.

We say an operator T is M-transitive if for any non-empty relatively open sets $U \subseteq M$ and $V \subseteq M$, there exists a non-negative integer n such that $T^{-n}U \cap V$ is non-empty and $T^n(M) \subseteq M$([11]). Subspace-transitive operators are subspace-supercyclic, since as it proved in [11, Theorem 3.5] subspace-transitive operators are subspace-hypercyclic. In the next example, we give an example of a subspace-supercyclic operator that is not subspace-transitive.

Example 2.2. *Let B be the backward shift on l^p, $p \geq 1$ and let $\{n_k\}_{k=1}^{\infty}$ and $\{m_k\}_{k=1}^{\infty}$ be two strictly increasing sequences of positive integers such that $n_k < m_k < n_k + 1$. Let M be the closed linear subspace that is generated by $\{e_j : n_k \leq j \leq m_k, k \geq 1\}$, where $\{e_j\}$ is the canonical basis for l^p. Le showed in* [9, Lemma 2.8 and Lemma 2.9] *that $T = 2B$ is an M-hypercyclic operator but it is not an M-transitive operator.*

So, T is M-supercyclic and is not M-transitive.

Hence, there are subspace-supercyclic operators that are not subspace-transitive.

Ansari proved in [1, Theorem 2] that if a vector x is supercyclic for T, then x is also a supercyclic vector for T^n for any $n \geq 1$. By this fact and Theorem 2.1, we can extend our theorem as follows.

Theorem 2.3. *Let $T \in B(X)$. If x is a supercyclic vector for T, then x is a subspace-supercyclic vector for T^n for any $n \geq 1$.*

Proof. Let x be a supercyclic vector for T. Then $\overline{\mathbb{C}.orb(T,x)} = X$. Let n be a positive integer. By what is said before the theorem, $\overline{\mathbb{C}.orb(T^n,x)} = X$. Now, by Theorem 2.1, there exists a closed and non-trivial subspace M_n of X such that $\overline{\mathbb{C}.orb(T^n,x) \cap M_n} = M_n$. Hence x is an M_n-supercyclic vector for T^n and this completes the proof. ∎

We can rewrite Theorem 2.3, as follows.

Corollary 2.4. *Let $T \in B(X)$ be a supercyclic operator. Then T^n is subspace-supercyclic for any $n \geq 1$.*

Example 2.3. *Let B_W be a weighted backward shift on $l^2(\mathbb{N})$ with a bounded and positive weight sequence $(w_n)_{n\geq 1}$, that defined by*

$$B_W(e_n) = w_n e_{n-1} (n \geq 1) \quad \textit{and} \quad B_W(e_0) = 0.$$

Then B_W is supercyclic[3, Example 1.15]. *Hence, $(B_W)^n$ is subspace-supercyclic for any $n \geq 1$ by Corollary 2.4.*

Theorem 2.3 and Corollary 2.4 lead us to the following question.

Question 1. *Let T be a subspace-supercyclic operator with respect to a closed and non-trivial subspace M. Can we deduce that T^n is M-supercyclic for any positive integer n?*

Zhang and Zhou asked this question in [16] that if T is invertible and subspace-supercyclic, can we deduce that T^{-1} is subspace-supercyclic too? In the following, we give partial answers to this question. First, we recall a corollary from [14].

Corollary 2.5. *(*[14]*) Let $T \in B(X)$ be an invertible operator. If $T \in B(X)$ is supercyclic, then T^{-1} is supercyclic.*

The proof of the next corollary is not hard by using Corollary 2.5 and Theorem 2.2.

Corollary 2.6. *Let $T \in B(X)$ be an invertible and supercyclic operator. Then both T and T^{-1} are subspace- supercyclic.*

Also, by Corollary 2.4, we can state that if T is an invertible supercyclic operator, then T^n and T^{-n} are subspace-supercyclic for any $n \in \mathbb{N}$.

In the next theorem, we give conditions that under them, both T and T^{-1} are subspace-supercyclic.

Theorem 2.7. *Let M be a closed and non-zero subspace of X. Let $T \in B(X)$ be an invertible operator such that, for any pair of non-empty and open sets $U \subseteq M$ and $V \subseteq M$, there exists $n \in \mathbb{N}$ and $\lambda \in \mathbb{C}$ such that $(\lambda T^n)^{-1}(U) \cap V \neq \phi$, $T^n(M) \subseteq M$ and $T^{-n}(M) \subseteq M$. Then, both T and T^{-1} are M-supercyclic.*

Proof. By Theorem 1.2, T is M-supercyclic. It remains that we show that T^{-1} is M-supercyclic. Let $U \subseteq M$ and $V \subseteq M$ be non-empty open subsets of M. By hypothesis, there exists $n \in \mathbb{N}$ and $\lambda \in \mathbb{C}$ such that $(\lambda T^n)^{-1}(V) \cap U \neq \phi$, $T^n(M) \subseteq M$ and $T^{-n}(M) \subseteq M$. So, there exists a vector x such that $x \in (\lambda T^n)^{-1}(V) \cap U$. Hence, $x \in U$ and $x \in (\lambda T^n)^{-1}(V)$. Therefore,

$$\exists y \in V; x = (\lambda T^n)^{-1}(y) = \frac{1}{\lambda} T^{-n}(y).$$

So, we can write $y = T^n(\lambda x) = \lambda^n T^n x$. Therefore $y \in (\frac{1}{\lambda^n} T^{-n})^{-1}(U) \cap V$. Hence, there exists $\mu \in \mathbb{C}$ such that $(\mu T^{-n})^{-1}(U) \cap V \neq \phi$. By hypothesis, $T^{-n}(M) \subseteq M$ and this shows that T^{-1} is M-supercyclic.

∎

We can also, state the following corollary.

Corollary 2.8. *Let $T \in B(X)$ be such that $\sigma_p(T^*) = \phi$. If $\{tT^n x : n \geq 0, t > 0\}$ is dense in X, then x is a subspace-supercyclic vector for T.*

Proof. By Theorem 1.1, $\{\lambda T^n x : n \geq 0, \lambda \in \mathbb{C}\}$ is dense in X. So, there exists a non-trivial and closed subspace M of X such that $\{\lambda T^n x : n \geq 0, \lambda \in \mathbb{C}\} \cap M$ is dense in M. Therefore x is a subspace-supercyclic vector for T. ∎

3. SUPERCYCLIC-SUPERCYCLICITY CRITERIA

Zhao, Shu and Zhou state a subspace-supercyclicity criteria in [17]. In this section, we present two subspace-supercyclicity criteria. The idea of first criteria is given from supercyclicity criteria in [12, Theorem 2.4] and the idea of the second criteria is given from [3, Theorem 1.14].

Theorem 3.1. *(Subspace-supercyclicity Criteria) Let $T \in B(X)$ and let M be a closed and non-zero subspace of X. Suppose that there exists a strictly increasing sequence $\{n_k\}$ such that $T^{n_k}(M) \subseteq M$. Consider there is an strictly increasing sequence $\{\lambda_{n_k}\}$ of positive integers and there exist dense subsets Z and Y of M and a mapping $S : Y \to Y$ such that:*

(i) $||\lambda_{n_k} T^{n_k} z|| \to 0$ *for any* $z \in Z$.
(ii) $||\frac{1}{\lambda_{n_k}} S^{n_k} y|| \to 0$ *for any* $y \in Y$.
(iii) $TS = I$ *on* Y.

Then T is M-supercyclic.

Proof. First, note that by (iii) and induction we can conclude that $T^n S^n = I$ on Y. Since we have $TS = I$ on Y and if we consider $T^k S^k = I$ on Y, then for any $y \in Y$,

$$T^{k+1} S^{k+1}(y) = T^k TSS^k(y) = T^k TS(S^k(y)) = T^k S^k(y) = y.$$

Now, let $U \subseteq M$ and $V \subseteq M$ be two open and non-empty sets. By hypothesis, Z and Y are dense in M. So, there exist $v \in Z \cap V$ and $u \in Y \cap U$. Since V and U are open, we can find $\varepsilon > 0$ such that

$$B(v, \varepsilon) \cap M \subseteq V \quad \text{and} \quad B(u, \varepsilon) \cap M \subseteq U. \tag{3.1}$$

Also, by conditions (i), (ii) and (iii) we can find large enough n_k such that

$$||\lambda_{n_k} T^{n_k} v|| < \frac{\varepsilon}{2}, \quad ||\frac{1}{\lambda_{n_k}} S^{n_k} u|| < \frac{\varepsilon}{2} \quad \text{and} \quad T^{n_k} S^{n_k}(u) = u. \tag{3.2}$$

Consider $w = v + \frac{1}{\lambda_{n_k}} S^{n_k} u$. It is not hard to see that $w \in M$, since $v \in M$ and $S^{n_k} u \in M$. Also, by (3.2)

$$||w - v|| = ||\frac{1}{\lambda_{n_k}} S^{n_k} u|| < \frac{\varepsilon}{2}.$$

So, we can deduce from (3.1) that $w \in V$. Hence, by (3.2)

$$T^{n_k} w = T^{n_k} v + \frac{1}{\lambda_{n_k}} T^{n_k} S^{n_k}(u) = T^{n_k} v + \frac{1}{\lambda_{n_k}} u.$$

We have $\lambda_{n_k} T^{n_k} w = \lambda_{n_k} T^{n_k} v + u$. Therefore another by (3.2),

$$||\lambda_{n_k} T^{n_k} w - u|| = ||\lambda_{n_k} T^{n_k} v|| < \frac{\varepsilon}{2}.$$

So, $\lambda_{n_k} T^{n_k} w \in U$ by (3.1). Hence $w \in (\lambda_{n_k} T^{n_k})^{-1}(U) \cap V$. Also, we know that $T^{n_k}(M) \subseteq M$. Therefore by Theorem 1.2, T is subspace-supercyclic with respect to M.

∎

By Theorem 3.1, we can say the following example.

Example 3.1. *Let B be the backward shift on l^3 and let*

$$M := \{\{a_n\}_{n=0}^{\infty} \in l^3 : a_{3k} = 0 \quad \textit{for all} \quad k\}.$$

Then B is subspace-supercyclic with respect to M by Theorem 3.1. Since it is sufficient to consider $n_k = 3k$ and consider S be the forward shift on l^3. Also, it is sufficient $\{\lambda_{n_k}\}$ be any strictly increasing sequence of positive integers tending to infinity.

Now, we state our second subspace-supercyclicity criteria as follows.

Theorem 3.2. *Let X be a Banach space and let $T \in B(X)$. Let M be a closed and non-zero subspace of X. Suppose that there exist a strictly increasing sequence $\{n_k\}$ of positive integers and two dense subsets Z and Y of M and a sequence of maps $S_{n_k} : Y \to Y$ such that*

(i) $||T^{n_k}x||||S_{n_k}y|| \to 0$ *for any $x \in Z$ and any $y \in Y$.*
(ii) $T^{n_k}S_{n_k}y \to y$ *for any $y \in Y$.*
(iii) $T^{n_k}(M) \subseteq M$.

Then, T is subspace-supercyclic with respect to M.

Proof. Let $U \subseteq M$ and $V \subseteq M$ be two non-empty open sets. By hypothesis, Z and Y are dense in M. So there exist $x \in Z \cap V$ and $y \in Y \cap U$. Since U and V are open, we can find $\varepsilon > 0$ such that

$$B(x, \varepsilon) \cap M \subseteq V \quad \text{and} \quad B(y, \varepsilon) \cap M \subseteq U. \tag{3.3}$$

For any $k \in \mathbb{N}$ we define λ_k as follows.

$$\lambda_k = \begin{cases} ||T^{n_k}(x)||^{-\frac{1}{2}}||S_{n_k}y||^{\frac{1}{2}}, & \text{if } T^{n_k}(x) \neq 0 \quad and \quad S_{n_k}y \neq 0\ ; \\ 2^k||S_{n_k}y||, & \text{if } T^{n_k}(x) = 0\ ; \\ 2^{-k}||T^{n_k}(x)||^{-1}, & \text{if } S_{n_k}y = 0. \end{cases}$$

It is not hard by definition of λ_k to see that

$$\lambda_k T^{n_k}x \to 0 \quad and \quad \frac{1}{\lambda_k}S_{n_k}y \to 0.$$

So, we can find k large enough such that

$$||\frac{1}{\lambda_k}S_{n_k}|| < \frac{\varepsilon}{2} \quad \text{and} \quad ||\lambda_k T^{n_k}x|| < \frac{\varepsilon}{2}. \tag{3.4}$$

If we consider $w := x + \frac{1}{\lambda_k}S_{n_k}y$, then $w \in M$ since $x \in M$ and $S_{n_k}y \in M$. Now, by (3.3), $w \in V$ since $w - x = \frac{1}{\lambda_k}S_{n_k}$ and by (3.4), $||\frac{1}{\lambda_k}S_{n_k}|| < \frac{\varepsilon}{2}$.

Also,

$$\begin{aligned} &\lambda_k T^{n_k}(w) = \lambda_k T^{n_k}(x + \frac{1}{\lambda_k}S_{n_k}y) \\ &= \lambda_k T^{n_k}(x) + T^{n_k}S_{n_k}(y) \\ &\to y \end{aligned}$$

Hence, we can find k large enough such that

$$||\lambda_k T^{n_k}(w) - y|| < \frac{\varepsilon}{2}.$$

So, $\lambda_k T^{n_k}(w) \in U$ by (3.3) and then $w \in (\lambda_k T^{n_k})^{-1}(U)$. Therefore $w \in (\lambda_k T^{n_k})^{-1}(U) \cap V$. Now, condition (iii) and Theorem 1.2 complete the proof.

∎

Now, the following question arises.

Question 2. *Are the two subspace-supercyclicity criteria that stated in this paper(Theorem 3.1 and Theorem 3.2) are equivalent? Are these theorems equivalent to subspace-supercyclicity criteria that is stated in* [17]*?*

In the next example, we make an M-supercyclic operator T that does not satisfy condition $T^{n_k}(M) \subseteq M$ of subspace-supercyclicity criteria. So, condition $T^{n_k}(M) \subseteq M$ is a sufficient condition but not a necessary condition for subspace-supercyclicity.

Example 3.2. *Let B be the backward shift on l^2. Madore and Martinez-Avendano showed in* [11, Example 3.8] *that λB is subspace-hypercyclic with respect to*

$$M := \{\{a_n\}_{n=0}^{\infty} \in l^2 : a_n = 0 \quad \textit{for} \quad n < m\}.$$

Hence, B is subspace-supercyclic with respect to M.
It is not hard to see that $T^n(M)$ is not a subspace of M for any $n \in \mathbb{N}$.

This example arises a question as follows.

Question 3. *Is there an operator that satisfies condition (i) and (ii) of Theorem 3.2 and not be subspace-supercyclic with respect to M?*

4. FINITE-DIMENTIONAL SPACES

Madore and Martinez-Avendano proved in [11, Theorem 4.10] that, if T is an M-hypercyclic operator, then M can not be finite-dimensional. But it is shown in [17] that an operator can be subspace-supercyclic with respect to a finite-dimensional subspace M.

Also, it is proved in [11, Theorem 4.9] that there are not any subspace-hypercyclic operator on a finite-dimensional Banach space X. But we prove in this section that subspace-supercyclic operators exist on finite-dimensional Banach spaces. First, we recall a theorem from [7].

Theorem 4.1. *Let X be a real separable Banach space. An operator $T \in B(X)$ has supercyclic vectors if and only if $dimX \in \{0, 1, 2\}$ or $dimX = \infty$.*

Similarly, for a complex separable Banach space, an operator has supercyclic vectors if and only if $dimX \in \{0, 1\}$ or $dimX = \infty$([7]).

By Theorem 4.1, we can state our theorem about existence of subspace-supercyclic operators on finite-dimensional spaces.

Theorem 4.2. *Subspace-supercyclic operators exist on finite-dimensional spaces.*

Proof. As we mentioned above, subspace-supercyclic operators exist on finite-dimensional spaces. Let T be a supercyclic operator on a finite-dimensional Banach space X and let x be a supercyclic vector for T. So $A := \{\lambda T^n x : n \geq 0, \lambda \in \mathbb{C}\}$ is dense in X. By Theorem 2.1, there exists a non-trivial and closed subspace M of X such that $\overline{A \cap M} = M$. Hence x is an M-supercyclic vector for T. ∎

In the next example, we present a subspace-supercyclic operator on a finite-dimensional space.

Example 4.1. *An example of subspace-supercyclic operators on finite-dimensional spaces are irrational rotations. A rotation T is defined as follows:*

$$T : \mathbb{T} \to \mathbb{T}, \quad z \to e^{i\alpha} z,$$

where $\mathbb{T} = \{z \in \mathbb{C} : |z| = 1\}$ and $\alpha \in [0, 2\pi)$. Irrational rotations are supercyclic([5]*). Hence, there are subspace-supercyclic by Theorem 2.2.*

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