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THE EFFECT OF HARVESTING ACTIVITIES ON PREY-PREDATOR FISHERY MODEL WITH HOLLING TYPE II IN TOXICANT AQUATIC ECOSYSTEM

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ABSTRACT. This paper discussed prey-predator fishery models, in particular by analysing the effects of toxic substances on aquatic ecosystems. It is assumed in this model, that the prey population is plankton and the predator population is fish. Interaction between the two populations uses the Holling type II function. The existence of local and global critical points of the system are shown and their stability properties are analysed. Furthermore, Bionomic equilibrium and optimal control of harvesting are discussed. Finally, numerical simulations have been carried out to show in the interpretation of results.

Key words and phrases: Dulac's criteria; Fishery model; Holling type II; Bionomic equilibrium; Control optimal.

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1. INTRODUCTION

One of the most significant problems in an aquatic ecological dynamic is the effect of poisons or toxic substances. Toxic substances have a real influence on the growth of living things in a food chain in an aquatic. The influence of toxic substances in the water on fish and other organisms can be in the form of lethality (mortality) and sublethal effects such as disruption of growth, development, reproduction, pathology, physiology, and behavior. This influence can be realized in measured parameters such as the number of dead organisms, hatchability, length and weight gain of fish, etc. Toxic substances come from the disposal of chemical compounds that are organic, inorganic or mineral that pollute into water. Toxic substances in the form of toxic chemicals can originate from industrial activity, mine wastewater, surface erosion at an open pit or the rupture of a chemical tanker in the sea [1].

East Kalimantan is one of the largest coal mining centers in Indonesia. Former coal mining areas that have been flooded will become giant pools because they form basins. The former mining basin that forms a giant pool contains several chemical elements in the form of heavy metals [2]. On the other hand, the giant pool of ex-mining has been functioned by the community as agricultural land for aquaculture. Fish that live in these waters will be polluted by heavy metals because the content of heavy metals in aquatic biota will increase over time which is bioaccumulative. In [3], if the metal content is too high in water, it gives danger and even death in the life of aquatic biota. Thus, further studies are needed to determine the development of aquatic biota life with the impact of polluted waters so that the aquatic ecosystem remains balanced.

Nowadays mathematical modeling of population dynamics has been developed in the field of fisheries. The mathematical model used to describe the dynamics of population growth of living things in the process of prey is called the predator-prey model. An example of a predator-prey model in the world of inland fisheries is carp (C*yprinus carpio*) - plankton, snapper (*Lates calcarifer*) - juvenile, catfish-small fish, etc.

Prey-predator models in the field of fisheries with polluted waters have been widely studied. [4] investigated the harvesting of fishery predator models by presenting toxic effects. [5] studied the predator-prey model of exploited Arowana (*Scleropages, spp.*) Fish populations. In 2013 Yumfei et al [6] presented a predator-prey model with harvesting for fisheries resources with a reserved area. Haque and Sarwardi [7] examined the toxic effects of fisheries models with logistical growth rates on two species. Ang et al [8] discuss the dynamic behavior of predator-prey fishery models with harvesting that is influenced by environmental toxicant.

In the research of Haque and Sarwardi [7] has two important points, namely the use of the response function and harvesting effects. This model on interactions between predator and prey species uses the type I Holling response function. The type I Holling response function is used when predator consumption increases linearly with the density of the prey but will be constant when the predator stops preying. The linear increase implies that the time needed by predators to process food is neglected or that the predator continues to search for food regardless of the level of food saturation. The Holing type I response function occurs when predators have passive characteristics or prefer to wait for their prey, for example in white sharks. Besides, the model added harvest parameters in predator populations because predator populations have economic value.

Based on the description that has been explained, this study modifies the research of Haque and Sarwadi [7]. The modification is from the competition model to the predator-prey model which is to replace the Holling type I response function to type II. Holling type II functions occur in predators that are active because most fish are active predators, therefore, the use of the Holling type II response function can describe the food chain phenomenon in aquatic ecosystems more realistic.

2. THE MATEMATICAL MODEL

In this study, the design of the model formed came from [7] model. The design of the model was done by modification, the initial concept was a model of competition between two species into a predator-prey model. Besides, the modification was done change the response function from Holling type I to type II and add harvesting effects to predator populations. In this case, we assume that prey is a plankton population and predators are fish populations. So that the modified model is

$$\frac{dx}{dt} = f(x,y) = r_1 x\left(1-\frac{x}{K}\right) - \frac{\gamma_1 xy}{A+x} - \alpha x^2 y \tag{2.1}$$

$$\frac{dy}{dt} = g(x,y) = \frac{c\gamma_2 xy}{A+x} - \beta xy^2 - (d+qE)y \tag{2.2}$$

with initial condition $x(0) \geq 0, y(0) \geq 0$.

In the systems (2.1)-(2.2), all parameters are positive. in order, $x(t)$ and $y(t)$ represents the population density of prey and predator population. r_1 and K are the rate of logistics growth and carrying capacity of the prey population. γ_1 and γ_2 represents the rate of interaction between prey populations with predatory fish and the rate of interaction between predatory fish populations and prey. The toxic coefficients in prey and predator fish populations are α and β. Then, A and c are the level of environmental protection to prey and the rate of maximum growth for predator. d and q ar the natural mortality rate of predatory fish populations and coefficient of predatory population capture activity. E is the harvesting rate of predatory fish population and simply $d + qE = r_2$.

Proposition 2.1. *The solution trajectories of system (2.1)-(2.2) with initial condition $x(0) \geq 0, y(0) \geq 0$ will be positive and boundedness.*

Proof. We will proven that $x(t) > 0, y(t) > 0, \forall t \geq 0$. Based on the system (2.1)-(2.2), we get solution of system (2.1)-(2.2) solution with the inequality Gronwall

$$x(t) = x_0 e^{\int_0^t r_1\left(1-\frac{x(s)}{K}\right) - \frac{\gamma_1 y(s)}{A+x(s)} - ax(s)y(s)ds} > 0,$$

$$y(t) = y_0 e^{\int_0^t \frac{c\gamma_2 x(s)}{A+x(s)} - \beta x(s)y(s) - r_2 ds} > 0.$$

it is clear that the solution trajectories of system (2.1)-(2.2) with initial condition $x(0) \geq 0, y(0) \geq 0$ will always be positive.

Then it will be proven that $x(t), y(t)$ is bounded.

(1) From system (2.1)-(2.2), we get $\frac{dx}{dt} = r_1 x\left(1-\frac{x}{K}\right) - \frac{\gamma_1 xy}{A+x} - \alpha x^2 y \leq r_1 x\left(1-\frac{x}{K}\right)$. if $\frac{dx}{dt} \leq r_1 x\left(1-\frac{x}{K}\right)$ then the solution is $x(t) \leq \frac{Kx_0}{x_0+(K-x_0)e^{-r_1 t}}$. Therefore, we can determine that $\lim_{t\to\infty}\sup x(t) \leq K+\varepsilon$.

(2) Then, we get $\frac{dy}{dt} = \frac{c\gamma_2 xy}{A+x} - \beta xy^2 - r_2 y \leq \frac{c\gamma_2 xy}{A+x} - \beta xy^2$ in systems (2.1)-(2.2). From the discussion of points (1), we get $x(t) \leq K+\varepsilon$ so that the equation $\frac{dy}{dt} \leq \left(\frac{c\gamma_2(K+\varepsilon)}{A+(K+\varepsilon)} - \beta(K+\varepsilon)y\right)y$ because the value $\varepsilon > 0$ is very small, it is ignored so that $\lim_{t\to\infty}\sup y(t) \leq \frac{c\gamma_2}{\beta(A+K)}$.

it can be seen that systems (2.1)-(2.2) is positive and bounded $\forall t \geq 0$. ∎

3. EQUILIBRIA AND STABILITY ANALYSIS

In this section, we discussed about the equilibria and stability properties of system (2.1)-(2.2). We have obtained three points of equilibrium. There are $E_0(0,0)$, $E_1(K,0)$ and $E_2(x^*,y^*)$ where $y^* = \frac{(c\gamma_2 - r_2)x^* - r_2 A}{\beta x^*(A+x^*)}$ and x^* is the positive root of equation (3.1),

$$x^4 + H_1x^3 + H_2x^2 + H_3x + H_4 = 0, \tag{3.1}$$

with $H_1 = (2A-K) + \frac{aK(c\gamma_2 - r_2)}{r_1\beta}$, $H_2 = \frac{\alpha AK(c\gamma_2 - 2r_2)}{r_1\beta}$, $H_3 = \frac{K}{r_1\beta}(\gamma_1(c\gamma_1 - r_2) - (\alpha r_2 + \beta r_1)A^2)$, and $H_4 = -\frac{KA\gamma_1 r_2}{r_1\beta}$.

3.1. **Local stability.** The Jacobian matrix of systems (2.1)-(2.2) for each equilibrium point (x^+, y^+) is

$$J(x^+,y^+) = \begin{pmatrix} r_1 - \dfrac{2r_1x^+}{K} - \dfrac{\gamma_1 Ay^+}{(A+x^+)^2} - 2\alpha x^+y^+ & -\dfrac{\gamma_1 x^+}{(A+x^+)^2} - \alpha {x^+}^2 \\ \dfrac{c\gamma_2 Ay^+}{(A+x^+)^2} - \beta {y^+}^2 & \dfrac{c\gamma_2 x^+}{A+x^+} - 2\beta x^+y^+ - r_2 \end{pmatrix}.$$

The Jacobian matrix at equilibrium poin E_0 is

$$J(E_0) = J_0 = \begin{pmatrix} r_1 & 0 \\ 0 & r_2 \end{pmatrix}.$$

Eigenvalues of matrix J_0 are $\lambda_1 = r_1$ and $\lambda_2 = -r_2$. Based on stability criteria, which $\lambda_1 > 0$ and $\lambda_2 < 0$ equilibrium point E_0 is saddle point.
Equilibrium point E_1 is substituted to the matrix J, we get the Jacobian matrix that

$$J(E_1) = \begin{pmatrix} -r_1 & -\dfrac{\gamma_1 K}{(A+K)^2} - \alpha K^2 \\ 0 & \dfrac{c\gamma_2 K}{A+K} - r_2. \end{pmatrix}.$$

Eigenvalues of matrix J_1 are $\lambda_1 = -r_1 < 0$ or $\lambda_2 = \frac{c\gamma_2 K}{A+K} - r_2$. If $r_2 > \frac{c\gamma_2 K}{A+K}$ then equilibrium point E_1 is a locally stable.
Equilibrium point $E_2(x^*,y^*)$ has the Jacobian matrix as follows

$$J(E_2) = J_2 = \begin{pmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{pmatrix},$$

where

$$c_{11} = r_1 - \frac{2r_1x^*}{K} - \frac{\gamma_1 Ay^*}{(A+x^*)^2} - 2\alpha x^*y^*,$$

$$c_{12} = -\frac{\gamma_1 x^*}{(A+x^*)^2} - 2\alpha {x^*}^2,$$

$$c_{21} = \frac{c\gamma_2 Ay^*}{(A+x^*)^2} - 2\beta {y^*}^2,$$

$$c_{22} = \frac{c\gamma_2 x^*}{A+x^*} - 2\beta x^*y^* - r_2.$$

.

The characteristic equation of matrix J_2 is

$$\lambda^2 - \omega_1\lambda + \omega_2 = 0,$$

where

$$\omega_1 = r_1 - r_2 - \frac{2r_1x^*}{K} - 2x^*y^*(\alpha+\beta) + \frac{c\gamma_2x^*(A+x^*) - \gamma_1Ay^*}{(A+x^*)^2},$$

$$\omega_2 = \left(r_1 - \frac{2r_1x^*}{K} - \frac{\gamma_1Ay^*}{(A+x^*)^2 - 2\alpha x^*y^*}\right)\left(\frac{c\gamma_2x^*}{A+x^*} - 2\beta x^*y^* - r_2\right) + \left(\frac{c\gamma_2Ay^*}{(A+x^*)^2} - \beta {y^*}^2\right)\left(\frac{\gamma_1x^*}{(A+x^*)^2} - \alpha {x^*}^2\right)$$

So the root characteristic matrix of J_2, that is

$$\lambda_1 = \frac{1}{2}(\omega_1 + \sqrt{W}), \quad \lambda_2 = \frac{1}{2}(\omega_1 - \sqrt{W}),$$

which $W = \omega_1^2 - 4\omega_2$. Equilibrium $E_2(x^*, y^*)$ is locally asymptotically stable if λ_1, λ_2 statisfies one of the conditions 3.1.

Conditions 3.1

(1) If $W = 0$ and $\omega_1 < 0$ then we get $\lambda_1, \lambda_2 < 0$.
(2) If $W > 0, \omega_1 < 0, \omega_2 > 0$ and $\sqrt{W} < |\omega_1|$ then we get $\lambda_1, \lambda_2 < 0$.

3.2. **Global Stability.** The equilibrium point E_2 is in an area R^2. If there is no limit cycle or periodic orbit around the equilibrium point then the solution orbit to the equilibrium point E_2 which is asymptotically stable so that the equilibrium point is globally stable. The existence of periodic orbits can be demonstrated using the Dulac criteria. A Dulac function in the area R^2 can be written as follows

$$D(x,y) = \frac{A+x}{xy}. \tag{3.2}$$

From equation (3.2), we get

$$\frac{\partial(Df(x,y))}{dx} + \frac{\partial(Dg(x,y))}{dy} = \frac{r_1}{r_2} - \left(\frac{r_1}{ky} + \alpha\right)(A+2x) - (A+x)\beta$$

If $\frac{r_1}{y} \leq \left(\frac{r_1}{ky} + \alpha\right)(A+2x) + (A+x)\beta$ then it's clear that $\frac{\partial(Df(x,y))}{dx} + \frac{\partial(Dg(x,y))}{dy} < 0$. Based on Dulac's criteria and proposition 2.1, the system (2.1)-(2.2) is not found periodic orbits so the orbits of the solutions that occur towards the equilibrium point that is locally stable. Therefore, the equilibrium point E_2 is globally asymptotically stable.

4. BIONOMIC EQUILIBRIUM

Bionomic point is a condition that occurs when the biological balance with the economic balance is the same. Biological balance occurs when $\frac{dx}{dt} = \frac{dy}{dt} = 0$. Economic equilibrium occurs when $TR = TC$ means that the value between total revenue and total costs is balanced. The function of economic benefits is

$$\pi(y, E) = (pqy - k_1)E.$$

Consecutive p and k_1 is the price per predator unit and the costs incurred when harvesting predators. Bionomic equilibrium points occur when

$$\frac{dx}{dt} = r_1 x\left(1 - \frac{x}{K}\right) - \frac{\gamma_1 xy}{A+x} - \alpha x^2 y = 0 \tag{4.1}$$

$$\frac{dy}{dt} = \frac{c\gamma_2 xy}{A+x} - \beta xy^2 - (d+qE)y = 0, \tag{4.2}$$

$$\pi(y) = pqy - k_1 = 0. \tag{4.3}$$

If $k_1 > pqy$ means that the cost of harvesting fish (predators) to exceeds income or suffers losses, then the harvesting efforts of fish must be stopped or $E = 0$. So in this case, bionomic equilibrium will not occur. If $k_1 \leq pqy$ can be interpreted as the cost of income for harvesting fish (predators) to exceed the selling price, then the existence of bionomic equilibrium remains or $E > 0$.

From (4.1), we get

$$y_\infty = \frac{k_1}{pq}. \tag{4.4}$$

Substitute equation (4.4) to (4.2), we obtained

$$B_1 x^3 + B_2 x^2 - B_3 x = 0, \tag{4.5}$$

where

$$B_1 = \frac{a}{\gamma_1} + \frac{pqr_1}{k_1\gamma_1 K}, \quad B_2 = \frac{Aa}{\gamma_1} + \frac{pqr_1}{k_1\gamma_1}\left(\frac{A}{K} - 1\right), \quad B_3 = \frac{Apqr_1}{k_1, \gamma_1, K} + 1.$$

The positive root of equation (4.5) is

$$x = \frac{-B + \sqrt{B_2^2 + 4B_{1B_3}}}{2B_1}. \tag{4.6}$$

Substitute equations (4.4) and (4.6) to equation (4.2), we get

$$E_\infty = \frac{1}{q}\left(\frac{c\gamma_2 x}{A+x} - \frac{\beta k_1}{pq}x - d\right). \tag{4.7}$$

So the bionimic equilibrium point in this model is (y_∞, E_∞) . The parameters in determining the bionomic equilibrium in this model are $\gamma_1 = 2.5, \gamma_2 = 5.5, \alpha = 0.2, \beta = 0.05, K = 50, c = 5.6, A = 4, d = 0.01, e = 0.1, r_1 - 0.9, p = 7, q = 2, k_1 = 1.2$. The results of the bionimic equilibrium are presented in table 4.1.

Table 4.1 Bionomic equilibrium in fishery models with the influence of toxic substances

Toxic substances		Bionomic equilibrium point (y^*, E^*)
$\alpha = 0.2$	$\beta = 0.05$	$(0.085, 12.97)$
$\alpha = 0$	$\beta = 0$	$(0.085, 14.17)$

Based on Table 4.1, the fish population density (predator) is the same. However, from equation (4.7), it can be seen that harvesting efforts with the influence of the presence of toxic substances are lower than in the absence of toxic substances. Thus, it is expected that fish (predators) can be harvested more if the toxic substances in the aquatic ecosystem have decreased.

5. OPTIMAL CONTROL

In this section, the optimal control with heat management in predator populations (fish) will be discussed. The heating control is expected to provide maximum profit./ The optimal control problems are as follows

$$\max \int_0^\infty [e^{-\delta t} \cdot \pi] dt,$$
$$\pi = (pqy - k_1)E,$$

where δ is annual discount rate, and the objective function or condition constraint is the equation systems (2.1)-(2.2). According to [9], this control optimal is bang-bang types. The parameter E as a harvest control is expressed as $E(t)$ because its value changes with time. Based on the objective function of the system of equations (2.1)-(2.2), the Hamilton function can be written as follows

$$H(E, x, y, \lambda) = (e^{\delta t}(pqy - k_1) - qy\lambda)E + \lambda\left(\frac{c\gamma_2 xy}{A + x} - \beta xy^2 - dy\right) \tag{5.1}$$

where λ is a costate variable. The switching function of the predator population is

$$\phi(t) = e^{\delta t}(pqy - k_1) - \lambda qy.$$

Optimal control must be a combination of bang-bang control and singular control. In bang-bang control, the optimal harvesting policy in the control variable must be fulfilled

$$E(t) = \begin{cases} e^{-\delta t}(pqy - k_1) - qy > 0, & E^* = E_{max} \\ e^{-\delta t}(pqy - k_1) - qy < 0, & E^* = 0 \\ e^{-\delta t}(pqy - k_1) - qy = 0, & \text{singular case} \end{cases}$$

or

$$E(t) = \begin{cases} E_{max}, & \text{if } \phi > 0 \text{ then } \lambda(t)e^{-\delta t} < p - \dfrac{k_1}{qx} \\ 0, & \text{if } \phi < 0 \text{ then } \lambda(t)e^{-\delta t} > p - \dfrac{k_1}{qx} \end{cases}$$

The $\lambda(t)e^{\delta t}$ is the shadow price of the harvest, while $p - \frac{k_1}{qx}$ is the net income of each fish harvesting unit. in this model, there are several possible cases. First, if the shadow price is greater than the net income per harvest unit, then the harvesting activity must be stopped so that there is no loss. Second, if the shadow price is smaller than the net income per harvest unit, then the harvesting activity must be increased. According to the Pontryagin Principle, Hamilton's function (5.1) reaches an optimal solution if the state, costate and stationary conditions satisfy as follows.

The state equation is

$$\frac{\partial H}{\partial \lambda} = \frac{c\gamma_2 xy}{A + x} - \beta xy^2 - (d + qE)y.$$

The costate equation is

$$\frac{d\lambda}{dt} = -\frac{\partial H}{\partial y} = -Epqe^{-\delta t} - \lambda\left(\frac{c\gamma_2 x}{A + x} - 2\beta xy - (d + qE)\right) \tag{5.2}$$

From equation (4.2), we get

$$E = \frac{1}{q}\left(\frac{c\gamma_2 x}{A + x} - \beta xy - d\right). \tag{5.3}$$

Substitute equation (5.3) to equation (5.2), we get

$$\frac{d\lambda}{dt} = -Epqe^{-\delta t} + \lambda\beta xy. \tag{5.4}$$

Equation (5.4) is a non-homogeneous ordinary differential equation. The general solution of equation (5.4) is

$$\lambda(t) = s_1 + s_2 e^{\beta xyt} + \frac{Epq}{\delta + \beta xy} e^{-\delta t}.$$

$\lambda(t)$ is bounded if $s_1 = s_2 = 0$ so the general solution of (5.4) is

$$\lambda(t) = \frac{Epq}{\delta + \beta xy} e^{-\delta t},$$

or

$$\lambda(t)e^{\delta t} = \frac{Epq}{\delta + xy} \tag{5.5}$$

Meanwhile the stationary conditions are written as follows

$$\frac{\partial H}{\partial E} = e^{-\delta t}(pqy - k_1) - qy\lambda = 0 \tag{5.6}$$

Substitute equations (5.5) to (5.6), we get

$$qy\left(p - \frac{Epq}{\delta + \beta xy}\right) - k_1 = 0. \tag{5.7}$$

From equation (4.3) or economic profit equation is

$$\begin{aligned} \pi(y, E) &= pqy - k_1, \\ &= \frac{Epq}{\delta + \beta xy}. \end{aligned}$$

If $\delta \to \infty$ then $\pi(y, E) \to 0$, it means that if the annual discount rate increases (towards infinity) then economic benefits will decrease. Conversely, if $\delta \to 0$ then π increase. In other words, profits depend on the discount rate. Harvesting activities will stop if the discount rate increase. Meanwhile, if the discount rate is going down, the economic profit will reach its maximum.

6. NUMERICAL SIMULATION AND DISCUSSION

To illustrate from system (2.1)-(2.2), we use numerical simulations with several estimator values of parameters. In the first simulation, we use the parameters in Table 1 and shown in figure 1. For the second simulation, we use parameter values in table 1 with some variation in the values of α and β, then shown in Table 2 and figure 2.

Table 1. Parameter values for simulation 1 and simulation 2.

Parameter	Simulation 1	Simulation 2
	Value	Value
γ_1	0.9	0.9
γ_2	0.2	0.2
α	0.006	Table 2
β	0.003	Table 2
K	50	50
A	0.3	0.3
d	0.001	0.001
e	0.01	0.01
r_1	5	5
c	0.9	0.9

The first simulation aims to show that the equilibrium point E_2 is globally asymptotically stable. The first simulation is shown in Figure 1 with some initial condition namely (40.20), (65.10), (30.30), (1.2), (10.10), (10.20). From figure 1, we can see that with several different initial condition over a long period the solution converges to the equilibrium point $E_2(46.54, 1.20)$. This shows that the equilibrium point E_2 is globally asymptotically stable.

Tabel 2. Population density with variations in toxic substances

α	β	color	Prey(x)	Predator (y)
4	1	Red	43.288	0.0038
0.4	0.1	Black	43.286	0.0387
0.04	0.01	Blue	43.253	0.3878
0.004	0.001	Magenta	42.928	3.9076

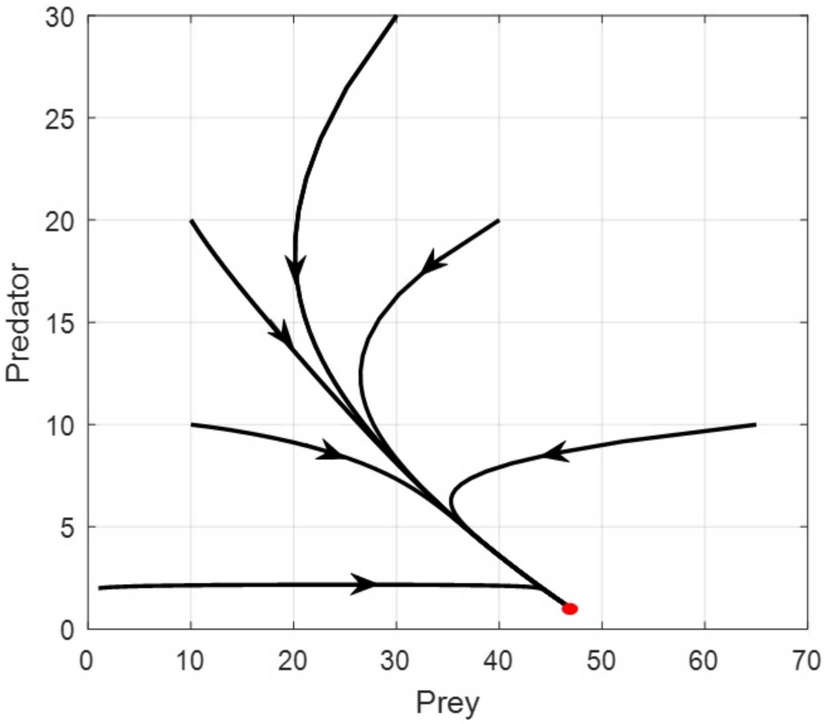
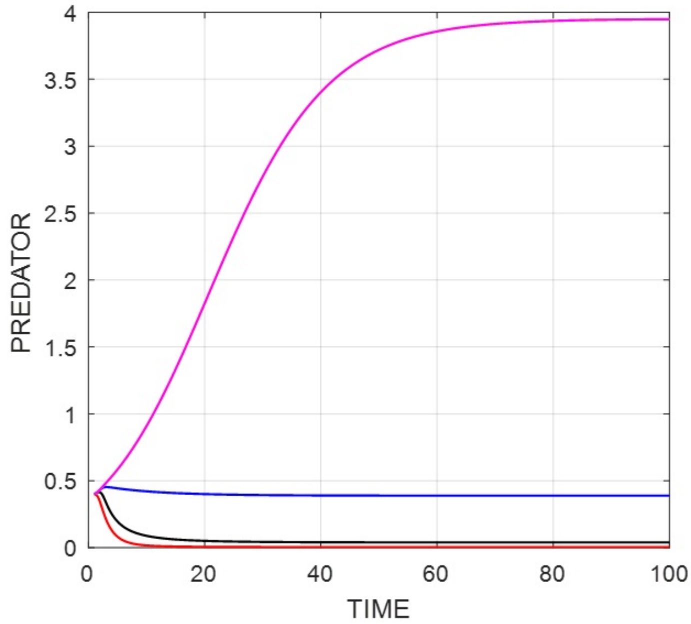
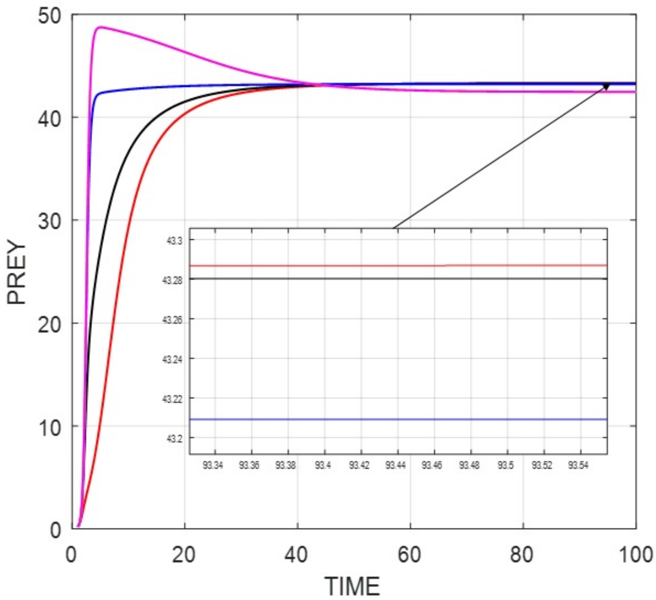


Figure 1. the global asymptotic stability of the interior equilibrium $E_2(46.54, 1.20)$

The second simulation aims to explain the effect of toxic levels on the density of the two species. The second simulation is shown in Figure 2 and the results are summarized in Table 2. From Figure 2, we can know that if the levels of toxic substances increase in the aquatic ecosystem, the density of predator population over time will decrease (figure 2a) and the population density of prey will increase (figure 2b). Thus, toxic substances influence the survival of both populations.



(a)



(b)

Figure 2. Effect of toxic substances on the prey and predator population density

7. CONCLUSION

This research discussed the dynamic analysis of fisheries predator-prey models with the influence of toxic substances. Dynamic analysis for local and global stability has been discussed fully with some properties. The existence of global stability for interior equilibrium points if it follow proposition 2.1 and Dulac's criteria. The bionomic balance in this model was studied with some of the possible cases. The Conditions that guarantee the existence of bionomic equilibrium have been analyzed. Optimal harvest control was studied to maximize income without causing extinction to predator population (fish). In addition, we was found that if the discount rate to zero results in maximum economic benefits. Based on the results of numerical simulations by changing some of the parameters of the toxic substances, if the decreased and the prey population has increased slowly. toxic substances in aquatic ecosystems gives influence population densities. Therefore, harvesting policy in polluted water ecosystems is very important to keep fisheries resources remain sustainable.

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