The Australian Journal of Mathematical Analysis and Applications

AJMAA

Volume 16, Issue 2, Article x, pp. 1-14, 2019

FORMULATION OF APPROXIMATE MATHEMATICAL MODEL FOR INCOMING WATER TO SOME DAMS ON TIGRIS AND EUPHRATES RIVERS USING SPLINE FUNCTION

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Received 28 December, 2018; accepted 27 May, 2019; published 31 July, 2019.

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ABSTRACT. In this paper, we formulate three mathematical models using spline functions, such as linear, quadratic and cubic functions to approximate the mathematical model for incoming water to some dams. We will implement this model on dams of both rivers; dams on the Tigris are Mosul and Amara while dams on the Euphrates are Hadetha and Al-Hindya.

Key words and phrases: Spline Function, Approximation mathematical model, Incoming water, Dams, Tigers and Euphrates Rivers.

2000 *Mathematics Subject Classification.* Primary 37MXX .

ISSN (electronic): 1449-5910

1. INTRODUCTION

Iraq is located in a dry to semi-dry area, with no more than an average annual rainfall of 200 mm a year. Half of the Iraqi areas is deserts or semi-deserts with no more than 50 mm of rainfall a year. The rest of Iraq has an average rain of 150-450 mm/year except for some mountain areas in the north-east of Iraq with rainfall of about 1000 mm/year. Most of the water revenue comes from the outer boundaries of the territorial regions through the Tigris and the Euphrates.

Based on the above, the associating factor between all water sources in Iraq is the amount of rain and snow in the main river banks (Tigris and its tributary and the Euphrates). Also, by the policy of running dams and reservoirs located at the upper river banks shared by Turkey, Syria and Iran. No agreement exists between Iraq and the mentioned states to share water. Thus, various activities have caused deterioration in the quality of water year after year; rendering the water contaminated making the provision of water to Iraq volatile.

Iraq will see a decrease in the provision of water resources and a decrease in their quality after the countries near the upper rivers finish their irrigation plan which will target farming more than 2.4 million hectares irrigating at the base of the Euphrates and about 1 million hectares are irrigating at the base of the Tigris. This will cause a deficit in the Tigris and Euphrates water supply by more than 43% in 2015 from the standard average which is 77 billion M^3 annually (50 billon M^3 for Tigris & 27 billion M^3 for Euphrates).
Dams and large or medium reservoirs are considered being a major investment of water resources. They are intended to ward off major danger of floods in main rivers, store flood peaks that are carried by the river during winter and spring, and organize its launch for the purposes of irrigation and power generation alongside other artificial and natural usages.

The annual earnings for rivers have proven to be volatile between the years as well as the pattern of consumption for the different sectors shall be the most severe in summer which highlights the status of the importance of setting up dams and reservoirs along the main rivers which are consistent with the needs of beneficial sectors.

The small dams along the valleys, especially the area of western desert, eastern areas and Kurdistan are working on taking advantage of water resources on the surface in the most efficient way which cannot be organized within the system of large dams. They offer water reservoirs to meet the needs of the citizens from drinking water and feeding animals which helps in the relocation of the population in the desert areas as the water from the reservoirs can be used in agriculture although in a limited manner. These Dams work on depositing the valleys water and keep it from overflowing, especially when valleys end at the sea which does not benefit them and makes up a source to nourish the ground water which is one of the tasks of sources of water.

The approximation theory is one of the main topics of numerical analysis. It is a foundation for numeral algorithms in the different fields of applied mathematics. Polynomial interpolation is especially important, since it offers an approximating function in a closed form, which is widely used in implementations. Polynomials are the most easily handled in practice, since they can be represented by restricted information, evaluated in limited number of basic operations and easily integrated or differentiated.

In recent years, splines have attracted attention of both researchers and users who need various approximation tools. To sum up, linear splines are a set of joint line segments, a continuous function with discontinuous first derivative at the knots. Quadratic splines have a continuous first derivative, cubic splines continuous first and second derivatives, and so on. Linear splines are evidently too rough of an approximation to a physical spline, a cubic spline is adequate, quadratic and higher order splines are possible but require more computation [1].

In this work, we find an approximate mathematical model depending on spline function including three types of polynomials: linear, quadratic and cubic for incoming water to two dams on Tigris river (Mosul and Amara) dams, and two dams on Euphrates river (Hadetha and Al-Hindya) dams. The remaining of the paper is organized as follows: In section 2, we will introduce the polynomial spline functions. In section 3, the mathematical models for incoming water are representing in tables and figures using MATLAB programming. Finally, the report ends with a brief conclusion.

1.1. **Spline Function.** Generally, we introduce a subdivision of the interval [a,b] as [2]:

$$\Delta : a = x_1 < x_2 < x_3 < \ldots < x_n = b$$

where $x_i = a + ih, h = (b\text{-}a)/n$, and *n* is the number of subinterval $[x_i, x_{i+1}]$, for all $i = 1, 2, \ldots$, *n*-1. We will define a class of functions:

$$S_m^k(h) = \{s : s \in C^k[a, b], s|_{[x_i,x_{i+1}]} \in \mathrm{P}_m, i = 1, 2, \ldots, n-1\} \tag{1.1}$$

where $m \geq 0,\ \ k \geq 0,\ \ S_m^k(h)$ the " spline functions of degree *m* and smoothness class k" relative to the subdivision *h*. Spline functions are defined to be piecewise polynomials of degree *n* joined together at the break points with *n*-1 continuous derivatives [3]. The break points of splines are called knot. Three types of spline functions have been considered, they are [4][5]:

1.2. **Linear Spline Function:** A function L is called linear spline if it satisfies:

1. There is a partition of the interval $a = x_0 < x_1 < \ldots < x_n = b$, such that L is polynomial of degree 1 on each subinterval $[x_i, x_{i+1}]$.
2. L is continuous on [*a* ,*b*], i.e.

$$L(x) = \begin{cases} l_0(x) & , x \in [x_0, x_1] \\ l_1(x) & , x \in [x_1, x_2] \\ & \vdots \\ l_{n-1}(x) & , x \in [x_{n-1}, x_n] \end{cases} \tag{1.2}$$

where $x_0,\ x_1,\ \ldots,\ x_n$ are called knots, and each piece of L(x) has the form: $l_{\mathrm{i}} = a_i(x - x_i) + b_i$ where $a_i$and $b_i$are the coefficients of liner spline function (1.2).

1.3. **Quadratic Spline Function:** A function Q is called quadratic spline if it satisfies:

1. There is a partition of the interval $a = x_0 < x_1 < \ldots < x_n = b$, such that Q is polynomial of degree 2 on each subinterval $[x_i, x_{i+1}]$.
2. Q and Q' are continuous on [a ,b],
i.e.

$$Q(x) = \begin{cases} q_0(x) & , x \in [x_0, x_1] \\ q_1(x) & , x \in [x_1, x_2] \\ & \vdots \\ q_{n-1}(x) & , x \in [x_{n-1}, x_n] \end{cases} \tag{1.3}$$

where $x_0,\ x_1,\ \ldots,\ x_n$are called knots, and each piece of Q(x) has the form:q_i=$a_i(x-x_i)^2$+$b_i(x-x_i)$+c_i where a_i,b_i and c_i are the coefficients of quadratic spline function(1.3).

1.4. **Cubic Spline Function:** The goal of cubic spline interpolation is to get an interpolation formula that is continuous in both the first and second derivatives, both within the intervals and at the interpolating nodes. This will give us a smoother interpolating function. The continuity of first derivative means that the graph y = F(x) will not have sharp corners. The continuity of second derivative means that the radius of curvature is defined at each point [6][7][8].

A function S is called cubic spline if it satisfies:

1. There is a partition of the interval $a = x_0<x_1<\ldots<x_n = b$, such that S is polynomial of degree 3 on each subinterval $[x_i, x_{i+1}]$
2. S, S' and S" are continuous on [a ,b], *i.e.*

$$S(x) = \begin{cases} s_0(x) & , x \in [x_0, x_1] \\ s_1(x) & , x \in [x_1, x_2] \\ \vdots & \\ s_{n-1}(x) & , x \in [x_{n-1}, x_n] \end{cases} \tag{1.4}$$

where $x_0,\ x_1,\ \ldots,\ x_n$are called knots, and each piece of S(x) has the form:s_i=$a_i(x-x_i)^3$ +$b_i(x-x_i)^2$+$c_i(x-x_i)$+d_i where a_i, b_i , c_i and $d_i$are the coefficients of cubic spline function (1.4).

2. MATHEMATICAL MODELS FOR DAMS

In this section, the approximate mathematical models for the chosen dams are considered using spline functions of three types. The data in the following table 2.1 is from the National Center for Water Resource Management-Water Control Section.

TABLE 2.1. Incoming water to some dams on Tigers and Euphrates

Month-year	Incoming Water (M^3/ Sec)			
	Tigris River		Euphrates River	
	Mosul Dam	Amara Dam	Hadetha Dam	Al-Hindya Dam
Feb-16	585	83	608	160
Mar-16	1110	74	402	170
Apr-16	1021	85	479	160
May-16	783	82	336	170
Jun-16	349	70	217	340
Jul-16	191	85	256	300
Aug-16	359	74	292	275
Sep-16	188	69	363	275
Oct-16	91	56	185	240
Nov-16	98	42	500	250
Dec-16	613	56	486	200
Jan-17	135	53	494	130
Feb-17	397	69	546	200

Table 2.1 continued from previous page

Month-year	Incoming Water (M^3/ Sec)			
	Tigris River		Euphrates River	
	Mosul Dam	Amara Dam	Hadetha Dam	Al-Hindya Dam
Mar-17	873	94	406	210
Apr-17	1934	90	228	165
May-17	857	131	192	170
Jun-17	210	121	608	320
Jul-17	265	85	104	280
Aug-17	223	75	295	255
Sep-17	228	65	334	220
Oct-17	64	53	326	180
Nov-17	116	34	422	150
Dec-17	135	41	321	185
Jan-18	463	44	382	165
Feb-18	351	65	377	210
Mar-18	789	72	235	195
Apr-18	268	72	213	135
May-18	650	65	213	100
Jun-18	508	65	235	122

Tigris River:

We choose two dams in Tigris River which are Mosul dam and Amara dam:

Mosul Dam:

After using the data and the proposed methods, we get three approximate polynomials of degree 1, 2 and 3 respectively, as we can see in the following Figures:

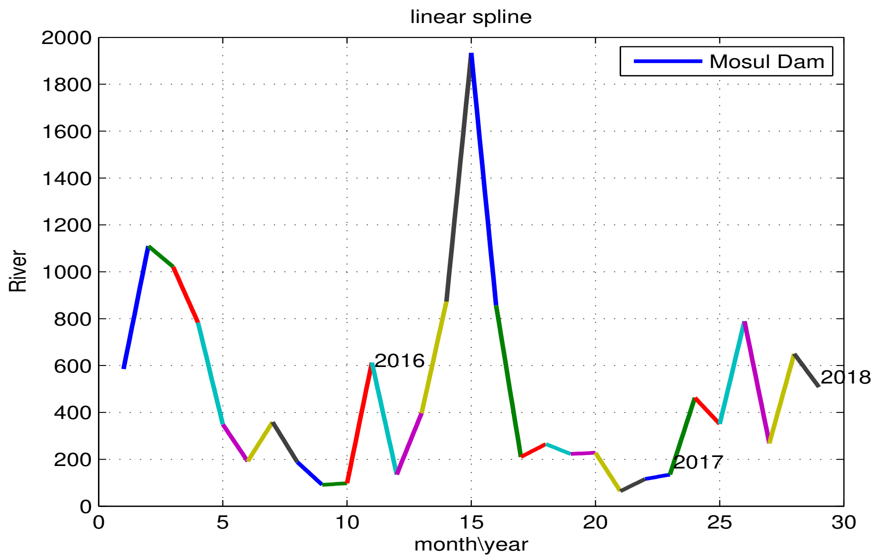

Figure 1: Approximate polynomial of order one

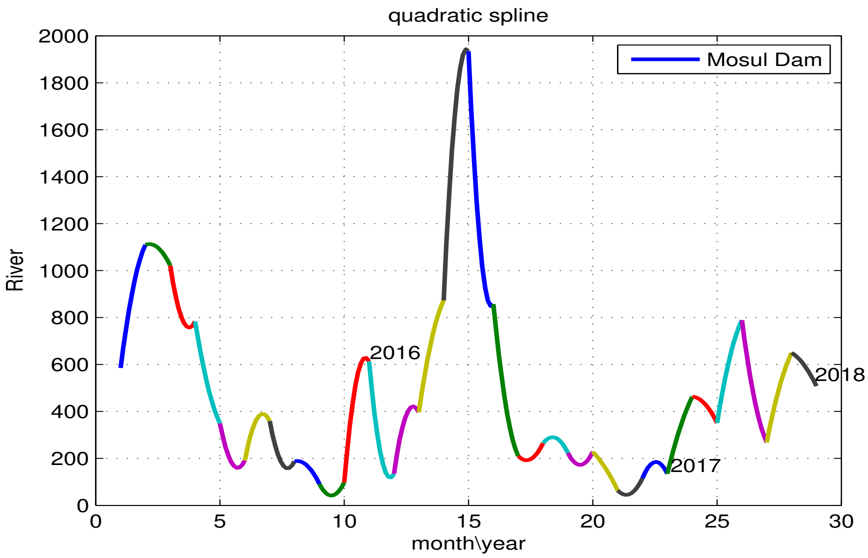

Figure 2: Approximate polynomial of order two

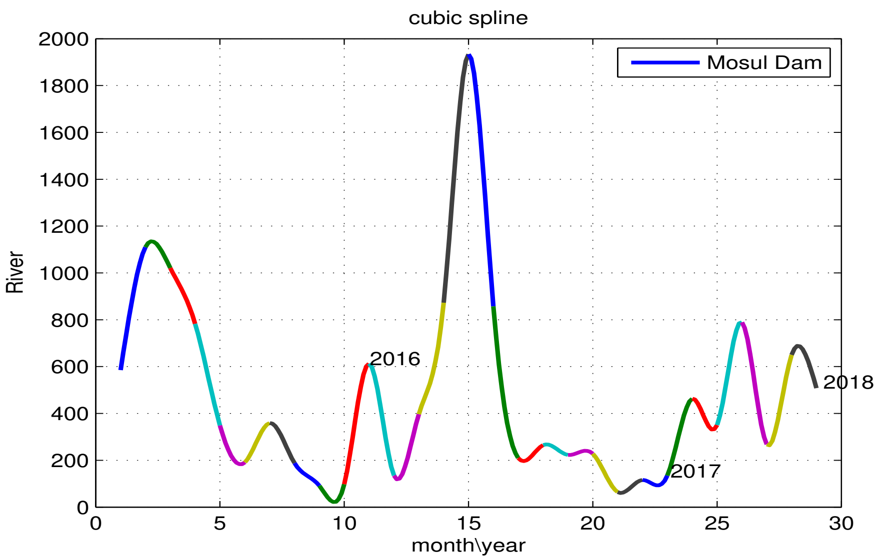

Figure 3: Approximate polynomial of order three

From Fig. 1, Fig. 2 and Fig. 3, we notice that in each figure the interval of data which include one month plot in a different color. Also, the same interval in each Figure is plotted in a same color to compare between them. Moreover, for Mosul dam we can see that the rate of incoming water increase in the months march, April in every year because it's the season of rain in Iraq. Finally, it is obvious that the cubic spline function is more accurate and smoothness than linear and quadratic one.

TABLE 2.2. The Coefficients of the Piecewise Cubic Spline Polynomial $d_i(x-x_i)^3 + c_i(x-x_i)^2 + b_i\,(x-x_i) + a_i$ for Mosul Dam

i	d_i	c_i	b_i	a_i
1	-158.430211411601	0	683.430211411601	585

Table 2.2 continued from previous page

i	d_i	c_i	b_i	a_i
2	178.151057058006	-475.290634234803	208.139577176798	1110
3	-89.1740168204210	59.1625369392133	-207.988520118792	1021
4	131.545010223679	-208.359513522050	-357.185496701629	783
5	34.9939759257064	186.275517148986	-379.269493074693	349
6	-221.520913926504	291.257444926105	98.2634690003989	191
7	186.089679780311	-373.305296853407	16.2156170730968	359
8	-109.837805194738	184.963742487524	-172.125937292786	188
9	283.261540998642	-144.549673096690	-131.711867901952	91
10	-619.208358799829	705.234949899236	428.973408900594	98
11	692.571894200676	-1152.39012650025	-18.1817677004233	613
12	-418.079218002875	925.325556101775	-245.246338098901	135
13	453.744977810823	-328.912097906849	351.167120096026	397
14	-1025.90069324042	1032.32283552562	1054.57785771480	873
15	926.857795150840	-2045.37924419563	41.5214490447879	1934
16	- 113.530487362945	735.194141256893	-1268.66365389395	857
17	-200.735845699061	394.602679168058	-138.866833468997	210
18	117.473870159189	-207.604857929125	48.1309877699361	265
19	-125.159634937695	144.816752548442	-14.6571176107472	223
20	167.164669591589	-230.662152264642	-100.502517326947	228
21	-158.499043428663	270.831856510126	-60.3328130814632	64
22	217.831504123063	-204.665273775863	5.83376965280000	116
23	-370.826973063590	448.829238593326	249.997734470263	135
24	516.476388131296	-663.651680597443	35.1752924661472	463
25	-705.078579461592	885.777483796444	257.301095665149	351
26	794.837929715074	-1229.45825458833	-86.3796751267408	789
27	-612.273139398704	1155.05553455689	-160.782395158185	268
28	227.254627879741	-681.763883639222	312.509255759481	650

Amara Dam:

After using the data and the proposed methods, we get three approximate polynomials of degree 1, 2 and 3 respectively, as we can see in the following Figures:

Figure 4: Approximate polynomial of order one

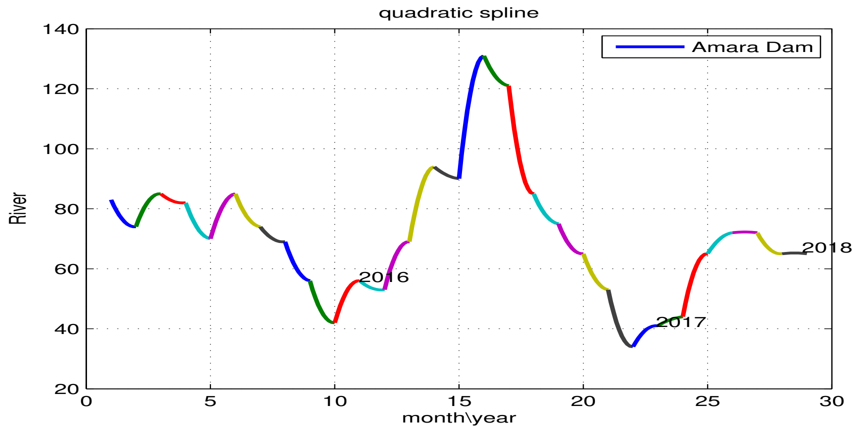

Figure 5: Approximate polynomial of order two

Figure 6: Approximate polynomial of order three

From Fig. 4, Fig. 5 and Fig. 6, in similar way as in Mosul dam each figure the interval of data includes one month plot in a different color. Also, the same interval in each Figure is plotted in a same color to compare between them. Moreover, for Amara dam we can see that the rate of incoming water increases in March and April only in year 2017 because in this year the rate of rain was more than the other years in the south of Iraq. In general, if we compare with Mosul dam we can see that the incoming water in Amara dam is less than in Mosul Dam because the resources of incoming water are less than in Amara dam. Finally, it is obvious that the cubic spline function is more accurate and smoothness than linear and quadratic one.

TABLE 2.3. The Coefficients of the Piecewise Cubic Spline Polynomial $d_i(x-x_i)^3 + c_i(x-x_i)^2 + b_i\,(x-x_i) + a_i$ for Amara Dam

i	d_i	c_i	b_i	a_i
1	6.01315832819634	0	-15.0131583281963	83
2	10.0657916409817-	18.0394749845890	3.02631665639268	74
3	0.250008235730517	-12.1578999383561	8.90789170262560	85
4	14.0657586980596	-11.4078752311646	-14.6578834668951	82
5	-20.5130430279691	30.7894008630144	4.72364216495472	70
6	14.9864134138167	-30.7497282208929	4.76331480707619	85

Table 2.3 continued from previous page

i	d_i	c_i	b_i	a_i
7	-7.43261062729765	14.2095120205572	-11.7769013932595	74
8	0.744029095373930	-8.08831986133579	-5.65570923403814	69
9	11.4564942458019	-5.8562325752140	-19.6002616705879	56
10	-17.5700060785817	28.5132501621918	3.05675591638988	42
11	13.8235300685248	-24.1967680735533	7.37323800502842	56
12	-1.72411419551762	17.2738221320212	0.450292063496397	53
13	-16.9270732864543	12.1014795454684	29.8255937409860	69
14	31.4324073413350	-38.6797403138947	3.24733297255967	94
15	-34.8025560788856	55.6174817101104	20.1850743687753	90
16	11.7778169742075	-48.7901865265466	27.0123695523390	131
17	12.6912881820554	1-3.4567356039239	-35.2345525781315	121
18	-10.5429697024293	24.6171289422424	-24.0741592398131	85
19	3.48059062766176	-7.01178016504553	-6.46881046261623	75
20	-5.37939280821774	3.42999171793975	-10.0505989097220	65
21	13.0369806052092	-12.7081867067135	-19.3287938984957	53
22	-13.7685296126190	26.4027551089141	-5.63422549629508	34
23	12.0371378452669	-14.9028337289430	5.86569588367605	41
24	-12.3800217684487	21.2085798068578	12.1714419615909	44
25	5.48294922852776	-15.9314854984882	17.4485362699605	65
26	-2.55177514566236	0.517362187095057	2.03441295856730	72
27	4.72415135412167	-7.13796324989202	-4.58618810422966	72
28	-2.34483027082433	7.03449081247300	-4.68966054164867	65

Euphrates River:

We choose two dams in Euphrates River which are Hadetha dam and Al-Hindya dam:

Hadetha Dam:

After using the data and the proposed methods, we get three approximate polynomials of degree 1, 2 and 3 respectively, as we can see in the following Figures:

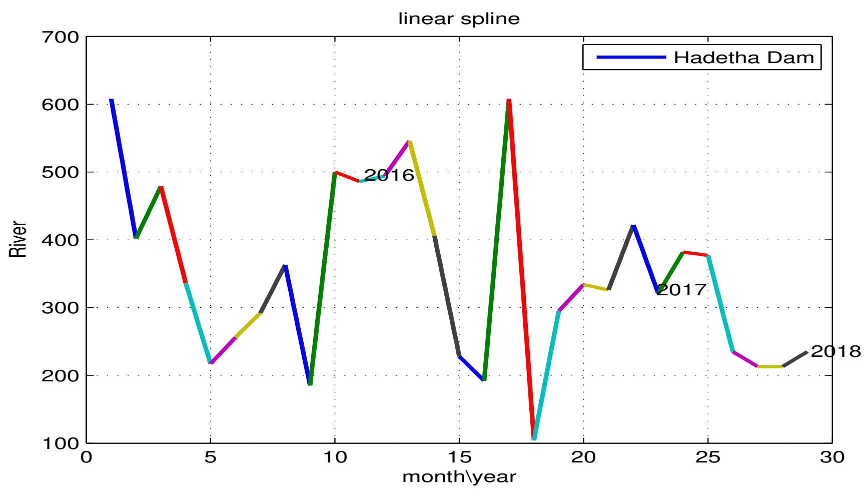

Figure 7: Approximate polynomial of order one

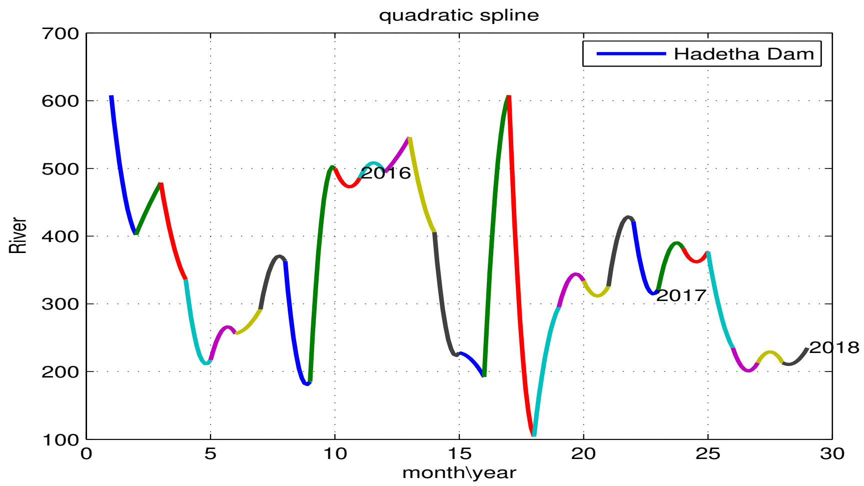

Figure 8: Approximate polynomial of order two

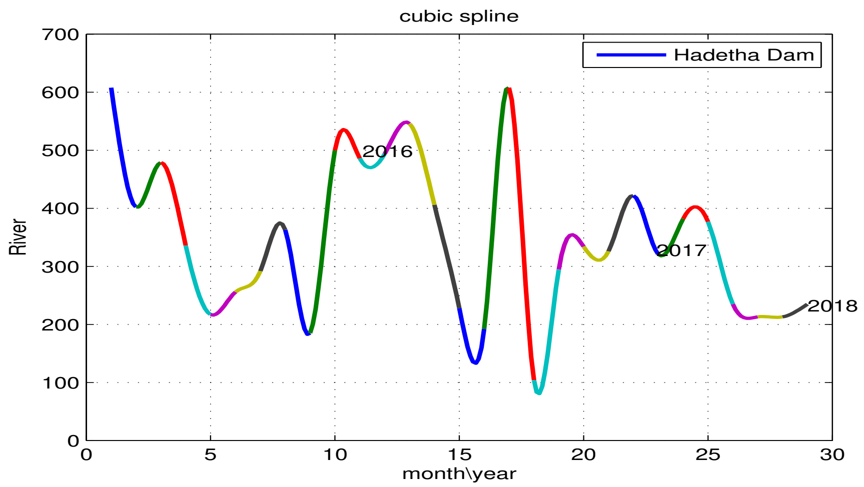

Figure 9: Approximate polynomial of order three

From Fig. 7, Fig. 8 and Fig. 9, in similar way as in Mosul and Amara dam each figure the interval of data includes one month plot in a different color. Also, the same interval in each Figure is plotted in a same color to compare between them. Moreover, for Hadetha dam we can see that the rate of incoming water increases in winter and decreases in the other seasons. Finally, it is obvious that the cubic spline function is more accurate and smoothness than linear and quadratic one.

TABLE 2.4. The Coefficients of the Piecewise Cubic Spline Polynomial $d_i(x-x_i)^3+c_i(x-x_i)^2+b_i\,(x-x_i)+a_i$ for Hadetha Dam

i	d_i	c_i	b_i	a_i
1	91.2149395489353	0	-297.214939548935	608
2	-173.074697744677	273.644818646806	-23.5701209021293	402
3	98.0838514297712	-245.579274587224	4.49542315745271	479
4	24.7392920255918	48.6722797020897	-192.411571727682	336
5	-63.0410195321384	122.890155778865	-20.8491362467268	217

Table 2.4 continued from previous page

i	d_i	c_i	b_i	a_i
6	66.4247861029616	-66.2329028175500	35.8081167145883	256
7	-164.658124879708	133.041455491335	102.616669388373	292
8	308.207713415871	-360.932919147790	-125.274794268081	363
9	-326.172728783777	563.690221099824	77.4825076839526	185
10	174.483201719235	-414.827965251506	226.344763532271	500
11	-20.7600780931635	108.621639906199	-79.8615618130358	486
12	-69.4428893465811	46.3414056267088	75.1014837198723	494
13	62.5316354794879	-161.987262413034	-40.5443730664534	546
14	-26.6836525713706	25.6076440254293	-176.923991454059	406
15	224.202974805995	-54.4433136886825	-205.759661117312	228
16	-560.128246652607	618.165610729301	357.962635923306	192
17	644.310011804435	-1062.21912922852	-86.0908825759138	608
18	-402.111800565131	870.710906184783	-277.599105619652	104
19	117.137190456090	-335.624495510611	257.487305054520	295
20	38.5630387407699	15.7870758576602	-62.3501145984301	334
21	-120.389345419170	131.476192079970	84.9131533392000	326
22	141.994342935910	-229.691844177540	-13.3024987583699	422
23	-88.5880263244689	196.291184630189	-46.7031583057204	321
24	-15.6422376380341	-69.4728943432174	80.1151319812515	382
25	80.1569768766054	-116.399607257320	-105.757369619286	377
26	-47.9856698683874	124.071323372496	-98.0856535041090	235
27	13.7857025969440	-19.8856862326657	6.09998363572166	213
28	-7.15714051938881	21.4714215581664	7.68571896122238	213

Al-Hindya Dam:

After using the data and the proposed methods, we get three approximate polynomials of degree 1, 2 and 3 respectively, as we can see in the following figures:

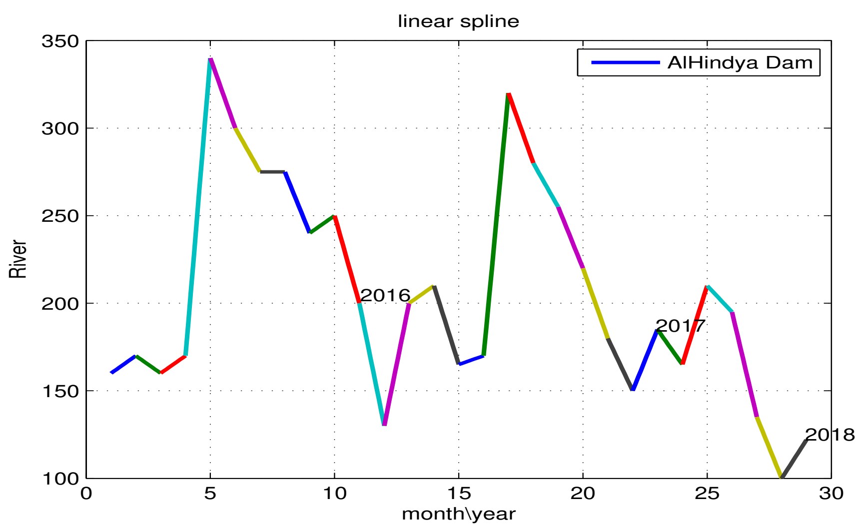

Figure 10: Approximate polynomial of order one

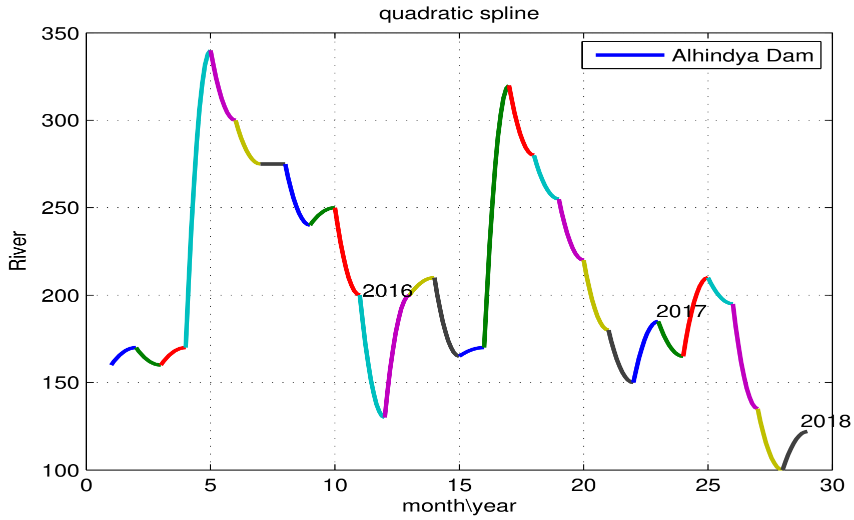

Figure 11: Approximate polynomial of order two

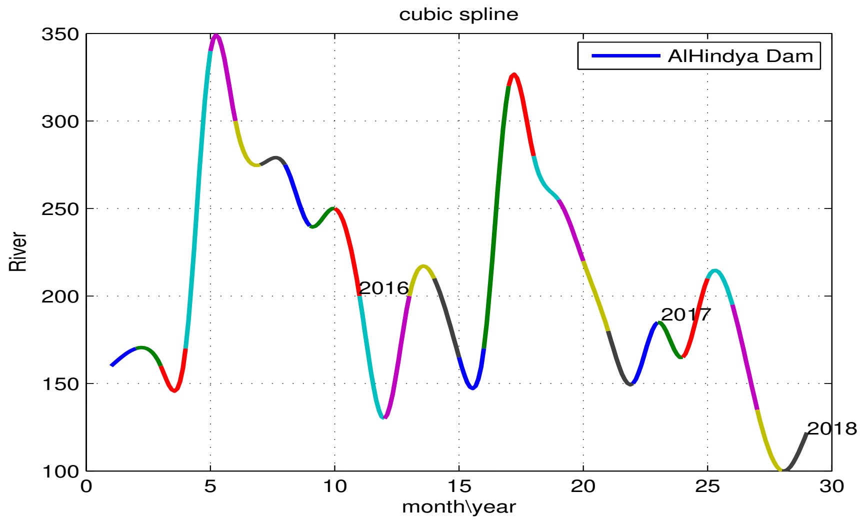

Figure 12: Approximate polynomial of order three

From Fig. 10, Fig. 11 and Fig. 12,in similar way as in Mosul ,Amara and Hadetha dam each figure the interval of data includes one month plot in a different color . Also, the same interval in each Figure is plotted in a same color to compare between them. Moreover, for Al Hindya dam we can see that the rate of incoming water is less than that of Hadetha dam because the site of dam. Finally, it is obvious that the cubic spline function is more accurate and smoothness than linear and quadratic one.

TABLE 2.5. The Coefficients of the Piecewise Cubic Spline Polynomial $d_i(x-x_i)^3 + c_i(x-x_i)^2 + b_i\,(x-x_i) + a_i$ for Al-Hindya Dam

i	d_i	c_i	b_i	a_i
1	2.62786636961609-	0	12.6278663696161	160
2	-6.86066815191956	-7.88359910884827	4.74426726076782	170
3	70.0705389772943	28.4656035646069-	-31.6049354126874	160

Table 2.5 continued from previous page

i	d_i	c_i	b_i	a_i
4	133.421487757258-	181.746013367276	121.675474389982	170
5	93.6154120517364	-218.518449904497	84.9030378527607	340
6	-16.0401604496881	62.3277862507122	-71.2876258010242	300
7	19.4547702529842-	14.2073049016481	5.24746535133613	275
8	33.8592414616249	-44.1570058573045	24.7022356043203-	275
9	-35.9821955935153	57.4207185275701	11.4385229340548-	240
10	5.06954091243644	-50.5258682529759	4.54367265946056-	250
11	55.7040319437696	35.3172455156666-	90.3867864281030-	200
12	-67.8856686875147	131.794850315642	6.09081837187259	130
13	15.8386428062893	71.8621557469020-	66.0235129406127	200
14	9.53109746235771	24.3462273280342-	30.1848701343235-	210
15	51.0369673442799	4.24706505903889	50.2840324033188-	165
16	-118.678966839477	157.357967091879	111.320999747599	170
17	88.6789000136299	-198.678933426554	70.0000334129238	320
18	-31.0366332150421	67.3577666143360	61.3211333992939-	280
19	10.4676328465385	-25.7521330307903	19.7154998157482-	255
20	-5.83389817111189	5.65076550882519	39.8168673377133-	220
21	27.8679598379090	-11.8509290045105	46.0170308333986-	180
22	-50.6379411805243	71.7529505092167	13.8849906713076	150
23	54.6838048841882	-80.1608730323563	5.47706814816808	185
24	-48.0972783562284	83.8905416202083	9.20673673602011	165
25	12.7053085407253	-60.4012934484769	32.6959849077515	210
26	12.2760441933270	-22.2853678263008	49.9906763670262-	195
27	8.19051468596658	14.5427647536802	-57.7332794396468	135
28	-13.0381029371933	39.1143088115799	-4.07620587438663	100

3. CONCLUSION

In this paper, spline functions are used to approximate the mathematical model for the chosen dams successfully. The proposed scheme is simple and computationally attractive and their accuracy is high and we can execute this method in a computer simply. From the resulting figures, we can notice that the cubic spline function is more efficient than the other methods that are used.

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