The Australian Journal of Mathematical Analysis and Applications

AJMAA

Volume 16, Issue 1, Article 8, pp. 1-9, 2019

THE INFLUENCE OF FLUID PRESSURE IN MACROMECHANICAL COCHLEAR MODEL

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Received 10 May, 2018; accepted 29 November, 2018; published 19 February, 2019.

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ABSTRACT. An increase of pressure in the structure of cochlea may cause a hearing loss. In this paper, we established the relationship between the fluid pressure and the amplitude of displacement of Basilar Membrane to clarify the mechanisms of hearing loss caused by increasing of this pressure. So, a mathematical cochlear model was formulated using finite difference method in order to explain and demonstrate this malfunction in passive model. Numerical simulations may be considered as helpful tools which may extend and complete the understanding of a cochlea dysfunction.

Key words and phrases: Mathematical model, Fluctuating hearing loss, Finite difference method, Fluid pressure.

2000 *Mathematics Subject Classification.* Primary: 93A30, 92B05. Secondary 00A69, 00A71.

ISSN (electronic): 1449-5910

1. INTRODUCTION

In the inner ear, vibrations are detected and converted into electrical signals. The special compartmentalization of the cochlea allows the function of this process. The cochlea is a complex three-dimensional fluid-mechanical structure that separates sounds on the basis of their frequency content and fine time structure [1] . Reissner and Basilar Membranes (BM) delineated three fluid-filled chambers within the cochlea, namely the scala vestibuli, scala media and scala tympani [2, 3].

The BM separates the scala media containing endolymph and the scala tympani containing perilymph, and acts as a spectral analyzer that translates vibration frequencies within the cochlear fluid pressure waves into positions of maximal displacement along its length [4, 5, 6]. A traveling wave initiated by the incoming sound travels rapidly from the base towards the apex of the BM, then, the amplitude of the wave increases at first and decreases quite abruptly, due to the dispersive nature of this membrane (see Figure 1).

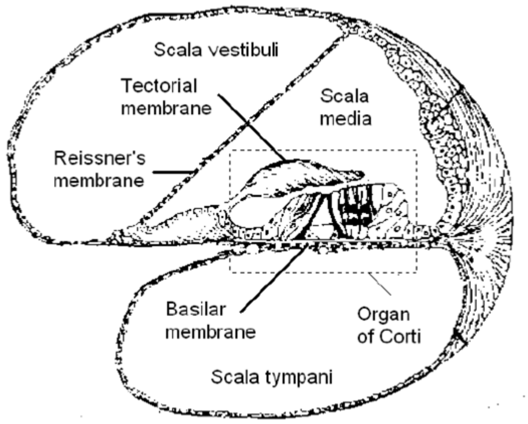

FIGURE 1. Cross section of the cochlea [7].

Hearing is the result of two kinds of processes, one mechanical and the other electrochemical, and any disharmony disrupt the ear's functionality [8, 9]. Hyper-pressure represents a pathological anatomy discovery in which the scala are distended by an enlargement of the fluid cochlear volume, resulting in an affection of scala and/or vestibular system [10]. The increase of the pressure affects the displacement of the BM and the structure of the cochlea, leading to a hearing loss [11].

In this paper, we developed a mathematical model using finite difference method with specified boundary conditions, in order to show the relation between the increase of fluid pressure on the displacement of the BM. Numerical simulations were obtained for fluid pressure and BM displacement to validate the theoretical results and demonstrate that a hearing loss might be caused by an increase of fluid pressure.

2. TWO-DIMENSIONAL MODEL OF THE COCHLEA

2.1. Conservation of mass and momentum equations.

The inner ear macro-mechanics couple hydrodynamics of the perilymph with deformation of oval window, round window and BM structures. The vibrations of the stapes of the oval window introduce a pressure wave in the cochlea. The wave deflects the BM resulting a local pressure changes in the perilymph [12]. This fluid in the cochlea surrounding the BM is incompressible and assumed to be inviscid. The equations of motion of this fluid are well known. We let $u = (u_1, u_2, u_3)$ be the fluid velocity, p the pressure and ρ the density of the fluid [13]. The conservation of mass equation can be written in the following form:

$$\int_V \left\{ \frac{\partial \rho}{\partial t} + div(\rho u) \right\} dV = 0 \tag{2.1}$$

Where V is the volume. Since the fluid is incompressible: $\rho = cst$, so $\dfrac{\partial \rho}{\partial t} = 0$

$$\int_V div(\rho u)\, dV = 0 \Rightarrow \int_V divu\, dV = 0 \tag{2.2}$$

According to the fundamental principle of dynamics, we obtain the conservation of momentum equation.

$$-\int_S p n_i \, dS = \int_V \rho \left(\frac{\partial u}{\partial t} + u \nabla . u \right) dV \tag{2.3}$$

Using the divergence theorem to convert surface integrals to volume integrals with $n = (n_1, n_2, n_3)$ is the outward unit normal to V, p is the pressure and $X = (x, y, z)$, we obtain;

$$\int_V \left\{ \rho \frac{\partial u}{\partial t} + \rho u \nabla . u + \frac{\partial p}{\partial X} \right\} dV = 0. \tag{2.4}$$

2.2. **Mathematical Cochlear model.** The mechanical characteristics of the cochlea have been modeled in two-dimensional by Lesser and Berkeley. According this model, perilymph is considered as an incompressible fluid with a fixed volume and can be moved inside due to the excitations transmitted by the oval window into the scala vestibuli [14]. This has the advantage of generalizing the behavior of the cochlea in two rectangular compartments which represent to the scala vestibuli and the scala tympani separated by the BM [15] (see Figure 2).

The two-dimensional mathematical model of the cochlea is presented by;

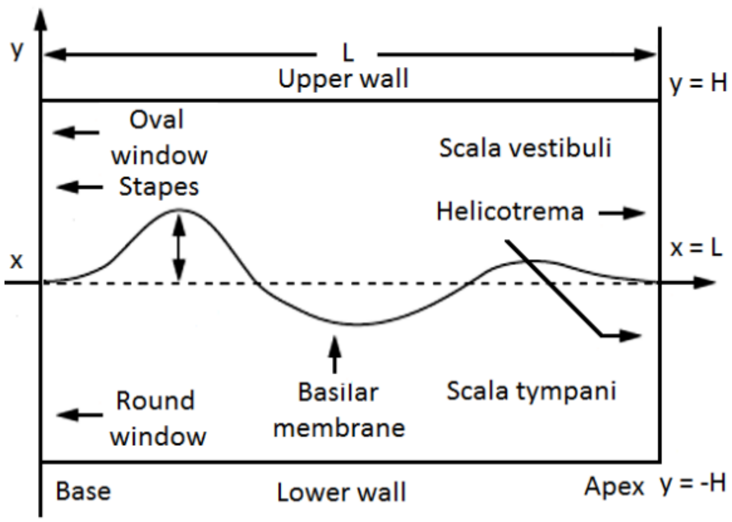

FIGURE 2. Two dimensional model of the cochlea [15].

where V is arbitrary, assuming that the density ρ is constant and according to conservation mass and moment equations, we obtain;

$$\begin{cases} \rho \dfrac{\partial u}{\partial t} + \nabla p & = 0 \\ \nabla . u & = 0 \end{cases} \tag{2.5}$$

So, from 2.5, we obtain;

$$\nabla p = -\rho \frac{\partial u}{\partial t} \tag{2.6}$$

The pressure function is required to satisfy Laplace's equation in the fluid;

$$\nabla^2 p = \Delta p = 0 \tag{2.7}$$

The upper-wall boundary is assumed to be perfectly rigid, i.e., this boundary has no motion. The upperwall boundary condition is;

$$\frac{\partial}{\partial y} p(x, H) = 0, \ 0 < x < L \tag{2.8}$$

The apical-end boundary represents the helicotrema and has no pressure difference across it. The apical boundary condition is;

$$p(L, y) = 0, \ 0 < y < H \tag{2.9}$$

The stapes and round window are assumed to move identically except in opposite directions. The basal boundary condition is;

$$\frac{\partial}{\partial x} p(0, y) = -2\rho a_s, \ 0 < y < H \tag{2.10}$$

where a_s is the acceleration of the stapes, we consider only sinusoidal excitation of the stapes at various frequencies, so $a_s = -w^2$;

The BM boundary condition is;

$$\frac{\partial}{\partial y} p(x, 0) = 2\rho a_b(x), \ 0 < x < L, \tag{2.11}$$

where $a_b(x)$ is the acceleration of the BM and ρ is the volume density of the fluid, with

$$a_b(x) = iwY(x)p(x, 0)$$

and

$$Y(x) = [k(x)/iw + c(x) + iwm(x)]^{-1}$$

Where $Y(x)$ is the acoustic admittance function, $k(x)$, $c(x)$, and $m(x)$ are the stiffness, damping and mass of the BM at position x, respectively.

So, the system of equation will be defined by;

$$\begin{cases} \Delta p(x, y) & = 0 \\ \dfrac{\partial}{\partial y} p(x, H) & = 0, \qquad 0 < x < L \\ \dfrac{\partial}{\partial x} p(0, y) & = -2\rho a_s, \quad 0 < y < H \\ \dfrac{\partial}{\partial y} p(x, 0) & = 2\rho a_b(x), \ 0 < x < L, \\ p(L, y) & = 0. \qquad 0 < y < H \end{cases} \tag{2.12}$$

The normal displacement of BM is modeled as a simple forced damped harmonic oscillator with mass $m(x)$, damping $c(x)$ and stiffness $k(x)$ that vary along of the length on the BM. The deflection of the membrane is then represented by a one-dimensional wave equation $\eta(x,t)$ which is the solution to the forced harmonic oscillator equation [16];

$$m(x)\frac{\partial^2\eta}{\partial t^2} + c(x)\frac{\partial\eta}{\partial t} + k(x)\eta = p. \tag{2.13}$$

The physical characteristic parameters of the BM proposed by Neely [16] are shown in the Table 1.

Table 1. Parameters of the BM proposed by Neely [16].

Parameter	symbol	Value	unit
mass	$m(x)$	0.15	g/cm^2
damping	$c(x)$	200	$dyn.s/cm^3$
stiffness	$k(x)$	$10^9\ e^{-2x}$	$dyn\ cm^3$

In order to solve the system of equations 2.12 with its boundary conditions numerically, we discretized the model using finite difference witch is a key and indispensable technology in the modelling and simulation procedures..

3. NUMERICAL RESULT

In the previous sections the model equations that describe the pressure of the fluid and of the BM displacement in a coiled cochlea have been formulated. To find solutions of this problem, we used the finite difference method to generate a discrete set of linear equations, the derivatives in equations are approximated by the conventional central differences:

$$\frac{\partial p}{\partial x} = \frac{p_{i+1}^j - p_{i-1}^j}{2\Delta x}; \qquad \frac{\partial p}{\partial y} = \frac{p_i^{j+1} - p_i^{j-1}}{2\Delta y}; \tag{3.1}$$

$$\frac{\partial^2 p}{\partial x^2} = \frac{p_{i+1}^j - 2p_i^j + p_{i-1}^j}{(\Delta x)^2}; \qquad \frac{\partial^2 p}{\partial y^2} = \frac{p_i^{j+1} - 2p_i^j + p_i^{j-1}}{(\Delta y)^2}; \tag{3.2}$$

This then turns system of equations 2.12 into a linear algebraic equation, which can be solved by straightforward matrix inversion. For all solutions the physical characteristic of BM is cited in the Table 1. The length and the height of the cochlea are, respectively, $L = 3.5\ cm$ and $H = 0.1\ cm$, density of the fluid $\rho = 1\ g/cm^3$. We divided the length of the cochlea into N_x elements and the height into the N_y elements, that gives N_xxN_y grid of points. At each point, we can write an equation for the pressure, in terms of this parameter at the neighboring points.

One goal of mathematical cochlear modelling is to replicate results observed in experiments. As it is already mentioned that a perturbation in the cochlear fluid leads to a decrease of the amplitude of the BM. So in this section we predict the effect of the increase of cochlear fluid pressure on the maximum amplitude displacement. First, we simulated the normal pressure of fluid cochlear, then we perturbed the system by an increase of the pressure value and we compare the both results.

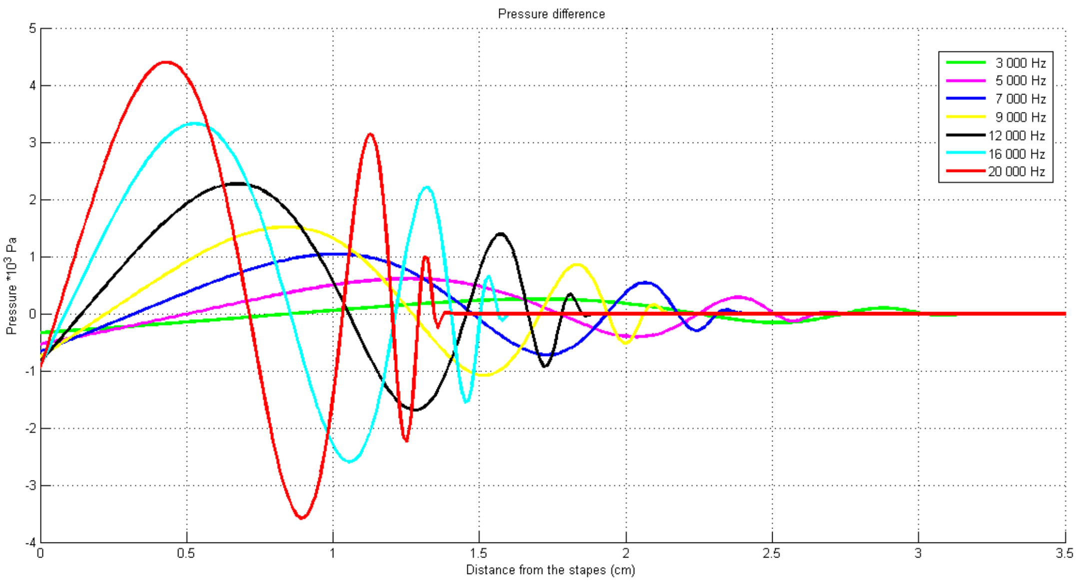

FIGURE 3. Variation of the pressure along the BM with different values of the frequencies .

The Figure 3 showed the results of the resolution of the system (12) using finite difference method. We taken different values of the frequencies, $3000Hz$, $5000Hz$, $7000Hz$, $9000Hz$, $12\ 000Hz$, $16\ 000Hz$, $20\ 000Hz$ and we obtained the variation of the pressure along the BM.

Our goal is to study the influence of the pressure on the maximum amplitude displacement of the BM, therefore, we simulated the normal pressure of each point with specific frequency and we obtained the amplitude of the displacement. For the same point of the BM, we applied a new perturbation that we called P_o (increasing of the fluid pressure) in order to show the difference between the normal and abnormal cases of the amplitude of the BM.

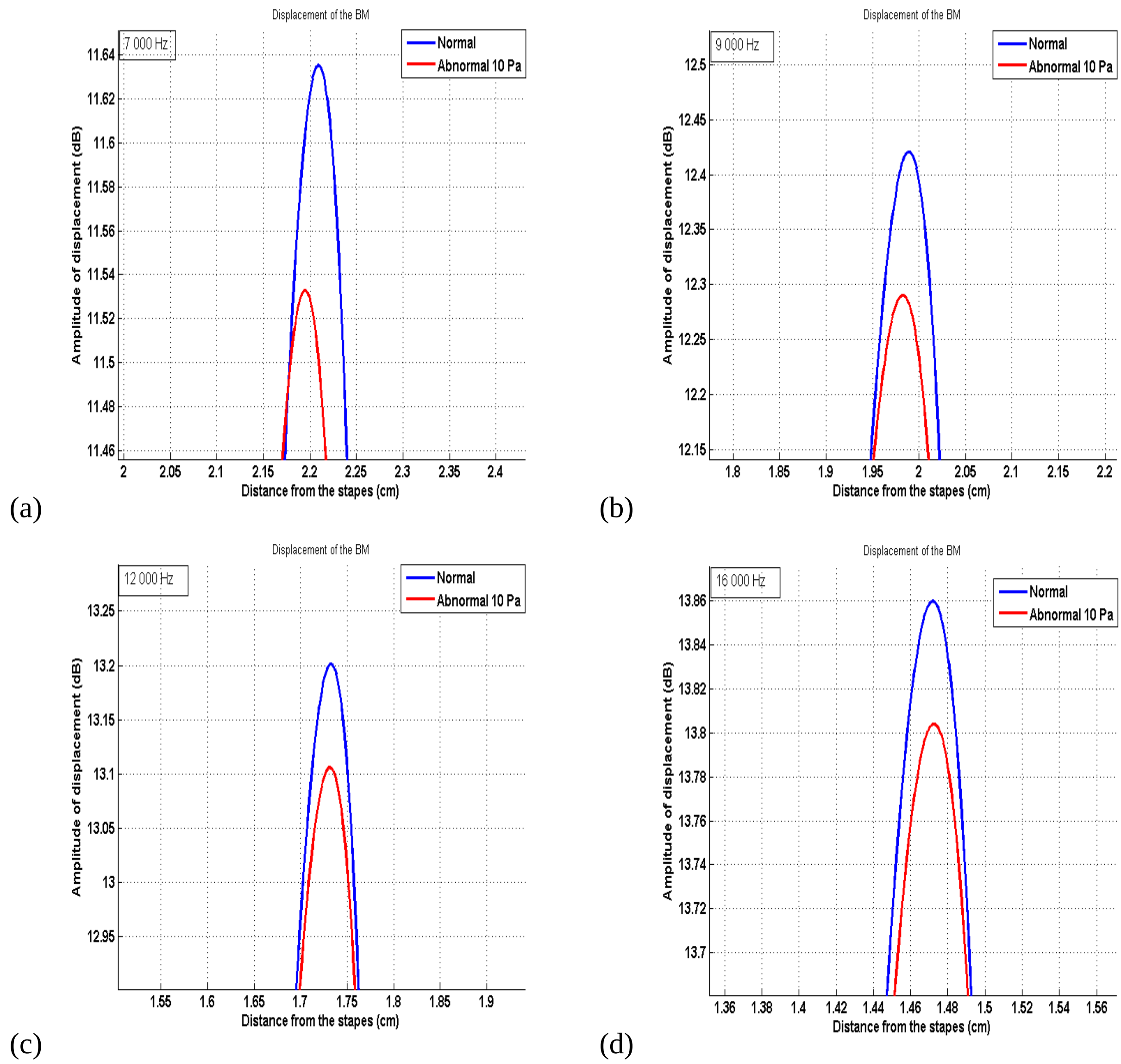

FIGURE 4. Displacement of the BM with $P_0 = 10$ Pa into different frequencies (a) $f = 7000\ Hz$, (b)$f = 9000\ Hz$,(c) $f = 12000\ Hz$ and (d) $f = 16000\ Hz$.

As shown in Figure 4, the increase of the pressure affects the maximum amplitude displacement of the BM. For example, we take $P_0 = 10$ Pa and we simulate the normal and abnormal cases, for different values of frequencies. Finally, we concluded that the amplitude decrease when the pressure of cochlear fluid increase.

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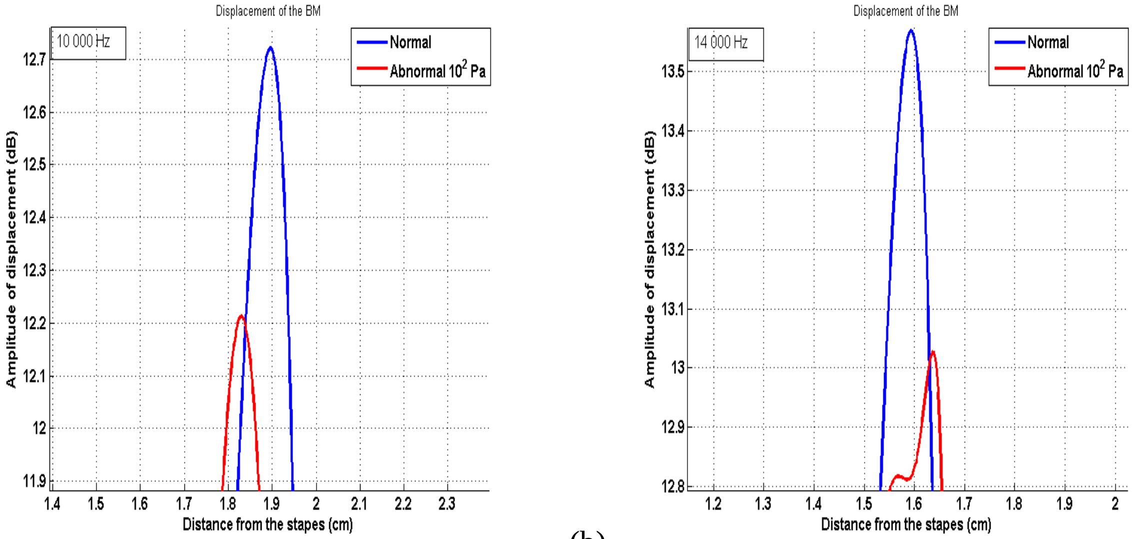

(a) (b)

FIGURE 5. Displacement of the BM with $P_0 = 10^2$ Pa into different frequency (a) $f = 10000\ Hz$ and (b) $f = 14000\ Hz$.

To confirm the previous results, we increased the perturbation P_0 to 10^2 Pa and we compared the results obtained for different frequencies.

The Figure 5 confirm that an increase of the pressure induced automatically a decrease of the amplitude. Therefore, this results suggests a relationship between the fluid pressure and the amplitude of the BM, which may lead to a hearing loss.

4. CONCLUSION

Our study is devoted to explain and analyze the dysfunction of the inner ear related to fluid pressure. In this paper, we have developed a mathematical model that established the effect of the increase of fluid pressure on the amplitude of the displacement of BM in order to validate the literature founding. Therefore, we added a new parameter that we called P_0 as a perturbation of pressure in the modeling equation and we solved the system using finite difference method. The numerical simulations obtained were helpful tools to compare the amplitude of BM in the both cases, normal and abnormal, with different values of the frequencies. Thus, when a perturbed pressure was applied on the BM, the amplitude of the BM decreased, these changes are considered to be the reasons of the hearing loss.

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