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AN ANALYTICAL SOLUTION OF PERTURBED FISHER'S EQUATION USING HOMOTOPY PERTURBATION METHOD (HPM), REGULAR PERTURBATION METHOD (RPM) AND ADOMIAN DECOMPOSITION METHOD (ADM)

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ABSTRACT. In this paper, Homotopy Perturbation Method (HPM), Regular Pertubation Method (RPM) and Adomian decomposition Method (ADM) are applied to Fisher equation. Then, the solution yielding the given initial conditions is gained. Finally, the solutions obtained by each method are compared.

Key words and phrases: Fisher equation, Homotopy Perturbation Method (HPM), Regular Pertubation Method (RPM), Adomian decomposition Method (ADM). .

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1. INTRODUCTION

The nonlinear reaction-diffusion equation

$$\frac{\partial u}{\partial t} = D\,\frac{\partial^2 u}{\partial x^2} + \varepsilon\, u(1-u)$$

was first introduced by Fisher as a model for the propagation of a mutant gene [9]. It has wide application in the fields of logistic population growth, flame propagation, neurophysiology, autocatalytic chemical reactions, branching Brownian motion processes, and nuclear reactor theory [18].

In chemistry the function $u(t,x)$ is the concentration of the reactant. D represents its diffusion coefficient, and the positive constant m specifies the rate of chemical reaction. In media of other natures, u might be temperature or electric potential, might be the thermal conductivity or specific electrical conductivity [18].

Several researches have been carried out for the resolution of Fisher's equation. Various powerful methods have been used to solve the Fisher's equation : Exp-function method in [18], Homotopy Perturbation method [6, 15], Haar wavelet method [10], Adomian Decomposition Method [5], Finite difference method [4], Variational Homotopy Perturbation Method [7].

In this work, we examine the following problem

$$\left\{\begin{array}{l} \dfrac{\partial u}{\partial t} = \dfrac{\partial^2 u}{\partial x^2} + \varepsilon\, u(1-u), \quad 0 \le x \le 1, \;\; 0 \le t \le T \\ \\ \qquad\qquad u(0,x) = \cos x \end{array}\right. \tag{1.1}$$

where $0 < \varepsilon \ll 1$

The paper is organised as follows : in section 1, we start with the solving (1.1) by HPM [11, 12, 13, 14, 17]. In Section 2 and section 3, we construct the solution of (1.1) respectively by RPM [8] and ADM [1, 2, 3, 16]. Section 4 contains the comparison of the solutions obtained by the different methods.

2. APPLICATION OF HPM TO FISHER'S EQUATION

In order to apply the HPM[11, 12, 13], we construct a homotopy $H(v,p)$ for equation (1.1) which satisfies :

$$H(v,p) = (1-p)\left[\frac{\partial v}{\partial t} - \frac{\partial u_0}{\partial t}\right] + p\left[\frac{\partial v}{\partial t} - \frac{\partial^2 v}{\partial x^2} + \varepsilon\, v(1-v)\right]$$

Where $p \in [0,\ 1]$.

As $H(v,p) = 0$, then we have :

$$\frac{\partial v}{\partial t} - \frac{\partial u_0}{\partial t} + p\frac{\partial u_0}{\partial t} - p\frac{\partial^2 v}{\partial x^2} - p\varepsilon\, v(1-v) = 0$$

Let as choose the initial approximation as $u_0 = \cos x$, thus $\dfrac{\partial u_0}{\partial t} = 0$

We have,

$$\frac{\partial v}{\partial t} - p\frac{\partial^2 v}{\partial x^2} - p\varepsilon\, v + p\varepsilon\, v^2 = 0 \tag{2.1}$$

Suppose that the solution of (1.1) is the form

$$v = v_0 + pv_1 + p^2 v_2 + p^3 v_3 + p^4 v_4 + p^5 v_5 + \cdots \tag{2.2}$$

In view of the HPM, substituting (2.2) into (2.1) and equating the coefficients of like power p, we get the following set of differential equations :

$$(2.3) \qquad p^0 \; : \; \begin{cases} \dfrac{\partial v_0}{\partial t} = 0 \\ v_0(0,x) = \cos x \end{cases}$$

$$(2.4) \qquad p^1 \; : \; \begin{cases} \dfrac{\partial v_1}{\partial t} - \dfrac{\partial^2 v_0}{\partial x^2} - \varepsilon v_0 + \varepsilon v_0^2 = 0 \\ v_1(0,x) = 0 \end{cases}$$

$$(2.5) \qquad p^2 \; : \; \begin{cases} \dfrac{\partial v_2}{\partial t} - \dfrac{\partial^2 v_1}{\partial x^2} - \varepsilon v_1 + 2\varepsilon v_0 v_1 = 0 \\ v_2(0,x) = 0 \end{cases}$$

$$(2.6) \qquad p^3 \; : \; \begin{cases} \dfrac{\partial v_3}{\partial t} - \dfrac{\partial^2 v_2}{\partial x^2} - \varepsilon v_2 + \varepsilon(2v_0 v_2 + v_1^2) = 0 \\ v_3(0,x) = 0 \end{cases}$$

$$(2.7) \qquad p^4 \; : \; \begin{cases} \dfrac{\partial v_4}{\partial t} - \dfrac{\partial^2 v_3}{\partial x^2} - \varepsilon v_3 + \varepsilon(2v_0 v_3 + 2v_1 v_2) = 0 \\ v_4(0,x) = 0 \end{cases}$$

$$(2.8) \qquad p^5 \; : \; \begin{cases} \dfrac{\partial v_5}{\partial t} - \dfrac{\partial^2 v_4}{\partial x^2} - \varepsilon v_4 + \varepsilon(2v_0 v_4 + 2v_1 v_3 + v_2^2) = 0 \\ v_5(0,x) = 0 \end{cases}$$

$$\vdots$$

Solve the system of (2.3) to (2.8) to get the solutions

$$\begin{aligned}
v_0(t,x) &= \cos x \\
v_1(t,x) &= \Bigg(\cos^3 x\,\varepsilon^2 - \frac{3\,\cos^2 x\,\varepsilon^2}{2} + \frac{\cos x\,\varepsilon^2}{2} - \sin^2 x\,\varepsilon + 2\,\cos^2 x\,\varepsilon \\
&\quad - \cos x\,\varepsilon + \frac{\cos x}{2}\Bigg)\, t^2 \\
v_2(t,x) &= \Bigg(-\cos^4 x\,\varepsilon^3 + 2\,\cos^3 x\,\varepsilon^3 - \frac{7\,\cos^2 x\,\varepsilon^3}{6} + \frac{\cos x\,\varepsilon^3}{6} \\
&\quad + \frac{8\,\cos x\,\sin^2 x\,\varepsilon^2}{3} - \frac{4\,\sin^2 x\,\varepsilon^2}{3} - 3\,\cos^3 x\,\varepsilon^2 + 3\,\cos^2 x\,\varepsilon^2 - \\
&\quad \frac{\cos x\,\varepsilon^2}{2} + 2\,\sin^2 x\,\varepsilon - \frac{8\,\cos^2 x\,\varepsilon}{3} + \frac{\cos x\,\varepsilon}{2} - \frac{\cos x}{6}\Bigg)\, t^3 \\
v_3(t,x) &= \Bigg(\cos^5 x\,\varepsilon^4 - \frac{5\,\cos^4 x\,\varepsilon^4}{2} + \frac{25\,\cos^3 x\,\varepsilon^4}{12} - \frac{5\,\cos^2 x\,\varepsilon^4}{8} + \frac{\cos x\,\varepsilon^4}{24} - \\
&\quad \frac{29\,\cos^2 x\,\sin^2 x\,\varepsilon^3}{6} + \frac{29\,\cos x\,\sin^2 x\,\varepsilon^3}{6} - \frac{11\,\sin^2 x\,\varepsilon^3}{12}
\end{aligned}$$

$$
\begin{aligned}
& +4\cos^4 x\,\varepsilon^3 - 6\cos^3 x\,\varepsilon^3 + \frac{7\cos^2 x\,\varepsilon^3}{3} - \frac{\cos x\,\varepsilon^3}{6} \\
& -\frac{32\cos x\,\sin^2 x\,\varepsilon^2}{3} + \frac{8\sin^2 x\,\varepsilon^2}{3} + \frac{37\cos^3 x\,\varepsilon^2}{6} - \frac{23\cos^2 x\,\varepsilon^2}{6} \\
& \left. +\frac{\cos x\,\varepsilon^2}{4} - \frac{7\sin^2 x\,\varepsilon}{3} + \frac{8\cos^2 x\,\varepsilon}{3} - \frac{\cos x\,\varepsilon}{6} + \frac{\cos x}{24}\right) t^4 \\
v_4(t,x) \;=\; & \left(\cos^5 x\,\varepsilon^4 - \frac{5\cos^4 x\,\varepsilon^4}{2} + \frac{25\cos^3 x\,\varepsilon^4}{12} - \frac{5\cos^2 x\,\varepsilon^4}{8} + \frac{\cos x\,\varepsilon^4}{24} - \right. \\
& \frac{29\cos^2 x\,\sin^2 x\,\varepsilon^3}{6} + \frac{29\cos x\,\sin^2 x\,\varepsilon^3}{6} - \frac{11\sin^2 x\,\varepsilon^3}{12} \\
& +4\cos^4 x\,\varepsilon^3 - 6\cos^3 x\,\varepsilon^3 + \frac{7\cos^2 x\,\varepsilon^3}{3} - \frac{\cos x\,\varepsilon^3}{6} \\
& -\frac{32\cos x\,\sin^2 x\,\varepsilon^2}{3} + \frac{8\sin^2 x\,\varepsilon^2}{3} + \frac{37\cos^3 x\,\varepsilon^2}{6} - \frac{23\cos^2 x\,\varepsilon^2}{6} \\
& \left. +\frac{\cos x\,\varepsilon^2}{4} - \frac{7\sin^2 x\,\varepsilon}{3} + \frac{8\cos^2 x\,\varepsilon}{3} - \frac{\cos x\,\varepsilon}{6} + \frac{\cos x}{24}\right) t^5 \\
v_5(t,x) \;=\; & \left(-\cos^6 x\,\varepsilon^5 + 3\cos^5 x\,\varepsilon^5 - \frac{13\cos^4 x\,\varepsilon^5}{4} + \frac{3\cos^3 x\,\varepsilon^5}{2} - \right. \\
& \frac{31\cos^2 x\,\varepsilon^5}{120} + \frac{\cos x\,\varepsilon^5}{120} + \frac{37\cos^3 x\,\sin^2 x\,\varepsilon^4}{5} \\
& -\frac{111\cos^2 x\,\sin^2 x\,\varepsilon^4}{10} + \frac{137\cos x\,\sin^2 x\,\varepsilon^4}{30} - \frac{13\sin^2 x\,\varepsilon^4}{30} \\
& -5\cos^5 x\,\varepsilon^4 + 10\cos^4 x\,\varepsilon^4 - \frac{\cos x\,\varepsilon^4}{24} - \frac{32\sin^4 x\,\varepsilon^3}{15} + \\
& \frac{422\cos^2 x\,\sin^2 x\,\varepsilon^3}{15} - \frac{189\cos x\,\sin^2 x\,\varepsilon^3}{10} + \frac{11\sin^2 x\,\varepsilon^3}{6} \\
& -\frac{163\cos^4 x\,\varepsilon^3}{15} - \frac{25\cos^3 x\,\varepsilon^4}{4} + \frac{5\cos^2 x\,\varepsilon^4}{4} + \frac{178\cos^3 x\,\varepsilon^3}{15} \\
& -\frac{43\cos^2 x\,\varepsilon^3}{15} + \frac{\cos x\,\varepsilon^3}{12} + \frac{364\cos x\,\sin^2 x\,\varepsilon^2}{15} - \frac{46\sin^2 x\,\varepsilon^2}{15} \\
& -\frac{317\cos^3 x\,\varepsilon^2}{30} + \frac{11\cos^2 x\,\varepsilon^2}{3} - \frac{\cos x\,\varepsilon^2}{12} + 2\sin^2 x\,\varepsilon - \\
& \left. \frac{32\cos^2 x\,\varepsilon}{15} + \frac{\cos x\,\varepsilon}{24} - \frac{\cos x}{120}\right) t^6
\end{aligned}
$$

The six-term HPM approximate solution of (1.1) is given by

$$
\begin{aligned}
u(t,x) & \simeq \lim_{p\to 1}\left[v_0(t,x) + p v_1(t,x) + \cdots + p^5 v_5(t,x)\right] \\
& \simeq v_0(t,x) + v_1(t,x) + \cdots + v_5(t,x)
\end{aligned}
$$

We gather according to the powers of ε,

$$
u(t,x) \;=\; \left[\cos x\left(1 - \frac{t^2}{2!} + \frac{t^4}{4!} - \frac{t^6}{6!} + \frac{t^8}{8!} - \frac{t^{10}}{10!} + \frac{t^{12}}{12!} - \frac{t^{14}}{14!} + \frac{t^{16}}{16!} - \right.\right.
$$

$$
\begin{aligned}
&\frac{t^{18}}{18!}+\frac{t^{20}}{20!}\Bigg)\Bigg]\\
&+\varepsilon\left[\cos^3 x\left(\frac{t^5}{20}-\frac{13\,t^7}{840}+\frac{71\,t^9}{30240}-\frac{491\,t^{11}}{3326400}+\frac{13711\,t^{13}}{1037836800}-\right.\right.\\
&\frac{28607\,t^{15}}{43589145600}+\frac{66811\,t^{17}}{2694601728000}-\frac{7198319\,t^{19}}{10137091700736000}+\\
&\frac{t^{21}}{13502619648000}-\frac{61\,t^{23}}{47826278793216000}+\\
&\left.\frac{547\,t^{25}}{28695767275929600000}\right)+\cos x\,\sin^2 x\left(\frac{t^7}{140}-\frac{t^9}{504}+\right.\\
&\frac{47\,t^{11}}{184800}-\frac{739\,t^{13}}{43243200}+\frac{34403\,t^{15}}{36324288000}-\frac{727\,t^{17}}{18712512000}+\\
&\frac{188947\,t^{19}}{153592298496000}-\frac{t^{21}}{4725916876800}\\
&\left.\left.+\frac{13\,t^{23}}{3416162770944000}-\frac{41\,t^{25}}{717394181898240000}\right)\right]\\
&+\varepsilon^2\left[\cos^5 x\left(-\frac{3\,t^8}{224}+\frac{439\,t^{10}}{100800}-\frac{23801\,t^{12}}{13305600}+\frac{743201\,t^{14}}{2421619200}-\right.\right.\\
&\frac{38239\,t^{16}}{931392000}+\frac{78121361\,t^{18}}{22230464256000}-\frac{1903\,t^{20}}{482236416000}+\\
&\left.\frac{5921\,t^{22}}{31191051386880}-\frac{297239\,t^{24}}{31304473391923200}\right)\\
&+\cos^3 x\,\sin^2 x\left(-\frac{13\,t^{10}}{2800}+\frac{3397\,t^{12}}{1663200}-\frac{352421\,t^{14}}{605404800}+\right.\\
&\frac{83371\,t^{16}}{825552000}-\frac{26118227\,t^{18}}{2223046425600}+\frac{1247\,t^{20}}{42195686400}-\\
&\frac{1191041\,t^{22}}{779776284672000}+\frac{11675131\,t^{24}}{215218254569472000}\\
&\left.+\frac{12517\,t^{25}}{1016715110400000}\right)+\cos x\,\sin^4 x\left(-\frac{13\,t^{12}}{61600}+\frac{1663\,t^{14}}{20180160}\right.\\
&-\frac{16447\,t^{16}}{864864000}+\frac{573889\,t^{18}}{205837632000}-\frac{2903\,t^{20}}{281304576000}\\
&+\frac{12517\,t^{23}}{20334302208000}-\frac{1191041\,t^{24}}{717394181898240000}-\\
&\left.\left.\frac{162721\,t^{25}}{12200581324800000}\right)\right]+\varepsilon^3\left[\cdots\right]+\cdots
\end{aligned}
$$

Or

$$
\begin{aligned}
u(t,x) \;=\;& e^t\cos x+\\
&\varepsilon\left[\cos^3 x\left(\frac{t^5}{20}-\frac{13\,t^7}{840}+\frac{71\,t^9}{30240}-\frac{491\,t^{11}}{3326400}+\frac{13711\,t^{13}}{1037836800}-\right.\right.\\
&\frac{28607\,t^{15}}{43589145600}+\frac{66811\,t^{17}}{2694601728000}-\frac{7198319\,t^{19}}{10137091700736000}+
\end{aligned}
$$

$$\frac{t^{21}}{13502619648000} - \frac{61\, t^{23}}{47826278793216000} + \frac{547\, t^{25}}{28695767275929600000}\Bigg) + \cos x\, \sin^2 x \left(\frac{t^7}{140} - \frac{t^9}{504} + \frac{47\, t^{11}}{184800} - \frac{739\, t^{13}}{43243200} + \frac{34403\, t^{15}}{36324288000} - \frac{727\, t^{17}}{18712512000} + \frac{188947\, t^{19}}{153592298496000} - \frac{t^{21}}{4725916876800} + \frac{13\, t^{23}}{3416162770944000} - \frac{41\, t^{25}}{717394181898240000}\right)\Bigg] + \cdots$$

3. THE REGULAR PERTURBATION METHOD

Let us suppose that the solution $u(t,x)$ of the initial value problem (1.1) has the following form [8] :

$$u(t,x) = \sum_{n=0}^{+\infty} \varepsilon^n u_n(t,x) \tag{3.1}$$

Putting (3.1) into (1.1), and collecting equal powers of ε we obtain a system of recurrent initial value problems

$$\varepsilon^0 \; : \; \begin{cases} \dfrac{\partial u_0}{\partial t} - \dfrac{\partial^2 u_0}{\partial x^2} = 0 \\ u_0(0,x) = \cos x \end{cases} \tag{3.2}$$

$$\varepsilon^1 \; : \; \begin{cases} \dfrac{\partial u_1}{\partial t} - \dfrac{\partial^2 u_1}{\partial x^2} - u_0 + u_0^2 = 0 \\ u_1(0,x) = 0 \end{cases} \tag{3.3}$$

$$\varepsilon^2 \; : \; \begin{cases} \dfrac{\partial u_2}{\partial t} - \dfrac{\partial^2 u_2}{\partial x^2} - u_1 + 2u_0u_1 = 0 \\ u_2(0,x) = 0 \end{cases} \tag{3.4}$$

$$\varepsilon^3 \; : \; \begin{cases} \dfrac{\partial u_3}{\partial t} - \dfrac{\partial^2 u_3}{\partial x^2} - u_2 + 2u_0u_2 + u_1^2 = 0 \\ u_3(0,x) = 0 \end{cases} \tag{3.5}$$

$$\varepsilon^4 \; : \; \begin{cases} \dfrac{\partial u_4}{\partial t} - \dfrac{\partial^2 u_4}{\partial x^2} - u_3 + 2u_0u_3 + 2u_1u_2 = 0 \\ u_4(0,x) = 0 \end{cases} \tag{3.6}$$

$$\varepsilon^5 \; : \; \begin{cases} \dfrac{\partial u_5}{\partial t} - \dfrac{\partial^2 u_5}{\partial x^2} - u_4 + 2u_0u_4 + 2u_1u_3 + u_2^2 = 0 \\ u_5(0,x) = 0 \end{cases} \tag{3.7}$$

$$\vdots$$

To solve the above equations, we use ADM and we obtain :

$$\begin{aligned}
u_0(t,x) &= e^t\cos x\\
u_1(t,x) &= 2\,\cos x\,e^{-t}+\sin^2 x\,e^{-2\,t}-2\,\cos^2 x\,e^{-2\,t}\\
u_2(t,x) &= -\frac{3\,\sin^2 x\,e^{-2\,t}}{2}-\frac{3\,\cos^2 x\,e^{-2\,t}}{2}+\frac{10\,\cos x\,\sin^2 x\,e^{-3\,t}}{9}\\
&\quad+\frac{10\,\cos^3 x\,e^{-3\,t}}{9}\\
u_3(t,x) &= \frac{5\,\sin^2 x\,e^{-2\,t}}{2}-\frac{19\,\cos^2 x\,e^{-2\,t}}{2}-\frac{644\,\cos x\,\sin^2 x\,e^{-3\,t}}{27}+\\
&\quad\frac{652\,\cos^3 x\,e^{-3\,t}}{27}-\frac{26\,\sin^4 x\,e^{-4\,t}}{9}+\frac{259\,\cos^2 x\,\sin^2 x\,e^{-4\,t}}{9}\\
&\quad-\frac{40\,\cos^4 x\,e^{-4\,t}}{3}\\
u_4(t,x) &= \frac{29\,\sin^2 x\,e^{-2\,t}}{2}-\frac{43\,\cos^2 x\,e^{-2\,t}}{2}-\frac{14132\,\cos x\,\sin^2 x\,e^{-3\,t}}{81}+\\
&\quad-\frac{926\,\sin^4 x\,e^{-4\,t}}{27}+\frac{13285\,\cos^2 x\,\sin^2 x\,e^{-4\,t}}{27}-\frac{536\,\cos^4 x\,e^{-4\,t}}{3}\\
&\quad+\frac{396\,\cos x\,\sin^4 x\,e^{-5\,t}}{5}-\frac{3388\,\cos^3 x\,\sin^2 x\,e^{-5\,t}}{9}\\
&\quad+\frac{3796\,\cos^5 x\,e^{-5\,t}}{45}\\
u_5(t,x) &= \frac{101\,\sin^2 x\,e^{-2\,t}}{2}-\frac{115\,\cos^2 x\,e^{-2\,t}}{2}-\\
&\quad\frac{267452\,\cos x\,\sin^2 x\,e^{-3\,t}}{243}+\frac{129124\,\cos^3 x\,e^{-3\,t}}{243}-\\
&\quad\frac{182585\,\sin^4 x\,e^{-4\,t}}{324}+\frac{906533\,\cos^2 x\,\sin^2 x\,e^{-4\,t}}{162}-\\
&\quad\frac{18787\,\cos^4 x\,e^{-4\,t}}{12}+\frac{201944\,\cos x\,\sin^4 x\,e^{-5\,t}}{75}\\
&\quad-\frac{6736804\,\cos^3 x\,\sin^2 x\,e^{-5\,t}}{675}+\frac{49544\,\cos^5 x\,e^{-5\,t}}{27}\\
&\quad+\frac{106682\,\sin^6 x\,e^{-6\,t}}{1215}-\frac{3500128\,\cos^2 x\,\sin^4 x\,e^{-6\,t}}{1215}\\
&\quad+\frac{6971032\,\cos^4 x\,\sin^2 x\,e^{-6\,t}}{1215}-\frac{903908\,\cos^6 x\,e^{-6\,t}}{1215}
\end{aligned}$$

Hence, the approximate solution of (1.1) is given by :

$$\begin{aligned}
u(t,x) &\simeq u_0(t,x)+\varepsilon u_1(t,x)+\cdots+\varepsilon^5 u_5(t,x)\\
&= e^t\cos x+\\
&\quad\varepsilon\left[\cos^3 x\left(\frac{t^5}{20}-\frac{13\,t^7}{840}+\frac{71\,t^9}{30240}-\frac{491\,t^{11}}{3326400}+\frac{13711\,t^{13}}{1037836800}-\right.\right.
\end{aligned}$$

$$\frac{28607\,t^{15}}{43589145600}+\frac{66811\,t^{17}}{2694601728000}-\frac{7198319\,t^{19}}{10137091700736000}+\frac{t^{21}}{13502619648000}-\frac{61\,t^{23}}{47826278793216000}+\frac{547\,t^{25}}{28695767275929600000}\Bigg)+\cos x\,\sin^2 x\left(\frac{t^7}{140}-\frac{t^9}{504}+\frac{47\,t^{11}}{184800}-\frac{739\,t^{13}}{43243200}+\frac{34403\,t^{15}}{36324288000}-\frac{727\,t^{17}}{18712512000}+\frac{188947\,t^{19}}{153592298496000}-\frac{t^{21}}{4725916876800}+\frac{13\,t^{23}}{3416162770944000}-\frac{41\,t^{25}}{717394181898240000}\right)\Bigg]+\cdots$$

4. APPLICATION OF ADM TO FISHER'S EQUATION

Defining the operators :

$$L_t(\bullet)=\frac{\partial}{\partial t}(\bullet),\quad L_x(\bullet)=\frac{\partial^2}{\partial x^2}(\bullet),\quad Nu=(u)^2\ \text{ et }\ L_t^{-1}(\bullet)=\int_0^t(\bullet)ds$$

Equation (1.1) can be written as :

$$L_tu-L_xu\ =\ \varepsilon u-\varepsilon Nu \tag{4.1}$$

Applying L_t^{-1} to (4.1), we obtain :

$$u(t,x)=u(0,x)+L_t^{-1}\left[L_xu(t,x)\right]+\varepsilon L_t^{-1}\left[u(t,x)\right]-\varepsilon L_t^{-1}\left[Nu(t,x)\right] \tag{4.2}$$

Assuming that the solution of (1.1) can be given by :

$$u(t,x)=\sum_{n=0}^{+\infty}u_n(t,x) \tag{4.3}$$

and

$$Nu(t,x)=\sum_{n=0}^{+\infty}A_n(t,x) \tag{4.4}$$

where A_n are the AdomianâĂŹs polynomials with

$$A_n=\frac{1}{n!}\left[\frac{d^n}{d\lambda^n}\left(N\left(\sum_{i=0}^{+\infty}\lambda^iu_i\right)\right)\right]_{\lambda=0}$$

By substituting (4.3) and (4.4) into (4.2), we obtain the Adomian algorithm

$$\begin{cases} u_0(t,x)=u(0,x)=\cos x \\ u_{n+1}(t,x)=L_t^{-1}\left[L_xu_n(t,x)\right]+\varepsilon L_t^{-1}\left[u_n(t,x)\right]-\varepsilon L_t^{-1}\left[A_n(t,x)\right],\ n\geq 0\end{cases}$$

with

$$\begin{aligned} A_0 &= u_0^2\\ A_1 &= 2u_0u_1\\ A_2 &= 2u_0u_2+u_1^2\\ A_3 &= 2u_0u_3+2u_1u_2\end{aligned}$$

$$\begin{aligned}
A_4 &= 2u_0u_4 + 2u_1u_3 + u_2^2 \\
A_5 &= 2u_0u_5 + 2u_1u_4 + 2u_2u_3 \\
A_6 &= 2u_0u_6 + 2u_1u_5 + 2u_2u_4 + u_3^2 \\
A_7 &= 2u_0u_7 + 2u_1u_6 + 2u_2u_5 + 2u_3u_4
\end{aligned}$$

Using the algorithm, we have :

$$\begin{aligned}
u_0(t,x) &= \cos x \\
u_1(t,x) &= \left(\cos^3 x\,\varepsilon^2 - \frac{3\,\cos^2 x\,\varepsilon^2}{2} + \frac{\cos x\,\varepsilon^2}{2} - \sin^2 x\,\varepsilon + 2\,\cos^2 x\,\varepsilon \right.\\
&\left. -\cos x\,\varepsilon + \frac{\cos x}{2}\right) t^2 \\
u_2(t,x) &= \left(-\cos^4 x\,\varepsilon^3 + 2\,\cos^3 x\,\varepsilon^3 - \frac{7\,\cos^2 x\,\varepsilon^3}{6} + \frac{\cos x\,\varepsilon^3}{6}\right.\\
&+\frac{8\,\cos x\,\sin^2 x\,\varepsilon^2}{3} - \frac{4\,\sin^2 x\,\varepsilon^2}{3} - 3\,\cos^3 x\,\varepsilon^2 + 3\,\cos^2 x\,\varepsilon^2 - \\
&\left.\frac{\cos x\,\varepsilon^2}{2} + 2\,\sin^2 x\,\varepsilon - \frac{8\,\cos^2 x\,\varepsilon}{3} + \frac{\cos x\,\varepsilon}{2} - \frac{\cos x}{6}\right) t^3 \\
u_3(t,x) &= \left(\cos^5 x\,\varepsilon^4 - \frac{5\,\cos^4 x\,\varepsilon^4}{2} + \frac{25\,\cos^3 x\,\varepsilon^4}{12} - \frac{5\,\cos^2 x\,\varepsilon^4}{8} + \frac{\cos x\,\varepsilon^4}{24} - \right.\\
&\frac{29\,\cos^2 x\,\sin^2 x\,\varepsilon^3}{6} + \frac{29\,\cos x\,\sin^2 x\,\varepsilon^3}{6} - \frac{11\,\sin^2 x\,\varepsilon^3}{12} \\
&+4\,\cos^4 x\,\varepsilon^3 - 6\,\cos^3 x\,\varepsilon^3 + \frac{7\,\cos^2 x\,\varepsilon^3}{3} - \frac{\cos x\,\varepsilon^3}{6} \\
&-\frac{32\,\cos x\,\sin^2 x\,\varepsilon^2}{3} + \frac{8\,\sin^2 x\,\varepsilon^2}{3} + \frac{37\,\cos^3 x\,\varepsilon^2}{6} - \frac{23\,\cos^2 x\,\varepsilon^2}{6} \\
&\left.+\frac{\cos x\,\varepsilon^2}{4} - \frac{7\,\sin^2 x\,\varepsilon}{3} + \frac{8\,\cos^2 x\,\varepsilon}{3} - \frac{\cos x\,\varepsilon}{6} + \frac{\cos x}{24}\right) t^4 \\
u_4(t,x) &= \left(\cos^5 x\,\varepsilon^4 - \frac{5\,\cos^4 x\,\varepsilon^4}{2} + \frac{25\,\cos^3 x\,\varepsilon^4}{12} - \frac{5\,\cos^2 x\,\varepsilon^4}{8} + \frac{\cos x\,\varepsilon^4}{24} - \right.\\
&\frac{29\,\cos^2 x\,\sin^2 x\,\varepsilon^3}{6} + \frac{29\,\cos x\,\sin^2 x\,\varepsilon^3}{6} - \frac{11\,\sin^2 x\,\varepsilon^3}{12} \\
&+4\,\cos^4 x\,\varepsilon^3 - 6\,\cos^3 x\,\varepsilon^3 + \frac{7\,\cos^2 x\,\varepsilon^3}{3} - \frac{\cos x\,\varepsilon^3}{6} \\
&-\frac{32\,\cos x\,\sin^2 x\,\varepsilon^2}{3} + \frac{8\,\sin^2 x\,\varepsilon^2}{3} + \frac{37\,\cos^3 x\,\varepsilon^2}{6} - \frac{23\,\cos^2 x\,\varepsilon^2}{6} \\
&\left.+\frac{\cos x\,\varepsilon^2}{4} - \frac{7\,\sin^2 x\,\varepsilon}{3} + \frac{8\,\cos^2 x\,\varepsilon}{3} - \frac{\cos x\,\varepsilon}{6} + \frac{\cos x}{24}\right) t^5 \\
u_5(t,x) &= \left(-\cos^6 x\,\varepsilon^5 + 3\,\cos^5 x\,\varepsilon^5 - \frac{13\,\cos^4 x\,\varepsilon^5}{4} + \frac{3\,\cos^3 x\,\varepsilon^5}{2} - \right.\\
&\frac{31\,\cos^2 x\,\varepsilon^5}{120} + \frac{\cos x\,\varepsilon^5}{120} + \frac{37\,\cos^3 x\,\sin^2 x\,\varepsilon^4}{5}
\end{aligned}$$

$$-\frac{111\cos^2 x\sin^2 x\,\varepsilon^4}{10}+\frac{137\cos x\sin^2 x\,\varepsilon^4}{30}-\frac{13\sin^2 x\,\varepsilon^4}{30}$$
$$-5\cos^5 x\,\varepsilon^4+10\cos^4 x\,\varepsilon^4-\frac{\cos x\,\varepsilon^4}{24}-\frac{32\sin^4 x\,\varepsilon^3}{15}+$$
$$\frac{422\cos^2 x\sin^2 x\,\varepsilon^3}{15}-\frac{189\cos x\sin^2 x\,\varepsilon^3}{10}+\frac{11\sin^2 x\,\varepsilon^3}{6}$$
$$-\frac{163\cos^4 x\,\varepsilon^3}{15}-\frac{25\cos^3 x\,\varepsilon^4}{4}+\frac{5\cos^2 x\,\varepsilon^4}{4}+\frac{178\cos^3 x\,\varepsilon^3}{15}$$
$$-\frac{43\cos^2 x\,\varepsilon^3}{15}+\frac{\cos x\,\varepsilon^3}{12}+\frac{364\cos x\sin^2 x\,\varepsilon^2}{15}-\frac{46\sin^2 x\,\varepsilon^2}{15}$$
$$-\frac{317\cos^3 x\,\varepsilon^2}{30}+\frac{11\cos^2 x\,\varepsilon^2}{3}-\frac{\cos x\,\varepsilon^2}{12}+2\sin^2 x\,\varepsilon-$$
$$\left.\frac{32\cos^2 x\,\varepsilon}{15}+\frac{\cos x\,\varepsilon}{24}-\frac{\cos x}{120}\right)t^6$$

Or

$$\begin{aligned}
u(t,x) &\simeq u_0(t,x)+u_1(t,x)+\cdots+u_5(t,x)\\
&= e^t\cos x+\\
&\quad \varepsilon\left[\cos^3 x\left(\frac{t^5}{20}-\frac{13\,t^7}{840}+\frac{71\,t^9}{30240}-\frac{491\,t^{11}}{3326400}+\frac{13711\,t^{13}}{1037836800}-\right.\right.\\
&\quad \frac{28607\,t^{15}}{43589145600}+\frac{66811\,t^{17}}{2694601728000}-\frac{7198319\,t^{19}}{10137091700736000}+\\
&\quad \frac{t^{21}}{13502619648000}-\frac{61\,t^{23}}{47826278793216000}+\\
&\quad \left.\frac{547\,t^{25}}{28695767275929600000}\right)+\cos x\sin^2 x\left(\frac{t^7}{140}-\frac{t^9}{504}+\right.\\
&\quad \frac{47\,t^{11}}{184800}-\frac{739\,t^{13}}{43243200}+\frac{34403\,t^{15}}{36324288000}-\frac{727\,t^{17}}{18712512000}+\\
&\quad \frac{188947\,t^{19}}{153592298496000}-\frac{t^{21}}{4725916876800}\\
&\quad \left.\left.+\frac{13\,t^{23}}{3416162770944000}-\frac{41\,t^{25}}{717394181898240000}\right)\right]+\cdots
\end{aligned}$$

5. COMPARISON OF THE APPROXIMATE SOLUTIONS

In this section we analyze the approximate solutions of (1.1) are obtained by the three numerical methods (ADM, RPM and ADM).

The solution in equation (1.1) which obtained by HPM is absolutely same as that of the solution obtained by ADM. But, the approximate solution obtained by RPM, differs from the one obtained by ADM and HPM.

The tables (5.1), (5.3) and (5.5) give some values of the approximate solutions obtained by the three methods. One notes a significant variation of the values obtained by RPM when ε becomes increasingly large.

The tables (5.2), (5.4) and (5.6)show absolute error between the various approximate solutions. It is noticed that the absolute error between the solutions of RPM and ADM just as RPM and HPM increases when ε becomes increasingly large.

x	u_{adm}	u_{rpm}	u_{hpm}
0	0.8187	0.8188	0.8187
0.1	0.8146	0.8147	0.8146
0.2	0.8146	0.8024	0.8146
0.3	0.7822	0.7822	0.7822
0.4	0.7541	0.7541	0.7541
0.5	0.7185	0.7186	0.7185
0.6	0.6757	0.6758	0.6757
0.7	0.6262	0.6263	0.6262
0.8	0.5704	0.5705	0.5704
0.9	0.5089	0.5090	0.5089
1	0.4424	0.4425	0.4424

Table 5.1: Approximate solutions by ADM, RPM and HPM, for $t = 0.2$ *and* $\varepsilon = 0.0001$

x	$\lvert u_{adm} - u_{rpm}\rvert$	$\lvert u_{adm} - u_{hpm}\rvert$	$\lvert u_{hpm} - u_{rpm}\rvert$
0	1×10^{-4}	0	1×10^{-4}
0.1	1×10^{-4}	0	1×10^{-4}
0.2	0	0	0
0.3	0	0	0
0.4	0	0	0
0.5	1×10^{-4}	0	1×10^{-4}
0.6	1×10^{-4}	0	1×10^{-4}
0.7	1×10^{-4}	0	1×10^{-4}
0.8	1×10^{-4}	0	1×10^{-4}
0.9	1×10^{-4}	0	1×10^{-4}
1	1×10^{-4}	0	1×10^{-4}

Table 5.2: Absolute error for variables x *from 0 to 1 and* $t = 0.2$ *and* $\varepsilon = 0.0001$

x	u_{adm}	u_{rpm}	u_{hpm}
0	0.6065	0.6066	0.6065
0.1	0.6035	0.6036	0.6035
0.2	0.5945	0.5945	0.5945
0.3	0.5795	0.5795	0.5795
0.4	0.5587	0.5588	0.5587
0.5	0.5323	0.5324	0.5323
0.6	0.5006	0.5007	0.5006
0.7	0.4639	0.4640	0.4639
0.8	0.4226	0.4227	0.4226
0.9	0.3770	0.3772	0.3770
1	0.3277	0.3279	0.3277

Table 5.3: Approximate solutions by ADM, RPM and HPM, for $t = 0.5$ and $\varepsilon = 0.0002$

x	$\lvert u_{adm} - u_{rpm}\rvert$	$\lvert u_{adm} - u_{hpm}\rvert$	$\lvert u_{hpm} - u_{rpm}\rvert$
0	0.1×10^{-3}	0	0.1×10^{-3}
0.1	0.1×10^{-3}	0	0.1×10^{-3}
0.2	0	0	0
0.3	0	0	0
0.4	0.1×10^{-3}	0	0.1×10^{-3}
0.5	0.1×10^{-3}	0	0.1×10^{-3}
0.6	0.1×10^{-3}	0	0.1×10^{-3}
0.7	0.1×10^{-3}	0	0.1×10^{-3}
0.8	0.1×10^{-3}	0	0.1×10^{-3}
0.9	0.2×10^{-3}	0	0.2×10^{-3}
1	0.2×10^{-3}	0	0.2×10^{-3}

Table 5.4: Absolute error for variables x from 0 to 1 and $t = 0.5$ and $\varepsilon = 0.0002$

x	u_{adm}	u_{rpm}	u_{hpm}
0	0.4495	0.4508	0.4495
0.1	0.4473	0.4486	0.4473
0.2	0.4406	0.4419	0.4406
0.3	0.4295	0.4308	0.4295
0.4	0.4141	0.4154	0.4141
0.5	0.3945	0.3959	0.3945
0.6	0.3710	0.3724	0.3710
0.7	0.3439	0.3453	0.3439
0.8	0.3132	0.3147	0.3132
0.9	0.2795	0.2809	0.2795
1	0.2429	0.2443	0.2429

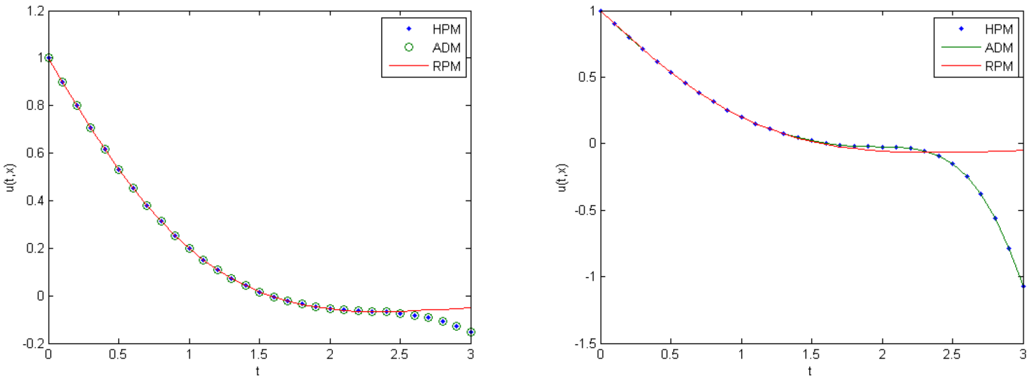
Table 5.5: Approximate solutions by ADM, RPM and HPM, for $t = 0.8$ *and* $\varepsilon = 0.003$

x	$\lvert u_{adm} - u_{rpm}\rvert$	$\lvert u_{adm} - u_{hpm}\rvert$	$\lvert u_{hpm} - u_{rpm}\rvert$
0	0.0013	0	0.0013
0.1	0.0013	0	0.0013
0.2	0.0013	0	0.0013
0.3	0.0013	0	0.0013
0.4	0.0013	0	0.0013
0.5	0.0014	0	0.0014
0.6	0.0014	0	0.0014
0.7	0.0014	0	0.0014
0.8	0.0015	0	0.0015
0.9	0.0014	0	0.0014
1	0.0014	0	0.0014

Table 5.6: Absolute error for variables x *from 0 to 1 and* $t = 0.8$ *and* $\varepsilon = 0.003$

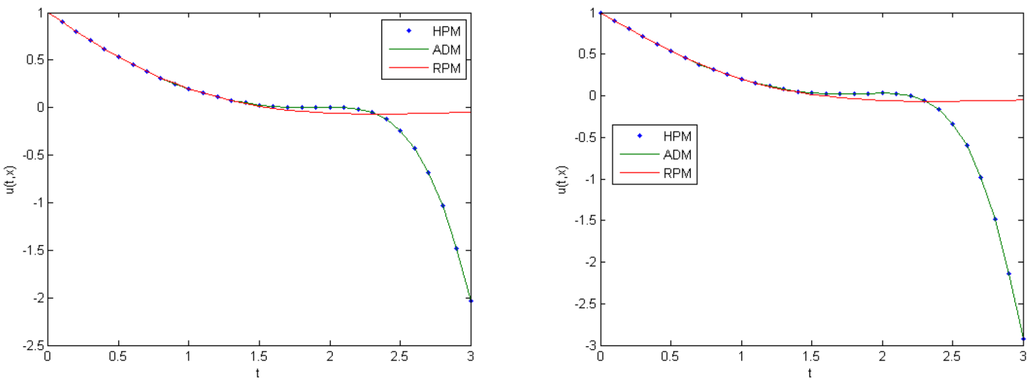
The figures (1) and (2) give the comparison of the approximate solutions in dimension 2, obtained by the three methods.

In dimension 3, we obtain the figures (3), (4),(5) and (6).



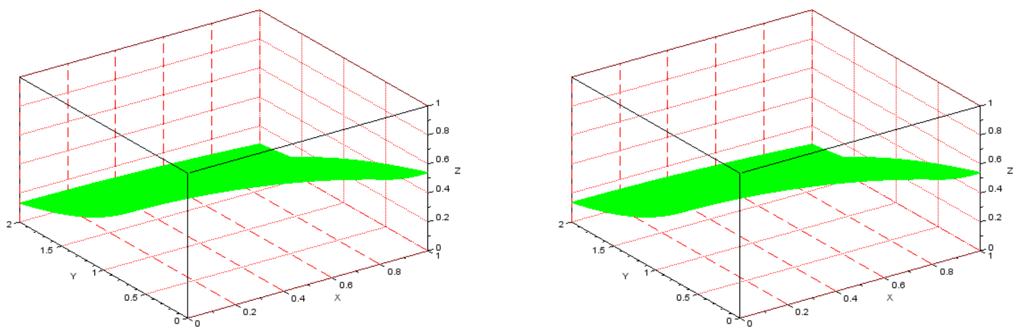
(a) $\varepsilon = 0.0001$ (b) $\varepsilon = 0.001$

Figure 1: Comparison of the HPM solution, ADM solution and RPM solution



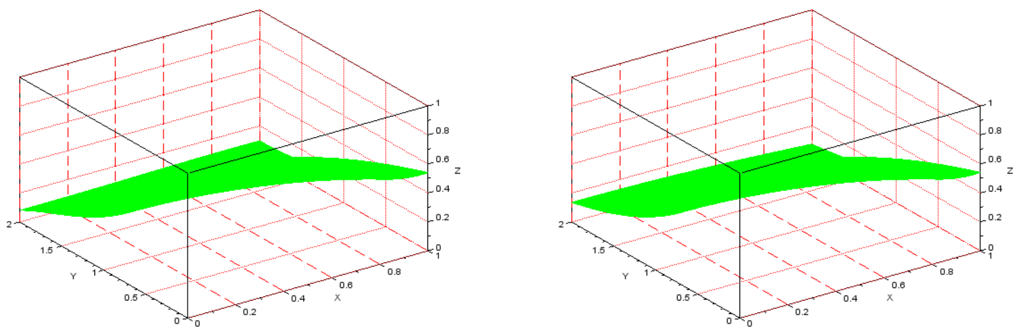
(a) $\varepsilon = 0.002$ (b) $\varepsilon = 0.003$

Figure 2: Comparison of the HPM solution, ADM solution and RPM solution



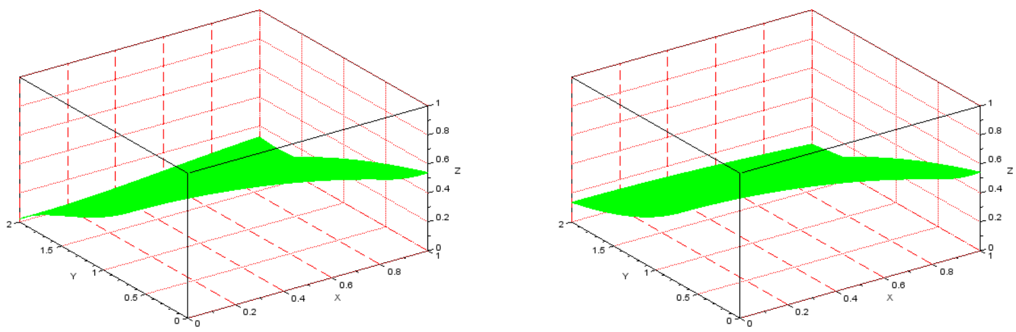
(a) HPM solution (b) RPM solution

Figure 3: Comparison of the HPM solution with RPM solution for $\varepsilon = 0.0001$



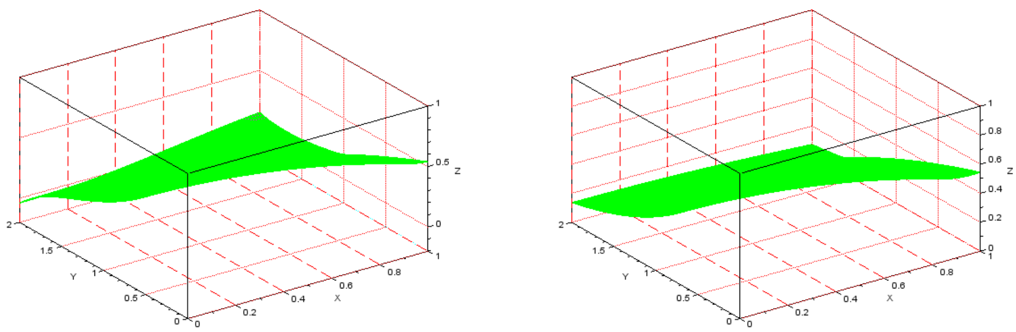
(a) ADM solution (b) RPM solution

Figure 4: Comparison of the HPM solution with RPM solution for $\varepsilon = 0.001$



(a) HPM solution (b) RPM solution

Figure 5: Comparison of the HPM solution with RPM solution for $\varepsilon = 0.002$



(a) ADM solution (b) RPM solution

Figure 6: Comparison of the HPM solution with RPM solution for $\varepsilon = 0.003$

6. CONCLUSION

In the present study, the HPM, ADM and RPM was applied on perturbed Fisher's equation. The results obtained by three methods were comapared. RPM solution differs from the ADM and HPM solution, but ADM and HPM give the same approximate solution. This research reveals that although the obtained results by HPM and ADM are the same, HPM are much easier, more convenient, and efficient in comparison. Different from ADM, where specific algorithms are usually used to determine the Adomian polynomials, HPM handle linear and nonlinear problem in simple manner by deforming a difficult problem into a simple one. In addition, the RPM solution is very sensitive to the variations of the perturbation parameter.

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