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STABILITY ANALYSIS EPIDEMIC MODEL OF SIR TYPE (SUSCEPTIBLE, INFECTIOUS, RECOVERED) ON THE SPREAD DYNAMIC OF MALARIA IN AMBON CITY

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ABSTRACT. This research discusses the spread of malaria in Ambon city through SIR (Susceptible, Infected, Recovered) model. This research analyzes the stability of equilibrium point on deterministic model and Basic Reproduction Ratio (R_0). The result shows that the epidemic model has two equilibrium points. They are disease free equilibrium, $[E_0 = [S, I, R] = [1, 0, 0]]$, and epidemic equilibrium, $E_i = [S, I, R] = [5, 497808; -3, 680341; -0, 818288]$. The Basic Reproduction Ratio of malaria disease in Ambon city is $0, 181891$. This result implies that malaria disease in Ambon will not be an endemic, because malaria disease in Ambon city will be eradicated slowly over time.

Key words and phrases: Basic Reproductive Ratio (R_0); Malaria; SIR Model.

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1. INTRODUCTION

The Research of Basic Health [4] from 2013 indicates that incidence malaria on the population in Indonesia is 1,9%. This is a lower number than in 2007 (2,9%). However, in West of Papua, the total number of malaria sufferers was increasing rapidly. Among the average population the incidence of malaria sufferer in 2013 is 6,0%. The five provinces with highest average of malaria sufferers are, in order: Papua (9,8% and 28,6%), East of Nusa Tenggara (6,8% and 23,3%), West of Papua (6,7% and 19,4%), Centre of Sulawesi (5,1% and 12,5%), and Moluccas (3,8% and 10,7%) [4].

Moluccas is one of the provinces in the East of Indonesia that have a higher average of malaria sufferer than the Indonesian national average. Its geographical condition is divided between coast and swamp lands, which cause all areas in Moluccas be the endemic(malaria disease spread space) [3].

Because contagious diseases can often be uncontrolled, it is important to research the spread patterns. Mathematical models allow a better understanding by use of an epidemic model. This research will discuss a mathematical model of malaria disease spread in Ambon City, Moluccas using epidemic model of SIR type (Susceptible, Infectious, Recovered). The assumptions we used in this research are: Birth and natural death rate are constants, There is homogeneous interaction in the population, There is no vaccination for the individual, and Population is continuous in time.

2. SIR EPIDEMIC MODEL

On the mathematical model of spread malaria disease, population (N) divide to be 3 sub population :Susceptible (S),Infected (I), and Recovered (R). We use the following notation and definitions for the human population [1]

Susceptible $[S(t)]$	:	The number of individuals who can be infected but have not yet contracted the malaria fever but may contract it if exposed to any mode of its transmission.
Infected $[I(t)]$	:	The number of individuals who have been infected or malaria fever.
Recovered $[R(t)]$	:	The number of individuals who are recovered after treatment and are immune to the disease.

Total individual of compartments in the time is $S(t), I(t)$ and $R(t)$. So, $N(t) = S(t)+I(t)+R(t)$, and can be shown as Figure 1 below:

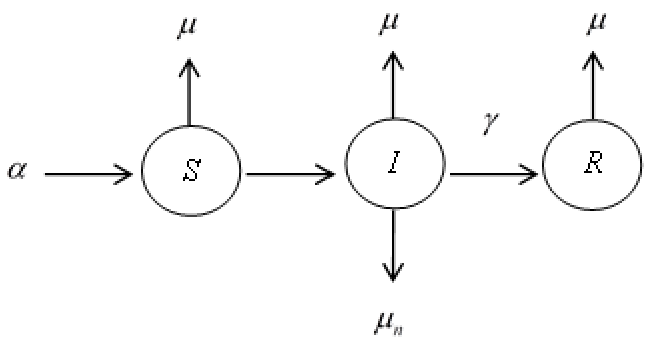

Figure 1: SIR (Susceptible, Infected, and Recovered) Epidemic Model

This model can be described by ordinary differential equations as follows:

$$\frac{dS}{dt} = \alpha - \beta SI - \mu S \tag{2.1}$$

$$\frac{dI}{dt} = \beta SI - I(\gamma + \mu_m + \mu) \tag{2.2}$$

$$\frac{dR}{dt} = \gamma I - \mu R \tag{2.3}$$

where,
$\frac{dS}{dt}$ = Total susceptible in the time,
$\frac{dI}{dt}$ = Total Infected in the time, and
$\frac{dR}{dt}$ = Total Recovered in the time
γ = The rate of recovery $(\gamma \geq 0)$
β = The rate of infection $(\beta \geq 0)$
α = Birth rate
μ = Death because natural factor
μ_m = Death because malaria disease

In determining the equilibrium points of the model, we will use the equations (2.1) – (2.3) with the constant position of the time.

3. Result And Discussion

In this section we formerly will determine the equilibrium of the SIR model of epidemic disease given by equations (2.1)-(2.3), with the constant position of the time, they are: $\frac{dS}{dt} = 0$, $\frac{dI}{dt} = 0$ and $\frac{dR}{dt} = 0$.So we get the following result:

$$\beta SI - I(\gamma + \mu_m + \mu) = 0 \; then \; I = 0 \; or \; \beta S - (\gamma + \mu_m + \mu) = 0 \tag{3.1}$$

$$\alpha - \beta SI - \mu S = 0 \; then \; S = \frac{\alpha}{\mu} \tag{3.2}$$

$$\gamma I - \mu R = 0 \; then \; R = 0 \tag{3.3}$$

Thus, the disease free equilibrium is $E_0 = [S, I, R] = [\frac{\alpha}{\mu}, 0, 0]$, which means that if the total susceptible is only depended on the total birth and the total death then there is no people who are infected so no people that recovered too. From the equation (3.1), we get :

$$S = \frac{\gamma + \mu_m + \mu}{\beta} \tag{3.4}$$

then, if we substitute equation (3.4) into equation (3.2) the we get:

$$\alpha - \beta SI - \mu S = 0$$

$$\alpha - (\gamma + \mu_m + \mu)I - \mu\Big[\frac{\gamma + \mu_m + \mu}{\beta}\Big] = 0$$

$$I = \Big[\frac{\alpha\beta - \mu((\gamma + \mu_m + \mu))}{\beta(\gamma + \mu_m + \mu)}\Big] \tag{3.5}$$

then we sustitute equation (3.5) into equation (3.3), then we get:

$$\gamma I - \mu R = 0$$

$$\gamma\Big[\frac{\alpha\beta - \mu(\gamma + \mu_m + \mu)}{\beta(\gamma + \mu_m + \mu)}\Big] - \mu R = 0$$

$$R = \gamma\Big[\frac{\alpha\beta - \mu(\gamma + \mu_m + \mu)}{\beta\mu(\gamma + \mu_m + \mu)}\Big] \tag{3.6}$$

so, we get the epidemic equilibrium from equation (3.4), (3.5) and (3.6), i.e:

$$E_i = [S, I, R] = \left[\Big(\frac{\gamma + \mu_m + \mu}{\beta}\Big), \Big(\frac{\alpha\beta - \mu(\gamma + \mu_m + \mu)}{\beta(\gamma + \mu_m + \mu)}\Big), \gamma\Big(\frac{\alpha\beta - \mu(\gamma + \mu_m + \mu)}{\beta\mu(\gamma + \mu_m + \mu)}\Big)\right]$$

The next step in this research is determining the stability of each equilibrium points. We use the Jacobian matrix of the model and the substitues the equilibrium points. the jacobian matrix of the SIR model is given as follow:

$$\mathbf{J} = \begin{bmatrix} -\beta I - \mu & -\beta S & 0 \\ \beta I & \beta S - (\gamma + \mu + \mu_m) & 0 \\ 0 & \gamma & -\mu \end{bmatrix}$$

From this jacobian matrix, then we substitute each g the equilibrium points, they are:

(1) Disease free equilibrium

We substitute the disease free equilibrium, $E_0 = [S, I, R] = [\frac{\alpha}{\mu}, 0, 0]$ into the jacobian matrix, J, above, and we get:

$$\mathbf{J_{E_0}} = \begin{bmatrix} -\mu & -\beta\frac{\alpha}{\mu} & 0 \\ 0 & \beta\frac{\alpha}{\mu} - (\gamma + \mu + \mu_m) & 0 \\ 0 & \gamma & -\mu \end{bmatrix}$$

Matrix J_{E_0} has the below eigen value:

$$\lambda_1 = -\mu, \lambda_2 = \beta\frac{\alpha}{\mu} - (\gamma + \mu + \mu_m)\ and\ \lambda_3 = -\mu \tag{3.7}$$

the parameter μ (number of the death because of natural factor) is always had the positive value. So, two of three the eigen values of matrix jacobian, J_{E_0}, have the negative value. They are $\lambda_1 < 0$ and $\lambda_3 < 0$. The system could be stable and finite to equilibrium point $E_0 = [S, I, R] = [\frac{\alpha}{\mu}, 0, 0]$ if $\beta\frac{\alpha}{\mu} < (\gamma + \mu + \mu_m)$.

(2) Epidemic equilibrium We substitute the disease free equilibrium, $E_i = [S, I, R] = \left[\Big(\frac{\gamma+\mu_m+\mu}{\beta}\Big), \Big(\frac{\alpha\beta-\mu(\gamma+\mu_m+\mu)}{\beta(\gamma+\mu_m+\mu)}\Big), \gamma\Big(\frac{\alpha\beta-\mu(\gamma+\mu_m+\mu)}{\beta\mu(\gamma+\mu_m+\mu)}\Big)\right]$ into the jacobian matrix, J, and we get:

$$\mathbf{J_{E_i}} = \begin{bmatrix} -\beta\Big(\frac{\alpha\beta-\mu(\gamma+\mu_m+\mu)}{\beta(\gamma+\mu_m+\mu)}\Big) - \mu & -\beta\Big(\frac{(\gamma+\mu_m+\mu)}{\beta}\Big) & 0 \\ \beta\Big(\frac{\alpha\beta-\mu(\gamma+\mu_m+\mu)}{\beta(\gamma+\mu_m+\mu)}\Big) & \beta\Big(\frac{(\gamma+\mu_m+\mu)}{\beta} - (\gamma + \mu + \mu_m) & 0 \\ 0 & \gamma & -\mu \end{bmatrix}$$

Matrix J_{E_i} has the below eigen value:

$$\lambda_1 = -\mu, \lambda_2 = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \; and \; \lambda_3 = \frac{-b - \sqrt{b^2 - 4ac}}{2a} \quad (3.8)$$

where, $a = 1$, $b = \left(\frac{\alpha\beta}{(\gamma+\mu_m+\mu)}\right)$ and $c = \alpha\beta - \mu(\gamma + \mu_m + \mu)$ The system will be stable if all of its eigen values have the negative value. λ_1 has the negative one, and the eigen value λ_2 and λ_3 has the negative one if it is reach the condition:
$\frac{-b+\sqrt{b^2-4ac}}{2a} < 0$ or $\frac{\alpha\beta}{\mu} > (\gamma + \mu_m + \mu)$.

From condition of stability on disease free equilibrium and the epidemic equilibrium shows that if the disease free equilibrium is stable, then the epidemic equilibrium is unstable, vice versa. Functional of Basic Reproduction Ratio on determining the infected rate is very urgent. In order to lost from infection of malaria, we have to have $R_0 < 1$. We have the second differential of I on variable t as follow:

$$\frac{d}{dI}\left(\frac{dI}{dt}\right) = \frac{d}{dI}\left(\frac{\beta S}{N} - (\gamma + \mu_m + \mu)\right)I$$

So, $C = \left(\frac{\beta S}{N} - (\gamma + \mu_m + \mu)\right)$. Then, we substitute the disease free equilibrium into C and we get $C = \beta - (\gamma + \mu_m + \mu)\big)$. If $C = P - U$, so $P = \beta$ and $U = \gamma + \mu_m + \mu$ [2]. The basic reproduction ratio, R_0, as follow:

$$R_0 = PU^{-1} = \frac{\beta}{\gamma + \mu_m + \mu}$$

Epidemic threshold is a value or number that there or no spread on the disease. From equation (2.2), infected can be prevented if $\frac{dI}{dt} < 0$ be constant on the variable time t. So, the value of epidemic threshold:

$$S_c = \frac{(\gamma + \mu_m + \mu)}{\beta}$$

The last step of this research is use the case study of malaria disease spread in Ambon. Here is the value of each variables:

Table 1. The value of Each Variables of the SIR model in Ambon

Data Type	**Total Number**	**Data Type**	**Total Number**
Susceptible (S)	383.286	Birth	31.857
Natural Death	31.857	Recovered (R)	7.083
Infected (I)	7.083	Population	383.286

From the data in Table 1, we can determine the following variables value:
$\alpha = 0.083115$, $\beta = 0.018479$, $\mu = 0.083115$,$\mu_m = 0$, and $\gamma = 0.018479$. Thus, the solution of equilibrium point as follows:

(1) Disease Free Equilibrium Point $E_0 = [S, I, R] = [\frac{\alpha}{\mu}, 0, 0]$, so we get that:

$$S = \frac{\alpha}{\mu} = \frac{0.083115}{0.083115} = 1; I = 0; R = 0$$

So, the disease free equilibrium is $E_0 = [1, 0, 0]$. The eigen value can be calculated with substitute the parameter into equation (3.7). So, we found that:
$\lambda_1 = -0.083115$, $\lambda_2 = -0.083115$ and $\lambda_3 = -0.083115$.

(2) Epidemic Equilibrium Point

$E_i = [S, I, R] = \left[\left(\frac{\gamma+\mu_m+\mu}{\beta}\right), \left(\frac{\alpha\beta-\mu(\gamma+\mu_m+\mu)}{\beta(\gamma+\mu_m+\mu)}\right), \gamma\left(\frac{\alpha\beta-\mu(\gamma+\mu_m+\mu)}{\beta\mu(\gamma+\mu_m+\mu)}\right) \right]$, so we get that:

$S = \frac{(\gamma+\mu_m+\mu)}{\beta} = 5.497808$, $I = \frac{\alpha\beta-\mu(\gamma+\mu_m+\mu)}{\beta(\gamma+\mu_m+\mu)} = -3.680341$ and

$R = \gamma\left(\frac{\alpha\beta-\mu(\gamma+\mu_m+\mu)}{\beta\mu(\gamma+\mu_m+\mu)}\right) = -0.818288$.

So, the epidemic equilibrium is $E_i = [5.497808, -3.680341, -0.818288]$. The eigen value can be calculated with substitute the parameter into equation (3.8). So, we found that:

$\lambda_1 = -0.083115$, $\lambda_2 = 0.0758985$ and $\lambda_3 = -0.0910175$.

From the result in case study, the analysis process shows that all the eigen values of disease free equilibrium is negative, but there is an eigen value on the epidemic equilibrium which value is positive . It is make epidemic equilibrium unstable. Thus, We know that eigen value that stable is eigen value on the disease free equilibrium. It means that the system will reach the equilibrium $(S = 1, I = 0, R = 0)$.

The basic reproduction ratio (R_0) can be calculated s follows:

$$R_0 = \frac{\beta}{\gamma + \mu_m + \mu} = 0.181891$$

It is showed that the number of $R_0 < 1$, it means that the infection of malaria will be decrease, so the spread of malaria disease in Ambon is not become to be an endemic. But, this condition can be constant in this area if the following condition is hold:

(1) There is no people who death naturally.
(2) Infected rate is same with recovered rate and death rate, for death cause malaria or natural death.

The malaria can be spread if the infected rate is more than total of recovered rate and death rate. Through the basic reproduction ratio, we found the value of threshold:

$$S_c = \frac{(\gamma + \mu_m + \mu)}{\beta} = 5.497808$$

where, $S(t) = S(0)e^{-\frac{R(t)}{\rho}} = 2.990251$ (use the formula in [5]).

It means that the value of S(t) is less than S_c . Hence, the spread of malaria disease on the population in Ambon will be decrease. It is the condition that be our hope because will going to the condition that free disease of malaria.

4. CONCLUSION

This research shows that the basic reproduction ratio $(R_0) < 1$, it means that the spread of malaria in Ambon City will be decrease, so Ambon City wouldn't have endemic of malaria disease. For the next research, we can use the other models, for example: SEIR, SIS, SIRS, etc.

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