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PARA-CHAOTIC TUPLES OF OPERATORS

BAHMANN YOUSEFI AND JAVAD IZADI

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DEPARTMENT OF MATHEMATICS, PAYAME NOOR UNIVERSITY, P.O. BOX 19395-3697, TEHRAN IRAN
b_yousefi@pnu.ac.ir
javadie2003@yahoo.com

ABSTRACT. In this paper, we introduce para-chaotic tuples of operators and we give some relations between para-chaoticity and Hypercyclicity Criterion for a tuple of operators.

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1. INTRODUCTION

By an n-tuple of operators we mean a finite sequence of length n of commuting continuous linear operators on a Banach space X.

Definition 1.1. Let $\mathcal{T} = (T_1, T_2, ..., T_n)$ be an n-tuple of operators acting on an infinite dimensional Banach space X. We will let

$$\mathcal{F} = \{{T_1}^{k_1}{T_2}^{k_2}...{T_n}^{k_n} : k_i \geq 0, i = 1, ..., n\}$$

be the semigroup generated by $\mathcal{T}$. For $x \in X$, the orbit of x under the tuple $\mathcal{T}$ is the set

$$Orb(\mathcal{T}, x) = \{Sx : S \in \mathcal{F}\}.$$

A vector x is called a hypercyclic vector for $\mathcal{T}$ if $Orb(\mathcal{T}, x)$ is dense in X and in this case the tuple $\mathcal{T}$ is called hypercyclic. The set of all hypercyclic vectors of $\mathcal{T}$ is denoted by $HC(\mathcal{T})$. Also, for all $k \geq 2$, by $\mathcal{T}_d^{(k)}$ we will refer to the set of all k copies of an element of $\mathcal{F}$, i.e.

$$\mathcal{T}_d^{(k)} = \{S_1 \oplus ... \oplus S_k : S_1 = ... = S_k \in \mathcal{F}\}.$$

We say that $\mathcal{T}_d^{(k)}$ is hypercyclic provided there exist $x_1, ..., x_k \in X$ such that

$$\{W(x_1 \oplus ... \oplus x_k) : W \in \mathcal{T}_d^{(k)}\}$$

is dense in the k copies of X, $X \oplus ... \oplus X$.

Note that if $T_1, T_2, ..., T_n$ are commutative bounded linear operators on a Banach space X, and $\{m_j(i)\}_j$, is a sequence of natural numbers for $i = 1, ..., n$, then we say

$$\{{T_1}^{m_j(1)}{T_2}^{m_j(2)}...{T_n}^{m_j(n)} : j \geq 0\}$$

is hypercyclic if there exists $x \in X$ such that

$$\{{T_1}^{m_j(1)}{T_2}^{m_j(2)}...{T_n}^{m_j(n)}x : j \geq 0\}$$

is dense in X.

Definition 1.2. We say that a tuple $\mathcal{T} = (T_1, T_2, ..., T_n)$ is topologically transitive with respect to a tuple of nonnegative integer sequences

$$(\{k_{j(1)}\}_j, \{k_{j(2)}\}_j, ..., \{k_{j(n)}\}_j),$$

if for every nonempty open subsets U, V of X there exists $j_0 \in \mathtt{N}$ such that

$$T_1^{k_{j_0(1)}}T_2^{k_{j_0(2)}}...T_n^{k_{j_0(n)}}(U) \cap V \neq \emptyset.$$

Also, we say that an n-tuple $\mathcal{T}$ is topologically transitive if it is topologically transitive with respect an n-tuple of nonnegative integer sequences. Similarly, We say that $\mathcal{T}_d^{(2)}$ is topologically transitive provided for any given nonempty open sets U_1, V_1, U_2, V_2 in X, there exist two positive integers m_i, $i = 1, ..., n$, such that

$$T_1^{m_1}T_2^{m_2}...T_n^{m_n}(U_1) \cap V_1 \neq \emptyset$$

and

$$T_1^{m_1}T_2^{m_2}...T_n^{m_n}(U_2) \cap V_2 \neq \emptyset.$$

Definition 1.3. Let $\mathcal{T} = (T_1, T_2, ..., T_n)$ be a tuple of bounded linear operators acting on a separable infinite dimensional Banach space X. Then, $x \in X$ is called a periodic point of $\mathcal{T}$ if there exists a tuple $(m_1, m_2, ..., m_n)$ of nonnegative integers such that

$$T_1^{m_1}T_2^{m_2}...T_n^{m_n}x = x.$$

Definition 1.4. Let $\mathcal{T} = (T_1, T_2, ..., T_n)$ be a tuple of continuous linear operators acting on a separable infinite dimensional Banach space X. A point $x \in X$ is said almost periodic for $\mathcal{T}$ if $Orb(\mathcal{T}, x)$ is precompact.

Definition 1.5. Let $\mathcal{T} = (T_1, T_2, ..., T_n)$ be a tuple of continuous linear operators acting on a separable infinite dimensional Banach space X. We say that $\mathcal{T}$ is chaotic if it is hypercyclic and admits a dense set of periodic points. Also, we say that $\mathcal{T}$ is para-chaotic if $\mathcal{T}$ contains a dense set of periodic points and for any positive integers $j_1, j_2, ..., j_n$, the sequence

$$\{T_1^{mj_1} T_2^{mj_2} ... T_n^{mj_n} : \ m = 0, 1, ...\}$$

is hypercyclic.

A nice criterion namely the Hypercyclicity Criterion was developed independently by Kitai [15], Gethner and Shapiro [12]. This criterion has used to show that hypercyclic operators arise within the class of composition operators [6], weighted shifts [18], adjoints of multiplication operators [7], and adjoints of subnormal and hyponormal operators [5], and hereditarily operators [4], and topologically mixing operators [8]. The formulation of the Hypercyclicty Criterion for a pair of operators was given by N. S. Feldman [11]. Here, we want to extend some properties of hypercyclic operators to a tuple of commuting operators. For some topics we refer to [1]–[29].

2. MAIN RESULTS

In this section we characterize the relation between the Hypercyclicity Criterion and a chaotic tuple.

Lemma 2.1. *Let $\mathcal{T} = (T_1, T_2, ..., T_n)$ be a tuple of continuous operators acting on a separable infinite dimensional Banach space X. Then $\mathcal{T}$ is topologically transitive if and only if it is hypercyclic.*

Proof. Let $\mathcal{T}$ be topologically transitive and fix an enumeration $\{B_n : n \in \mathbb{N}\}$ of the open balls in X with rational radii, and centers in a countable dense subset of X. By the continuity of the operators $T_i, \ i = 1, ..., n$, the sets

$$G_j = \bigcup \{T_1^{-k(1)} T_2^{-k(2)} ... T_n^{-k(n)}(B_j) : k(i) \geq 0; \ i = 1, ..., n\}$$

are open. Clearly, HC($\mathcal{T}$) is equal to $\bigcap\{G_j : j \in \mathbb{N}\}$. Now, let W be an arbitrary nonempty open set in X. Then, for all $m \in \mathbb{N}$ and $i = 1, ..., n$, there exist $k_m(i)$ in $\mathbb{N}$ such that

$$T_1^{k_m(1)} {T_2}^{k_m(2)} ... {T_n}^{k_m(n)}(W) \cap B_m \neq \emptyset,$$

which implies that $W \cap G_m \neq \emptyset$ for all m. Thus each G_m is dense and so $HC(\mathcal{T})$ is also dense in X. In particular, $HC(\mathcal{T})$ is nonempty and so $\mathcal{T}$ is hypercyclic. Conversely, let $\mathcal{T}$ be hypercyclic and (U, V) be a pair of nonempty open subsets of X. Let $x \in HC(\mathcal{T})$. Since $Orb(\mathcal{T}, x) \subset HC(\mathcal{T})$, thus $HC(\mathcal{T})$ is dense and so the sets $U \cap HC(\mathcal{T})$ and $V \cap HC(\mathcal{T})$ are nonempty. Choose $x \in U \cap HC(\mathcal{T})$ and $y \in V \cap HC(\mathcal{T})$. Since $V \cap Orb(\mathcal{T}, x)$ is nonempty, thus there there exists a tuple $m_1, m_2, ..., m_n$ of integers such that

$$T_1^{m_1} {T_2}^{m_2} ... {T_n}^{m_n}(x) \in V \cap HC(\mathcal{T}),$$

and so

$$T_1^{m_1} {T_2}^{m_2} ... {T_n}^{m_n}(U) \cap V \neq \emptyset.$$

Thus $\mathcal{T}$ is topologically transitive. This completes the proof. ∎

Note that, $\mathcal{T}$ is said to satisfy the Hypercyclicity Criterion if it satisfies the hypothesis of the following theorem.

Theorem 2.2. *(Hypercyclicity Criterion for tuples) Suppose that X is a separable infinite dimensional Banach space and $\mathcal{T} = (T_1, T_2, ..., T_n)$ be the n-tuple of operators T_1, T_2, ..., T_n acting on X. If there exist two dense subsets Y and Z in X, and strictly increasing sequences $\{m_{j(i)}\}_j$ for $i = 1, ..., n$ such that :*
1. $T_1^{m_{j(1)}}...T_n^{m_{j(n)}}y \to 0$ for all $y \in Y$
2. There exist a sequence of functions $\{S_j : Z \to X\}$ such that for every $z \in Z$, $S_j z \to 0$, and $T_1^{m_{j(1)}}...T_n^{m_{j(n)}}S_j z \to z$,
then $\mathcal{T}$ is a hypercyclic tuple.

Proof. Let U and V be two nonempty open sets in X and choose $y \in Y \cap U$ and $z \in Z \cap V$. Define $x_j = y + S_j z$. Then $x_j \to y$ and we have

$$T_1^{m_{j(1)}}...T_n^{m_{j(n)}}x_j = T_1^{m_{j(1)}}...T_n^{m_{j(n)}}y + T_1^{m_{j(1)}}...T_n^{m_{j(n)}}S_j z$$

which tends to z as $j \to \infty$. Thus for large j, we have $x_j \in U$ and

$$T_1^{m_{j(1)}}...T_n^{m_{j(n)}}x_j \in V.$$

Hence we get

$$T_1^{m_{j(1)}}...T_n^{m_{j(n)}}(U) \cap V \neq \emptyset$$

and so $\mathcal{T}$ is topologically transitive. Thus by Lemma 2.1, $\mathcal{T}$ is a hypercyclic tuple. This completes the proof. ∎

Proposition 2.3. *Let $\mathcal{T} = (T_1, T_2, ..., T_n)$ be a tuple of bounded linear operators acting on a separable infinite dimensional Banach space X and $HC(\mathcal{T}) \neq \emptyset$. If there exists a dense set of almost periodic points for $\mathcal{T}$, then $\mathcal{T}$ satisfies the Hypercyclicity Criterion.*

Proof. Denote the dense subset of almost periodic points of $\mathcal{T}$ by D. Let $x \in HC(\mathcal{T})$ and define $U_k = B(0, 1/k)$ and $V_k = B(x, 1/k)$. Since $\mathcal{T}$ is hypercyclic, thus there exists a tuple

$$(\{m_k^{(1)}\}_k, \{m_k^{(2)}\}_k, ..., \{m_k^{(n)}\}_k)$$

of integer sequences such that

$$T_1^{m_k^{(1)}}T_2^{m_k^{(2)}}...T_n^{m_k^{(n)}}(U_k) \cap V_k \neq \emptyset.$$

Since $\overline{HC(\mathcal{T})} = X$ and

$$T_1^{-m_k^{(1)}}T_2^{-m_k^{(2)}}...T_n^{-m_k^{(n)}}(V_k) \cap U_k \neq \emptyset,$$

thus there exists a hypercyclic vector u_k such that $u_k \to 0$ and

$$T_1^{m_k^{(1)}}T_2^{m_k^{(2)}}...T_n^{m_k^{(n)}}(u_k) \in V_k$$

for all k. Also, note that since $\overline{D} = X$, thus $D \cap B(x, 1/2k) \neq \emptyset$. Now choose $y \in D \cap B(x, 1/2k)$. Since $Orb(\mathcal{T}, y)$ is precompact, there exist a tuple of subsequences

$$(\{m_{k_j}^{(1)}\}_j, \{m_{k_j}^{(2)}\}_j, ..., \{m_{k_j}^{(n)}\}_j)$$

of the tuple of sequences

$$(\{m_k^{(1)}\}_k, \{m_k^{(2)}\}_k, ..., \{m_k^{(n)}\}_k),$$

and $y_0 \in X$, $k_0 \in \mathbb{N}$ such that

$$\|T_1^{m_{k_j}^{(1)}}T_2^{m_{k_j}^{(2)}}...T_n^{m_{k_j}^{(n)}}(y) - y_0\| \leq 1/2k$$

for all $j \geq k_0$.

But, $Orb(\mathcal{T}, x)$ is dense in X, thus there exists a tuple of integers $(m_0^{(1)}, m_0^{(2)}, ..., m_0^{(n)})$ such that
$$\|T_1^{m_0^{(1)}} T_2^{m_0^{(2)}} ... T_n^{m_0^{(n)}}(x) + y_0\| \leq 1/2k.$$
Note that
$$T_1^{m_k^{(1)}} T_2^{m_k^{(2)}} ... T_n^{m_k^{(n)}} u_k \to x,$$
thus we can choose a tuple of large enough integers
$$(m_{k_{j_0}}^{(1)}, m_{k_{j_0}}^{(2)}, ..., m_{k_{j_0}}^{(n)}) \in (\{m_{k_j}^{(1)}\}_j, \{m_{k_j}^{(2)}\}_j, ..., \{m_{k_j}^{(n)}\}_j)$$
with $j_0 \geq k_0$ such that
$$\|u_{k_{j_0}}\| \leq 1/(2k\|T_1^{m_0^{(1)}} T_2^{m_0^{(2)}} ... T_n^{m_0^{(n)}}\|)^{-1},$$
and
$$\|T_1^{m_0^{(1)}} T_2^{m_0^{(2)}} ... T_n^{m_0^{(n)}} T_1^{m_{k_{j_0}}^{(1)}} T_2^{m_{k_{j_0}}^{(2)}} ... T_n^{m_{k_{j_0}}^{(n)}} u_{k_{j_0}} + y_0\| \leq 1/2k.$$
Set
$$w = y + T_1^{m_0^{(1)}} T_2^{m_0^{(2)}} ... T_n^{m_0^{(n)}} u_{k_{j_0}}.$$
Thus,
$$\|w - x\| \leq \|y - x\| + \|T_1^{m_0^{(1)}} T_2^{m_0^{(2)}} ... T_n^{m_0^{(n)}} u_{k_{j_0}}\| < 1/k$$
and so, $w \in V_k$. Therefore, we have
$$\begin{aligned}
\|T_1^{m_{k_{j_0}}^{(1)}} T_2^{m_{k_{j_0}}^{(2)}} ... T_n^{m_{k_{j_0}}^{(n)}} w\| &= \|T_1^{m_{k_{j_0}}^{(1)}} ... T_n^{m_{k_{j_0}}^{(n)}} y + T_1^{m_{k_{j_0}}^{(1)}} ... T_n^{m_{k_{j_0}}^{(n)}} T_1^{m_0^{(1)}} ... T_n^{m_0^{(n)}} u_{k_{j_0}}\| \\
&\leq \|T_1^{m_{k_{j_0}}^{(1)}} ... T_n^{m_{k_{j_0}}^{(n)}}(y) - y_0\| \\
&\quad + \|T_1^{m_{k_{j_0}}^{(1)}} ... T_n^{m_{k_{j_0}}^{(n)}} T_1^{m_0^{(1)}} ... T_n^{m_0^{(n)}}(u_{k_{j_0}}) + y_0\| \\
&< 1/2k + 1/2k = 1/k.
\end{aligned}$$
So
$$T_1^{m_{k_{j_0}}^{(1)}} T_2^{m_{k_{j_0}}^{(2)}} ... T_n^{m_{k_{j_0}}^{(n)}} w \in U_k.$$
Thus we get
$$T_1^{m_{k_{j_0}}^{(1)}} T_2^{m_{k_{j_0}}^{(2)}} ... T_n^{m_{k_{j_0}}^{(n)}}(V_k) \cap U_k \neq \emptyset$$
and this completes the proof. ∎

Corollary 2.4. *Let $\mathcal{T} = (T_1, T_2, ..., T_n)$ be a tuple of bounded linear operators acting on a separable infinite dimensional Banach space X. Then $\mathcal{T}_d^{(2)}$ is hypercyclic if and only if for given four nonempty open subsets U_1, U_2, V_1, V_2 of X, there exists a tuple of integers $(m_1, m_2, ..., m_n)$ such that the sets $T_1^{m_1} T_2^{m_2} ... T_n^{m_n}(U_1) \cap V_1$ and $T_1^{m_1} T_2^{m_2} ... T_n^{m_n}(U_2) \cap V_2$ are nonempty.*

Theorem 2.5. *Let $\mathcal{T} = (T_1, T_2, ..., T_n)$ be a tuple of continuous linear operators acting on a separable infinite dimensional Banach space X. If $\mathcal{T}$ is para-chaotic, then $\mathcal{T}_d^{(2)}$ is topologically transitive.*

Proof. To show that $\mathcal{T}_d^{(2)}$ is topologically transitive, consider the nonempty open sets U_1, U_2, V_1 and V_2 in X. Since $\mathcal{T}$ is hypercyclic, by Lemma 2.1, there exists a tuple $(m_1, m_2, ..., m_n)$ of integers large enough such that

$$T_1^{m_1} T_2^{m_2} ... T_n^{m_n}(U_1) \cap V_1 \neq \emptyset.$$

So,

$$T_1^{-m_1} T_2^{-m_2} ... T_n^{-m_n}(V_1) \cap U_1$$

is a nonempty open set, and since $\mathcal{T}$ contains a dense set of periodic points, thus there exists a periodic point $u_1 \in U_1$ such that

$$T_1^{m_1} T_2^{m_2} ... T_n^{m_n} u_1 \in V_1$$

and

$$T_1^{p_1} T_2^{p_2} ... T_n^{p_n} u_1 = u_1$$

for some $p_1, ..., p_n \in \mathbb{N}$.

On the otherhand, since

$$\{T_1^{ip_1} T_2^{ip_2} ... T_n^{ip_n} : i = 0, 1, ...\}$$

is hypercyclic, thus there exists $i \in \mathbb{N}$ such that

$$T_1^{ip_1} T_2^{ip_2} ... T_n^{ip_n}(U_2) \cap T_1^{-m_1} T_2^{-m_2} ... T_n^{-m_n}(V_2) \neq \emptyset.$$

Define $r_j = m_j + ip_j$ for $j = 1, ..., n$, then

$$T_1^{r_1} T_2^{r_2} ... T_n^{r_n}(U_2) \cap V_2 = T_1^{m_1+ip_1} T_2^{m_2+ip_2} ... T_n^{m_n+ip_n}(U_2) \cap V_2$$

is nonempty, since

$$T_1^{ip_1} T_2^{ip_2} ... T_n^{ip_n}(U_2) \cap T_1^{-m_1} T_2^{-m_2} ... T_n^{-m_n}(V_2)$$

is nonempty. Also, since

$$\begin{aligned} T_1^{r_1} T_2^{r_2} ... T_n^{r_n} u_1 &= T_1^{m_1} T_2^{m_2} ... T_n^{m_n} T_1^{ip_1} T_2^{ip_2} ... T_n^{ip_n} u_1 \\ &= T_1^{m_1} T_2^{m_2} ... T_n^{m_n} u_1 \in V_1, \end{aligned}$$

thus

$$T_1^{r_1} T_2^{r_2} ... T_n^{r_n}(U_1) \cap V_1 \neq \emptyset.$$

This completes the proof. ∎

Theorem 2.6. *Let $\mathcal{T} = (T_1, T_2, ..., T_n)$ be a tuple of continuous linear operators acting on a separable infinite dimensional Banach space X. If $\mathcal{T}$ is para-chaotic, then $\mathcal{T}$ satisfies the Hypercyclicity Criterion.*

Proof. By Theorem 2.5, $\mathcal{T}_d^{(2)}$ is topologically transitive. This implies that $\mathcal{T}$ satisfies the Hypercyclicity Criterion. Indeed, let $x \oplus y$ be a hypercyclic vector for $\mathcal{T}_d^{(2)}$. In particular, x and y are hypercyclic for $\mathcal{T}$. Thus for all tuple of nonnegative integers $(m_1, m_2, ..., m_n)$, the vector $T_1^{m_1} T_2^{m_2} ... T_n^{m_n} y$ is hypercyclic for $\mathcal{T}$ and so

$$(x, T_1^{m_1} T_2^{m_2} ... T_n^{m_n} y)$$

is a hypercyclic vector for $\mathcal{T}_d^{(2)}$. This implies that for all nonempty open subset U of X, there is $u \in U$ such that (x, u) is a hypercyclic vector for $\mathcal{T}_d^{(2)}$. Fix now $\{U_k\}_{k \geq 1}$ a decreasing 0-basis in X. Proceeding by induction we find $u_k \in U_k$ for all $k \in \mathbb{N}$, and increasing sequences $\{m_k^{(i)}\}_k$ $(i = 1, ..., n)$ of natural numbers satisfying

$$T_1^{m_k^{(1)}} T_2^{m_k^{(2)}} ... T_n^{m_k^{(n)}} x \in U_k$$

and

$$T_1^{m_k^{(1)}} T_2^{m_k^{(2)}} ... T_n^{m_k^{(n)}} u_k \in x + U_k$$

for all $k \in \mathtt{N}$. Let $X_0 = Orb(\mathcal{T}, x)$ which is dense in X. Then we have that

$$T_1^{m_k^{(1)}} T_2^{m_k^{(2)}} ... T_n^{m_k^{(n)}} x \to 0$$

and so

$$T_1^{m_k^{(1)}} T_2^{m_k^{(2)}} ... T_n^{m_k^{(n)}} v \to 0$$

for all $v \in X$. Define

$$S_k(T_1^{m_1} T_2^{m_2} ... T_n^{m_n} x) = T_1^{m_1} T_2^{m_2} ... T_n^{m_n} u_k$$

for all m_i, $i = 1, ..., n$, and all k in $\mathtt{N}$. Then $S_k v \to 0$ for all $v \in X_0$. Finally, given $m_0^{(1)}, m_0^{(2)}, ..., m_0^{(n)} \in \mathtt{N}$, we get

$$T_1^{m_k^{(1)}} ... T_n^{m_k^{(n)}} S_k(T_1^{m_0^{(1)}} ... T_n^{m_0^{(n)}} x) = T_1^{m_0^{(1)}} ... T_n^{m_0^{(n)}} (T_1^{m_k^{(1)}} ... T_n^{m_k^{(n)}} u_k)$$

which tends to $T_1^{m_0^{(1)}} T_2^{m_0^{(2)}} ... T_n^{m_0^{(n)}} x$ as $k \to \infty$, so the proof is complete. ∎

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