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ON STAN ULAM AND HIS MATHEMATICS

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ABSTRACT. In this note we give a glimpse of the curriculum vitae of Stan Ulam, his personality and some of the mathematics he was involved in.

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To write about Stan Ulam and his incredible achievements, an article is not enough; it is necessary to write a whole book. In fact, one does not need to do it; two such books have been already published. One of these books is a wonderful mathematical autobiography by Ulam ([50]), the other one is a special issue of *Los Alamos Science* published after his death ([18]). So, why write something again?

The purpose of this note is to give just a glimpse of the curriculum vitae of Ulam, his personality and some of his mathematics he was involved with. For an extensive account on his life and works the reader is referred to the books mentioned above and several articles (cf. [12], [14], [16], [17], [22], [36], [46], [47], [48]).

Stan Ulam (right) with his father Józef, a lawyer

Stanisław Marcin Ulam (later known as Stan Ulam) was born on April 3, 1909 in the city of Lvov, Poland. In 1933 he received his M.Sc from the Technical University of Lvov and in the same year he also received from the same university a doctorate in mathematics. He had written both his theses under the supervision of the famous topologist Kazimierz Kuratowski. The atmosphere of the Lvov School of mathematics was great at that time because outstanding mathematicians were working there. The *Scottish Café* (cf. [5], [6], [7], [28], [31]) provided an influence upon his career, since a number of eminent Polish mathematicians (such as Stefan Banach and Stanisław Mazur) met there and discussed new mathematics. It was there that they posed a number of research problems, some of which still remain unsolved today (cf. [34]). In the thirties Ulam visited several other mathematical centres. In 1935 Ulam received an invitation from John von Neumann to visit the Institute for Advanced Study in Princeton, USA for three months, which he accepted. The following year he again visited the United States, this time for a longer period. From 1936–1940 he was a member of the Society of Fellows in Harvard, returning to Lvov annually for the summer. At the onset of World War II he remained in the USA. Ulam was appointed a lecturer at Harvard University in 1940 and in 1941–1943 he was an assistant professor at the University of Wisconsin in Madison. It was around that time that he received an invitation from John von Neumann to "...do some very important war work in a place which he couldn't name...", and, as he mentioned ([14]) "...I was to meet him in Chicago in some railroad station to learn a little bit more about it. I went there; and he couldn't tell me where he was going; and there were two guys, sort of guards, looking like gorillas, with him. He discussed with me some mathematics, some interesting physics, and the importance of this work. And that was Los Alamos at the very start..."; Ulam worked in Los Alamos during 1944–1967. In 1965–1977 he was a professor at the University of Colorado. In 1941 he married Françoise, who was a great influence and inspiration. They had one daughter, Claire. Ulam died suddenly in May 13, 1984 in Santa Fe after a heart attack.

John von Neumann (left), Richard Feynmann (center), Stan Ulam (right)
A photo taken near Los Alamos sometime in 1950s by a Los Alamos staff photographer

Ulam was a mathematician, but the work in Los Alamos was mainly concentrated in physics. In this area, he was very much interested in the theory of relativity as well as quantum theory. Even though his interest in physics and astronomy (from which his interest in science began) remained, he did much more work in pure mathematics than in the applied sciences or in theoretical physics. His broad scientific interest was not limited to mathematics and physics; it included also technology, computer science and biology. He is now famous for his research in nuclear physics that was performed in Los Alamos, but one could mention here also other non-mathematical achievements; in particular, he developed original methods of propulsion of vessels moving above the earth's atmosphere. Some of Ulam's results were kept secret and were not presented to the public even for some time after his death.

Now, we will devote a few words to some of his mathematics. His broad interest in mathematics and his results in various branches of science are really incredible. In fact, for someone to describe his achievements even in brief would take several monographs. Thus, let us only concentrate on some mathematical disciplines in which he obtained essential new results. He started his mathematical research with set theory and in his Ph.D. thesis he succeeded in proving three fundamental theorems that are considered classical results today and still provide inspiration for further new research. With respect to set theory, his invention of infinite games resulted in the formulation of the axiom of determinity by hismself and Jan Mycielski. Another important theme in which Ulam was involved was ergodic theory. Ulam and John C. Oxtoby proved beautiful and fundamental theorems on measure-preserving homeomorphisms ([38], [39]). Ulam wrote ([18]) the following about the paper [39]: "...which I consider one of the more important results that I had a part in." Some other results which are due to Ulam and C. J. Everett concern multiplicative systems in several variables. He was also influential in geometric topology and authored some essential results dealing with computers and the Monte Carlo method. Together with Enrico Fermi and John Pasta, he initialised investigations into different kinds of non-linear systems in [15]. This paper has served as the basis for various important investigations by several mathematicians and physicists, especially for the mathematical theory of solitons. The list of his publications contains more than 150 papers ([18], [36]). The majority of his works were joint publications. Among his coauthors, besides the ones already mentioned above, are Stefan Banach, Karol Borsuk, Paul Erdős (which means that Ulam has Erdős number 1), Mark Kac, Kazimierz Kuratowski, Max Planck, Julian Schreier and Edward Teller.

Stan Ulam shaking hands with John F. Kennedy

At the beginning of March 2009 the Internet database of Mathematical Reviews contained an amazing number of 697 papers with the name "Ulam" in their titles.

Several titles of research publications include phrases such as "a problem of S. Ulam" or "Ulam's conjecture". This provides a sample of the impact of Ulam's work in science. He formulated original research problems, new conjectures and talked about mathematics with others. This led to the formulation and derivation (or genesis) of a number of results by several researchers worldwide, which were published later in a number of scientific journals. Another aspect to remark on is the very frequent appearance of his name in connection with other names. Sometimes this is due to the fact that the paper is concerned with some results that were obtained jointly with Ulam and someone else, results that have been proven so important that they are used with specific names. Very frequently this is also due to the fact that some results or problems considered by Ulam provided impulses for future research to other scientists and students. This is not an exception in mathematics, as an example we mention the famous Stone-Weiestrass Theorem (Marshall H. Stone was born in 1903, just a few years after Karl Weierstrass' death in 1897). There is an impressive list of combinations of names which appear together with Ulam's name (we drop out terms such as "theorem"). These are the following: Borsuk-Ulam, Mazur-Ulam, Auerbach-Banach-Mazur-Ulam, Everett-Ulam-Harris, Mycielski-Ulam, Mackey-Ulam, von Neumann-Ulam, Schreier-Ulam, Baer-Schreier-Ulam, Oxtoby-Ulam, Hyers-Ulam, Hyers-Ulam-Rassias, Kuratowski-Ulam, Ulam-Zahorski, Ulam-Li, Beyer-Stein-Ulam, Ulam-Rényi, Borsuk-Ulam-Munkholm-Fenn-Connett-Cohen-Lusk (the one with the greatest number of names), Fermi-Ulam, Lorenz-Fermi-Pasta-Ulam, and finally Fermi-Pasta-Ulam.

We will refer here to two terms which appear most frequently. The first one is the *Borsuk-Ulam Theorem*. The term appeared for the first time in the title of a paper of Hirsch in 1944 ([21]). Since then, the term Borsuk-Ulam Theorem has been repeatedly cited in the titles of several research publications in a number of contexts in mathematical analysis and topology. The *Borsuk–Ulam theorem about antipodes* has a very simple formulation. It states that for any continuous mapping $f : S^n \to \mathbb{R}^n$ there is an $x \in S^n$ such that $f(-x) = f(x)$. This theorem has an amazingly large variety of applications, one of which can be stated as follows: *at every moment there are always two antipodal points on the face of the earth with the same temperature and atmospheric pressure*. The classical Borsuk-Ulam antipodal theorem was proved by Karol Borsuk and was published in the year 1933 ([3]). However, the theorem is known with two names (as Borsuk-Ulam), because the problem was formulated for the first time by Ulam. The proof of the theorem is not simple and requires some advanced tools from topology which were considered very advanced in the thirties.

The Borsuk-Ulam theorem led to several further investigations and various generalizations, some of which influence research even today. For example, in his paper [3] Borsuk obtained the so-called Lyusternik-Shnirelman Theorem as a nontrivial consequence of the theorem about antipodes. The Lyusternik-Shnirelman Theorem asserts that if an $n-$ dimensional sphere is covered by $n+1$ closed sets, then at least one of them contains a pair of antipodal points. This theorem was proved slightly earlier in [33], however, Borsuk did not know their results and he had applied different methods. On the other hand, it is a known fact that the Borsuk-Ulam Theorem can also be proved by means of the Lyusternik-Shnirelman Theorem.

Another result connected with the theorem about the antipodes is a famous "sandwich theorem" (the problem was posed by Hugo Steinhaus and the proof was given later by Stefan Bamach). For a popular description of the Borsuk-Ulam Theorem and a discussion of a number of problems connected with the antipodal property, one is referred to [8].

The second term which is connected with Ulam and appears very frequently is *Hyers-Ulam-Rassias stability* (sometimes it is also called *Hyers-Ulam stability* or *generalized Hyers-Ulam stability*). The term "Hyers-Ulam-Rassias stability" appeared for the first time in 1994 in the title of a paper written by Pasc Găvruta [19]. Since then, several results for the Hyers-Ulam-Rassias stability of functional equations have been proved by many mathematicians. The interested reader is referred to [1], [2], [4], [9], [13], [19], [20], [24], [25], [27], [29], [30], [32], [35], [37], [42], [43], [44], [45].

Following Ulam ([51]), for very general functional equations, one can ask the following question: "When is it true that the solution of an equation (e.g. $f(x+y) = f(x) + f(y)$) differing slightly from a given one, must of necessity be close to the solution of the given equation?" Similarly, if we replace a given functional equation by a functional inequality, when can one assert that the solutions of the inequality lie near the solutions of the strict equation? The stability problem of functional equations originated from the question of Ulam concerning the stability of group homomorphisms. In 1941, a partial solution of the problem was provided by Donald H. Hyers ([23]). Hyers' theorem was generalized for approximate additive mappings by T. Aoki ([1]) and for approximate linear mappings by Th.M.Rassias ([41]). In 1978, Themistocles M. Rassias considered the unbounded Cauchy difference inequality $||f(x+y) - f(x) - f(y)|| \leq \varepsilon(||x||^p + ||y||^p)$, where $\varepsilon > 0$ and $p \in [0, 1)$ and proved the stability of linear mappings between Banach spaces. Thus the Ulam problem for approximate homomorphisms was brought into a new general context. This has led to the stability phenomenon that is known today as the Hyers-Ulam-Rassias stability.

Th. M. Rassias recalls his personal contact with Professor Stan Ulam in July 1976, just after he had received his Ph.D in mathematics from the University of California at Berkeley under the supervision of Professor Stephen Smale. Ulam had given him the open problem of the stability of approximate homomorphisms and encouraged him as follows: "Themistocles, if you do something towards the solution of this problem, it will be useful for your career". So he did, and he feels grateful to Professor Stan Ulam for his encouragement. An amusing coincidence is that D. H. Hyers' birthday is April 1, Th. M. Rassias' birthday is April 2 and S. M. Ulam's birthday is April 3.

The third term which has appeared very frequently, especially during the last number of years, is "Fermi-Pasta-Ulam", after the famous paper of Enrico Fermi, John Pasta and Stan Ulam "Studies of nonlinear problems. I" ([15]). Since this result is less mathematical, we will not discuss it here. The reader is referred to [10] for some further details. Here, let us only point out that the combination of the three names: E. Fermi, J. Pasta and S. Ulam, introduced by the Los Alamos report of the year 1940 [15], appears with several terms, such as: *Fermi-Pasta-Ulam systems, lattices, chains, problem, phenomenon, q-breather, model, arrays, recurrence, equation* and *experiment*.

Stan Ulam standing on the piece of land in Santa Fe
that he bought for 150 US dollars in 1947

For further information and a number of anecdotes referring to Ulam, especially going back to the period of the time he lived in Lvov, the interested reader is referred to his book "Adventures of a Mathematician" ([50]) that has been translated to several languages, as well to his article "An Anecdotal History of the Scottish Book" regarding the Scottish Café that is contained in [34]. Referring to [34], one could probably wonder how it really happened that such a book was published in English after World War II. This was mainly thanks to the efforts of Stan Ulam and Dan Mauldin. The Scottish Book survived during the war and was well preserved by Stefan Banach's wife. Later on, the book was translated to English by Hugo Steinhaus. In the following years Ulam circulated some of the problems from the Scottish Book in a list of problems that he published in the United States. In Los Alamos, Ulam in talking with Mauldin used to say that the atmosphere there was quite similar to what he remembered from his Lvov time which was connected to the Scottish Café. Mauldin was fascinated by this and decided to prepare the publication of “The Scottish Book. Mathematics from the Scottish Café” ([34]). Mauldin's publication of the collected problems has proved to be remarkably valuable for mathematicians and graduate students alike because, besides the problems and solutions (if there are given), it also includes many useful comments and interesting remarks concerning the progress that has been made for the individual problems originating from the Scottish Book. This still serves today as a large and important masterpiece of a live mathematics.

Ulam was a great personality, a man of great culture with a deep knowledge of literature and history. In his youth he was a pupil of the very good Lvov Gymnasium Nr. VII. Ulam was a great influence to mathematicians of all ages and he exerted a very special magnetism, encouraging them to do original and interdisciplinary research. We conclude this short article with a statement from Mark Kac ([14]): “Stan generates problems and conjectures at probably the highest rate in the world. It is very difficult to find anybody in his class in that.”

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