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ON POSITIVE ENTIRE SOLUTIONS OF SECOND ORDER QUASILINEAR ELLIPTIC EQUATIONS

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ABSTRACT. In this paper, our main purpose is to establish the existence theorem of positive entire solutions of second order quasilinear elliptic equations under new conditions. The main results of the present paper are new and extend the previously known results.

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1. INTRODUCTION

In this paper we are concerned with the existence of positive entire solutions of second order quasilinear elliptic equations of the type

$$\text{div}(|\nabla u|^{p-2}\nabla u) \pm f(x,u) = 0, \quad x \in \mathbb{R}^N \tag{1.1}$$

where $f(x,u)$ is a continuous function on $\mathbb{R}^N \times (0,\infty)$. By an positive entire solution of equation (1.1) we mean a function $u \in C^1(\mathbb{R}^N)$ which satisfies (1.1) at every point of $\mathbb{R}^N$ in a weak sense with $u > 0$ in $\mathbb{R}^N$ (See [4] and references therein).

This problem appears in the study of non-Newtonian fluids ([1], [12]) and non-Newtonian filtration ([2]). The quantity p is a characteristic of the medium. Media with $p > 2$ are called dilatant fluids and those with $p < 2$ are called pseudoplastics. If $p = 2$, they are Newtonian fluid.

The existence and non-existence of entire solutions, existence of multiple positive entire solutions of (1.1) have been studied in previous papers(see [15]-[17], [19]). Other some problems have also been treated by many other authors. See, for example, [4]-[6], [8], [11].

When $f : (0,\infty) \to (0,\infty)$ and $q : \mathbb{R}^N \to (0,\infty)$ are continuous functions, and

$$\int_1^\infty (\int_0^u f(s)ds)^{-1/p} du = \infty,$$

it has been shown in [15] that there exists entire radially symmetric solutions of the problem

$$\text{div}(|\nabla u|^{p-2}\nabla u) = q(x)f(u), \ \ x \in \mathbb{R}^N. \tag{1.2}$$

When $f : (0,\infty) \to (0,\infty)$ is locally Lipschitz continuous and strictly increasing, and

$$\int_1^\infty (\int_0^u f(s)ds)^{-1/p} du < \infty, \quad \int_0^1 (\int_0^u f(s)ds)^{-1/p} du = \infty,$$

with q is positive and continuous in $\mathbb{R}^N$, and

$$\lim_{|x|\to\infty} \inf |x|^p q(x) > 0,$$

it has been shown in [15] that Eq.(1.2) has no positive entire solutions.

On the other hand, it was shown in [17] that problem

$$\text{div}(|\nabla u|^{p-2}\nabla u) + f(x,u) = 0, \ \ x \in \mathbb{R}^N,$$

possesses infinitely many positive entire solutions when $f(x,u)$ is defined on $\mathbb{R}^N$ is locally Hölder continuous in x and is locally Lipschitz continuous in u; there exist a locally Hölder continuous function $\psi(r) \geq 0$ on $[0,\infty)$, $\int_0^\infty (\int_0^s \psi(t)dt)^{1/(p-1)} ds < \infty$, and locally Lipschitz continuous function $F(u) > 0$ on $(0,\infty)$ such that

$$f(x,u) \leq \psi(|x|)F(u), \ \ (x,u) \in \mathbb{R}^N \times (0,\infty),$$

and $\lim_{u\to 0} \dfrac{F(u)}{u^{p-1}} = 0$.

Moreover, it was also shown in [16] that problem

$$\text{div}(|\nabla u|^{p-2}\nabla u) + q(x)u^{-\gamma} = 0, \ \ x \in \mathbb{R}^N.$$

has a positive entire solution if $q \in C(\mathbb{R}^+), 0 \leq \gamma < p-1$, for any
$0 < \varepsilon < (N-p)(p-1-|r|)/(p-1)$,

$$\int_1^\infty r^{p+\varepsilon-1+[(N-p)|r|/(p-1)]} q(r)dr < \infty,$$

and for $r \in (0,1), \delta < 1, q(r) = O(r^{-\delta})$.

Furthermore, it was shown in [11] that problem

$$\operatorname{div}(|\nabla u|^{p-2}\nabla u) + q(x)u^{\gamma} = 0, \quad x \in \mathbb{R}^N, \quad \lim_{|x|\to\infty} u(x) = \infty,$$

has an entire explosive positive radial solution if $q(x) \in C(\mathbb{R}^N), q(x) = q(|x|) \geq C > 0, \gamma > p-1$ and

$$\int_0^\infty (t^{1-N}\int_0^t s^{N-1}q(s)ds)^{1/(p-1)}dt < \infty.$$

Motivated by the results of the above cited papers, we further study the existence of positive entire solutions for (1.1), the results of the semilinear equations are extended to the quasilinear ones. We can find the related results for $p = 2$ in [7], [9], [13]-[14], [10], [18]. The main differences between $p = 2$ and $p \neq 2$ are known in [5]-[6]. When $p = 2$, it is well known that all the positive solutions in $C^2(B_R)$ of the problem

$$\triangle u + f(u) = 0 \text{ in } B_R,$$

$$u(x) = 0 \text{ on } \partial B_R.$$

are radially symmetric solutions for very general f(see [3]). Unfortunately, this result does not apply to the case $p \neq 2$. Kichenassary and Smoller showed that there exist many positive nonradial solutions of the above problem for some f (see [8]). The major stumbling block in the case of $p \neq 2$ is that certain nice features inherent to the case $p = 2$ seem to be lost or at least difficult to verify. In [19], we study the existence of positive entire solutions of (1.1). In this paper, we obtain more results under new conditions. Therefore the following results obtained complement conrresponding results in [14]-[16], [19], and extended to results in [7], [9], [18] and complement to the results by [14]-[16], [11].

2. PRELIMINARIES

From [17], [19], we give the following lemmas

Lemma 2.1. *Suppose that $f(x,u)$ is defined on $\mathbb{R}^{N+1}$ and is locally Hölder continuous (with exponent $\lambda \in (0,1)$) in x. Suppose moreover that there exist functions $v, w \in C^{1+\lambda}_{loc}(\mathbb{R}^N)$ such that*

$$\textit{div}(|\nabla v|^{p-2}\nabla v) \pm f(x,v) \leq 0, \quad x \in \mathbb{R}^N, \tag{2.1}$$

$$\textit{div}(|\nabla w|^{p-2}\nabla w) \pm f(x,w) \geq 0, \quad x \in \mathbb{R}^N, \tag{2.2}$$

and

$$w(x) \leq v(x), \quad x \in \mathbb{R}^N, \tag{2.3}$$

and that $f(x,u)$ is locally Lipschitz continuous in u on the set

$$\{(x,u) : x \in \mathbb{R}^N, w(x) \leq u \leq v(x)\}.$$

Then, equation (1.1) possesses an entire solution $u(x)$ satisfying

$$w(x) \leq u(x) \leq v(x), \quad x \in \mathbb{R}^N.$$

Remark 2.1. A function $v(x)$(resp.$w(x)$) satisfying the differential inequality (2.1)(resp.(2.2)) is referred to as a supersolution (resp. subsolution) of (1.1) in $\mathbb{R}^N$.

Lemma 2.2. *(Weak Comparison Principle) Let Ω be a bounded domain in $\mathbb{R}^N (N \geq 2)$ with smooth boundary $\partial\Omega$ and $\theta : (0, \infty) \to (0, \infty)$ is continuous and nondecreasing. Let $u_1, u_2 \in W^{1,p}(\Omega)$ satisfy*

$$\int_\Omega |\nabla u_1|^{p-2}\nabla u_1 \nabla\psi dx + \int_\Omega \theta(u_1)\psi dx \leq \int_\Omega |\nabla u_2|^{p-2}\nabla u_2 \nabla\psi dx + \int_\Omega \theta(u_2)\psi dx,$$

for all non-negative $\psi \in W_0^{1,p}(\Omega)$. Then the inequality

$$u_1 \leq u_2 \quad \textit{on} \quad \partial\Omega,$$

implies that

$$u_1 \leq u_2 \quad \textit{in} \quad \Omega.$$

Lemma 2.3. *Suppose that $\rho(x) \in C(\mathbb{R}^N)$ is nonnegative and*

$$H_\infty = \int_0^\infty (s^{1-N} \int_0^s t^{N-1}\psi(t)dt)^{\frac{1}{p-1}} ds < \infty, \tag{2.4}$$

where $\psi(r) = \max_{|x|=r} \rho(x)$. Then the equation

$$\textit{div}(|\nabla U|^{p-2}\nabla U) + \rho(x) = 0, \tag{2.5}$$

has a ground state solution $U(x)$ in $\mathbb{R}^N$, which is bounded and $\lim_{|x|\to\infty} U(x) = 0$.

Proof. Because

$$V(x) = \int_{|x|}^\infty (\frac{1}{s^{N-1}} \int_0^s \sigma^{N-1}\psi(\sigma)d\sigma)^{\frac{1}{p-1}} ds,$$

which is a solution for the $-\text{div}(|\nabla V|^{p-2}\nabla V) = \psi(r)$ in $\mathbb{R}^N$ and $\lim_{|x|\to\infty} V(x) = 0$, so V is a supersolution for (2.5). On the other hand, 0 is a subsolution for (2.5), then (2.5) exists bounded entire solution. ∎

Remark 2.2. If $N \geq 3, N > p$, then condition (2.4) of Lemma 2.3 is replaced by

$$0 < \int_1^\infty r^{\frac{1}{p-1}}\psi(r)^{\frac{1}{p-1}} dr < \infty, \quad \text{if } 1 < p \leq 2, \tag{2.6}$$

$$0 < \int_1^\infty r^{\frac{(p-2)N+1}{p-1}}\psi(r) dr < \infty, \quad \text{if } p \geq 2. \tag{2.7}$$

Let

$$J(r) = \int_0^r (t^{1-N} \int_0^t s^{N-1}\psi(s)ds)^{\frac{1}{p-1}} dt.$$

If fact, if $1 < p \leq 2$, by estimating above the integral

$$J(r) \leq C_1 + \int_1^r t^{\frac{1-N}{p-1}} [\int_0^t s^{N-1}\psi(s)ds]^{1/(p-1)} dt.$$

Using the assumption $N \geq 3$ in the computation of the first integral above and Jensen's inequality to estimate the last one,

$$J(r) \leq C_2 + C_3 \int_1^r t^{\frac{3-N-p}{p-1}} \int_1^t s^{\frac{N-1}{p-1}}\psi(s)^{\frac{1}{p-1}} ds dt.$$

Computing the above integral, we obtain

$$J(r) \leq C_2 + C_4 \int_1^r t^{\frac{1}{p-1}}\psi(t)^{\frac{1}{p-1}} dt.$$

Applying (2.6) in the integral above we infer that $H_\infty = \lim_{r\to\infty} J(r) < \infty$. On the other hand, if $p \geq 2$, set

$$H(t) = \int_0^t s^{N-1}\psi(s)ds.$$

and note that either, $H(t) \leq 1$ for $t > 0$ or $H(t_0) = 1$ for some $t_0 > 0$. In the first case, $H^{\frac{1}{p-1}} \leq 1$, and hence,

$$J(r) = \int_0^r t^{\frac{1-N}{p-1}} H(t)^{\frac{1}{p-1}} dt \leq C_5 + \int_1^r t^{\frac{1-N}{p-1}} dt$$

so that $J(r)$ has a finite limit because $p < N$. In the second case, $H(s)^{\frac{1}{p-1}} \leq H(s)$ for $s \geq s_0$ and hence,

$$J(r) \leq C_6 + \int_1^r t^{\frac{1-N}{p-1}} \int_0^s s^{N-1}\psi(s)dsdt.$$

Estimating and integrating by parts, we obtain

$$\begin{aligned} J(r) &\leq C_6 + \frac{p-1}{N-p}\int_0^1 t^{N-1}\psi(t)dt + \frac{p-1}{N-p}[\int_1^r t^{\frac{(p-2)N+1}{p-1}}\psi(t)dt - r^{\frac{p-N}{p-1}}\int_0^r t^{N-1}\psi(t)dt] \\ &\leq C_7 + C_8\int_1^r t^{\frac{(p-2)N+1}{p-1}}\psi(t)dt. \end{aligned}$$

By (2.7), $H_\infty = \lim_{r\to\infty} J(r) < \infty$.

3. MAIN RESULTS

By a modification of the method given in [18]-[19], we obtain the following results.

Theorem 3.1. *Let F be a positive, non-decreasing and locally Lipschitz continuous function defined on $(0,\infty)$, let $\rho \geq 0$ and $\rho \in C(\mathbb{R}^N)$ satisfies*

$$0 < \int_1^\infty r^{\frac{1}{p-1}}\psi(r)^{\frac{1}{p-1}}dr < \infty, \quad \textit{if } 1 < p \leq 2,$$

$$0 < \int_1^\infty r^{\frac{(p-2)N+1}{p-1}}\psi(r)dr < \infty, \quad \textit{if } p \geq 2$$

where $\psi(r) = \max_{|r|=r}\rho(x), N \geq 3, N > p$. Then for any $\alpha > 0$, the equation

$$\textit{div}(|\nabla u|^{p-2}\nabla u) = \rho(x)F(u) \tag{3.1}$$

has a solution u in $\mathbb{R}^N$ such that $\lim\limits_{|x|\to\infty} u(x) = \alpha$.

Proof. From Lemma 2.2, Eq.(2.5) has a bounded solution U, since $\rho(x) \geq 0$, so U is nonnegative in $\mathbb{R}^N$. Then for any $\alpha > 0$, $\rho(x)F(\alpha) \geq 0$, so $u_1 = \alpha$ is a super-solution of (3.1), then we can set $u_2 = \alpha - [F(\alpha)]^{\frac{1}{p-1}}U$, so we have $u_2 \leq \alpha$ and

$$\text{div}(|\nabla u_2|^{p-2}\nabla u_2) = \rho(x)F(\alpha) \geq \rho(x)F(u_2),$$

which means that u_2 is a sub-solution. Hence, from Lemma 2.1, we get a solution u satisfying (3.1) in $\mathbb{R}^N$ and $u_1 \geq u \geq u_2$, for $\lim_{|x|\to\infty} U(x) = 0$, we have $\lim_{|x|\to\infty} u(x) = \alpha$. That's end the proof. ∎

Theorem 3.2. *Let F be a positive, non-increasing and continuous function defined on $(0,\infty)$, let $\rho \geq 0$ such that $\rho \in C(\mathbb{R}^N)$. Then the equation*

$$\textit{div}(|\nabla u|^{p-2}\nabla u) + \rho(x)F(u) = 0 \ \textit{ in } \ \mathbb{R}^N, \quad \lim_{|x|\to\infty} u(x) = 0, \tag{3.2}$$

admits a solution if and only if the linear (2.5) has a solution U.

Proof. Let U be the ground state solution of (2.5) and $c = \sup_{\mathbb{R}^N} U$. Using the monotonicity, we have $\lim_{t\to\infty} \frac{F(t)^{\frac{1}{p-1}}}{t} = 0$, so $\lim_{x\to\infty} \frac{1}{x}\int_0^x \frac{t}{F(t)^{\frac{1}{p-1}}} = +\infty$. Therefore, we obtain some $x_0 > 0$, such that

$$cx_0 \le \int_0^{x_0} \frac{t}{F(t)^{\frac{1}{p-1}}} dt$$

We define a function $v(x)$ by

$$U(x) = \frac{1}{x_0}\int_0^{v(x)} \frac{t}{F(t)^{\frac{1}{p-1}}} dt, \quad \forall x \in \mathbb{R}^N.$$

Then $v(x) > 0$ and is bounded from above, since $v(x) \le x_0$ in $\mathbb{R}^N$, and we claim that v is a super-solution of (3.2). In fact

$$-\rho(x) = \text{div}(|\nabla U|^{p-2}\nabla U) = (\frac{v}{x_0})^{p-1}\frac{\text{div}(|\nabla v|^{p-2}\nabla v)}{F(v)} + \frac{|\nabla v|^p}{x_0^{p-1}}(\frac{t^{p-1}}{F(t)})'|_{t=v}$$

$$\ge (\frac{v}{x_0})^{p-1}\frac{\text{div}(|\nabla v|^{p-2}\nabla v)}{F(v)}$$

so $-\text{div}(|\nabla v|^{p-2}\nabla v) \ge (\frac{x_0}{v})^{p-1}\rho(x)F(v) \ge \rho(x)F(v)$

Since $\lim_{|x|\to\infty} U(x) = 0$ implies that $\lim_{|x|\to\infty} v(x) = 0$ and 0 is a subsolution, we get then a solution for (3.2).

Inversely, if a solution u of (3.2) exists, we have $F(u) \ge F(\max_{R^N} u) = a^{p-1} > 0$, it follows that

$$-\text{div}(|\nabla(u/a)|^{p-2}\nabla(u/a)) = -\frac{1}{a^{p-1}}\text{div}(|\nabla u|^{p-2}\nabla u) = \frac{1}{a^{p-1}}\rho(x)F(u) \ge \rho(x).$$

So $\frac{u}{a}$ is a supersolution for (2.5), the proof is done. ∎

Theorem 3.3. *Let ρ, F be as in Theorem 3.1. If*

$$\mathit{div}(|\nabla u|^{p-2}\nabla u) + \rho(x)F(u) = 0$$

has a entire solution which is bounded from below and above by positive constants, then the solution of problem (2.5) exists and satisfies

$$\|U\|_\infty < \int_0^\infty \frac{dt}{[F(t)]^{\frac{1}{p-1}}}. \tag{3.3}$$

Proof. Suppose that there exists a positive entire solution u for

$$\text{div}(|\nabla u|^{p-2}\nabla u) + \rho(x)F(u) = 0.$$

If the right side of (3.3) is infinite, since u is bounded, there exist $a, b > 0$ such that $a \le u \le b$, then we have $F(u) \ge C$ for some positive constant C, since F is nondecreasing. Obviously, $C^{\frac{-1}{p-1}}u$ is a super-solution for (2.5), the claim is true.

Now we suppose that the integral is finite and define a function w by

$$w(x) = \int_0^{u(x)} \frac{dt}{(F(t))^{\frac{1}{p-1}}} \qquad \forall x \in \mathbb{R}^N.$$

it follows that

$$-\text{div}(|\nabla w|^{p-2}\nabla w) = -\frac{\text{div}(|\nabla u|^{p-2}\nabla u)}{F(u)} + |\nabla u|^p\frac{F'(u)}{F^2(u)} \ge \rho(x).$$

Since $w > 0$, it is a supersolution of (2.5). Then we obtain a solution U of (2.5), verifying $0 \le U \le w$, the proof is end. ∎

Now we consider the following problem($N \ge 3$)

$$\mathrm{div}(|\nabla u|^{p-2}\nabla u) = \rho(x)f(u) \ \text{ in } \ \mathbb{R}^N, \quad \lim_{|x|\to\infty} u(x) = \infty. \tag{3.4}$$

Such a solution is called a explosive solution.

Theorem 3.4. *Suppose that f is a nondecreasing, locally Lipschitz function defined on $[0, +\infty)$ such that $f(t) > 0$ in $(0, \infty)$. Suppose moreover that satisfies the condition*

$$\int_1^\infty [\int_0^t f(s)ds]^{-1/p} dt < \infty. \tag{3.5}$$

Let $\rho \ge 0$ and $\rho \in C(\mathbb{R}^N)$ satisfies

$$0 < \int_1^\infty r^{\frac{1}{p-1}} \psi(r)^{\frac{1}{p-1}} dr < \infty, \ \ \textit{if } 1 < p \le 2,$$

$$0 < \int_1^\infty r^{\frac{(p-2)N+1}{p-1}} \psi(r) dr < \infty, \quad \textit{if } p \ge 2$$

where $\psi(r) = \max_{|r|=r} \rho(x), N \ge 3, N > p$. Then equation (3.4) admits a positive solution.

Proof. By Lemma 2.3 of [11], we know that for any $k \in \mathbf{N}$, there exists a positive solution v_k of equation

$$\mathrm{div}(|\nabla v_k|^{p-2}\nabla v_k) = \rho(x)f(v_k) \ \text{ in } \ B_k, \quad \lim_{|x|\to k} v_k(x) = \infty. \tag{3.6}$$

According to the Lemma 2.2(weak comparison principle), it is clear that $v_k \ge v_{k+1}$ in B_k. Therefore $v = \lim_{k\to\infty} v_k$ exists and div$(|\nabla v|^{p-2}\nabla u) = \rho(x)f(v)$ in $\mathbb{R}^N$. For estimating v, define

$$w_k(x) = \int_{v_k(x)}^\infty [f(s)]^{-1/(p-1)} ds \quad \text{in } B_k, \ \ \forall k \in \mathbf{N}.$$

By the condition (3.6) and the monotonicity of f, we see that w_k is well defined. A simple calculus shows that $-$div$(|\nabla w_k|^{p-2}\nabla w_k) \le \rho$ in B_k and $w_k = 0$ on ∂B_k, which yields $w_k(x) \le U(x)$ on B_k by the weak comparison principle, where U is the solution of (2.5)(By Lemma 2.3, equation (2.5) has a bounded solution U). Thus

$$\int_{v(x)}^\infty [f(s)]^{-1/(p-1)} ds \le U(x) \ \text{ in } \ \mathbb{R}^N.$$

Thus, v is positive in $\mathbb{R}^N$ and $\lim_{|x|\to\infty} U(x) = 0$ implies that $\lim_{|x|\to\infty} v(x) = \infty$. ∎

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