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# A MULTI-STAGE DIFFERENTIAL TRANSFORM APPROACH FOR SOLVING DIFFERENTIAL ALGEBRAIC SYSTEMS WITHOUT INDEX REDUCTION

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ABSTRACT. This paper aims to solve differential algebraic systems without the need to reduce the index, which causes a defect in the behavior of the approximate solution. The differential transform method was developed to solve differential algebraic systems. The differential algebraic system is transferred to the algebraic system by applying the differential transform method. Then the Multi-stage differential transform method is applied to extend the interval of the convergence. The numerical results show the new technique is an efficient and flexible tool to obtain accurate results that meet the initial conditions and keep the behavior of the approximate solution consistent.

*Key words and phrases:* Differential Algebraic Equations; Differential Transform Method; Multi-stage Differential Transform Method.

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## 1. INTRODUCTION

Differential Algebraic Equations (DAE) systems play a crucial role in modeling various real-life problems across a board spectrum of fields such as physics, chemistry, mechanics, and multi-body dynamics [6],[15]. These systems arise naturally in situation where constraints must be incorporated into the dynamics, such as in mechanical systems with joints or electrical circuits with Kirchhoff's laws. Given their wide applicability, solving DAE systems has been the focus of significant research, with numerous methods developed to tackle the challenges they present. Traditionally, many approaches for solving DAE systems rely on techniques like index reduction, which involves reformulating the DAE into a lower-index system to simplify its numerical treatment [4],[5]. Index reduction methods, such as those based on the pantelides algorithm or structural analysis, have proven effective in making DAE more tractable for numerical solvers. Another common approach is the use of backward differential formulas (BDF), which are implicit time-stepping methods that have shown robustness in dealing with stiff nature of many DAE problems [7]. However, while these methods have yielded good results in terms of obtaining solutions, they are not without their drawbacks. One significant issue is the process of index reduction or the application of drawback differentiation can alter the behavior of the approximate solution, particularly near the constraints. These alteration can lead to inconsistencies or defects in solution, where the approximate solution no loner faithfully represents the original problem, especially in maintaining the integrity of the constraints over time. This can be particularly problematic in sensitive applications where accurate adherence to physical constraints is essential. To address these shortcomings, a new technique is proposed that leverages the differential transform method (DTM). Unlike traditional methods, the DTM presents a direct approach to solving DAEs by transforming them into a system of algebraic equations (AE). This transformation simplifies the problem while maintaining the essential characteristics of the original DAE system. By applying the DTM directly to the DAE, the method preserves the system's inherent constraints and ensure that the approximate solution retains its correct behavior, even in the presence of complex interactions between differential and algebraic components. This approach not only overcomes the limitation of traditional methods but also provides a more robust and accurate framework for solving DAE systems in various scientific and engineering applications.

## 2. DIFFERENTIAL TRANSFORM METHOD (DTM)

**Definition 2.1.** [1, 2] If a function $v\left(t\right)$ is analytical with respect to $t$ in the domain of interest, then

$$V(n) = \frac{1}{n!}[\frac{d^n v(t)}{dt^n}]_{t=t_0} \tag{2.1}$$

is the transformed function of $v(t)$.

**Definition 2.2.** [1, 2] The differential inverse transforms of the $set$ $\{V(n)\}_{n=0}^{n}$ is defined by

$$v\left(t\right) = \sum_{n=0}^{\infty} V\left(n\right)\left(t - t_0\right)^n . \tag{2.2}$$

If the equation (2.1) is substituted in the equation (2.2), then the following is obtained

$$v\left(t\right) = \sum_{n=0}^{\infty} \frac{1}{n!}\left[\frac{d^n v\left(t\right)}{dt^n}\right]_{t=t_0} \left(t - t_0\right)^{\mathrm{n}}. \tag{2.3}$$

It is clear that the principle of the DTM is generated from the expansion of the power series according to the definitions 2.1 and 2.2 above. To explain how the DTM is applied to solve a system of ordinary differential equations (ODEs), Suppose the nonlinear system of ODE equations as follows:

$$\frac{dv(t)}{dt}=f(v(t),t),\ \ t\geq t_0, \tag{2.4}$$

with initial conditions

$$v(t_0)=v_0. \tag{2.5}$$

The DTM generates an approximate solution of (2.4), which is formulated as

$$v(t)=\sum_{n=0}^{\infty}V(n)(t-t_0)^n, \tag{2.6}$$

to determine the values of unknowns $V(0), V(1), V(2), \ldots$. The DTM is applied to the initial conditions (2.4) and (2.5) consecutively, then the following is obtained

$$V(0)=v_0, \tag{2.7}$$

and

$$(1+n)V(n+1)=F(V(0),\ldots,V(n),n),\ \ \ n=0,1,2,\ldots, \tag{2.8}$$

where the differential of $f(v(t),t)$ is $F(V(0),\ldots,V(n),n)$. From (2.7) and (2.8), the values of $V(n), n=0,1,2,\ldots$ are computed. Then, the set of values $\{V(n)\}_{m=0}^{n}$ generates the solution as follows:

$$v(t)=\sum_{n=0}^{m}V(n)(t-t_0)^n, \tag{2.9}$$

the exact solution of problem (2.4)-(2.5) is obtained from equation (2.6).
If $X(n)\ and\ Y(n)$ are the transform function of $x(t)\ and\ y(t)$ respectively, then the basic operations of the DTM are presented in the following table:

*Table 2.1: Main Operations of DTM.*

| Original Function | Transformed Function |
|---|---|
| $\alpha x(t)\pm\beta y(t)$ | $\alpha X(k)\pm\beta Y(k)$ |
| $x(t)y(t)$ | $\sum_{r=0}^{n}X(n)Y(n-r)$ |
| $x(t)y(t)z(t)$ | $\sum_{r=0}^{n}\sum_{l=0}^{r}X(l)Y(r-l)Z(n-r)$ |
| $\frac{d^n}{dt^n}[x(t)]$ | $(k+1)\ldots(k+n)X(k+n)$ |
| $e^{\mu t}$ | $\frac{\mu e^{\mu t_0}}{n!}$ |
| $\sin(\phi t)$ | $\frac{\phi^n}{n!}\sin\left(\phi t_0+\frac{\pi n}{2}\right)$ |
| $\cos(\phi t)$ | $\frac{\phi^n}{k!}\cos\left(\phi t_0+\frac{\pi n}{2}\right)$ |

A recursion system of unknowns $V(0), V(1), V(2), \ldots$ is obtained by applying the DTM to the initial conditions to equations (2.4) and (2.5). Although the DTM converges over small intervals, this is enough reason to think about a technique that is able to enlarge the convergence to conclude larger intervals. The new technique is MsDTM, which will be illustrated in the next section.

## 3. MULTI-STAGE DIFFERENTIAL TRANSFORM METHOD (MSDTM)

The first time which MsDTM was applied by [8]. The main aim of applying the MsDTM is to extend the limitations of convergence over larger intervals. The MsDTM is applied by [8]-[11]. The essential principle of the MsDTM is based on the division of the study interval into small sub-intervals. To apply the MsDTM to equation (2.2), the following steps will state that as follows:

- The main interval $[0, T]$ is divided into N sub-intervals.
- Apply DTM to the first sub-interval $[0, t_1]$, then $[t_1, t_2]$.
- Repeat step 2 until to get the last sub-interval $[t_{N-1}, T]$.
- The formula of the solution obtained for each subinterval is as follows: $v_i(t) = \sum_{l=0}^{K} V_l(t-t_i)^l$, $i = 2, 3..., N-1$.
- The approximate solution over the whole study interval is obtained:

$$v(t) = \begin{cases} v_1(t) & 0 \le t \le t_1, \\ v_2(t) & t_1 \le t \le t_2, \\ \vdots \\ v_n(t) & t_{N-1} \le t \le T. \end{cases} \tag{3.1}$$

## 4. SOLVING NONLINEAR SYSTEMS OF DIFFERENTIAL ALGEBRAIC EQUATIONS

Assume that the formula of the system of nonlinear index-2 DAEs is as follows:

$$\begin{cases} M(v).v' = f(v, v') - G^T(v).\lambda, \\ 0 = g(v), \end{cases} \tag{4.1}$$

where $v$ is the vector of differential state variables, $M(v)$ is a square coefficient matrix (depending on $v$), $f(v, v')$ is a vector of known functions of $v$ and $v'$, $G(v) = \frac{dg(v)}{dv}$ is the Jacobian matrix of the constraint functions $g(v)$, and $v(t_0) = v_0$ are the initial conditions. Applying DTM to both sides of the system (4.1):

$$DT[M(v).v']_{n-1} = DT[f(v, v') - G^T(v).\lambda]_{n-1}, n \ge 1, \tag{4.2}$$

then the following is obtained based on the properties of the power series and Adomain polynomials mentioned in [12, 14, 15]:

$$\begin{cases} \sum_{l=0}^{n-1} M_{n-1-l}.(l+1)V_{l+1} = f_{n-1} - \sum_{l=0}^{n-1} G^T_{n-1-l}.\lambda_l. \\ 0 = g_n \end{cases} \tag{4.3}$$

for $l = n-1$, then the following equation is obtained:

$$M_0 n V_n = f_{n-1} - \sum_{l=0}^{n-2} M_{n-1-l}.(l+1)V_{l+1} - G_0^T \lambda_{n-1} - \sum_{l=0}^{n-2} G^T_{n-1-l}.\lambda_l. \tag{4.4}$$

From equation (4.4) the following equation is obtained:

$$M_0 n V_n + G_0^T \lambda_{n-1} = f_{n-1} - \sum_{l=0}^{n-2} [M_{n-1-l}.(l+1)V_{l+1} + G^T_{n-1-l}.\lambda_l], \tag{4.5}$$

$$nV_n + M_0^{-1} G_0^T \lambda_{n-1} = M_0^{-1}[f_{n-1} - \sum_{l=0}^{n-2} [l(l+1) M_{n-1-l} V_{l+1} + G^T_{n-1-l}.\lambda_l]], \tag{4.6}$$

assume $G_0V_n = S_n$,then $S_n = -g_n + G_0V_n, n \geq 1$, then the following is obtained:

$$G_0M_0^{-1}G_0^T\lambda_{n-1} = -nS_n + G_0M_0^{-1}[f_{n-1} - \sum_{l=0}^{n-2}[l(l+1)M_{n-1-l}V_{l+1} + G_{n-1-l}^T.\lambda_l]]. \tag{4.7}$$

From equation (4.7) $\lambda_{n-1}$ is calculated as follows:

$$\lambda_{n-1} = (G_0M_0^{-1}G_0)^{-1}(-nS_n + G_0r_{n-1}), n \geq 1, \tag{4.8}$$

where $r_{n-1} = f_{n-1} - \sum_{l=0}^{n-2}[l(l+1)M_{n-1-l}V_{l+1} + G_{n-1-l}^T.\lambda_l]$, then from equation (4.4) $V_n$ is evaluated:

$$V_n = \frac{1}{n}[-M_0^{-1}G_0^T\lambda_{n-1} + r_{n-1}]. \tag{4.9}$$

The system (4.1) is converted into an algebraic system after applying the DTM method. Finally, the inverse transform function is used to obtain the approximate solution as follows:

$$\begin{cases} v(t) = \sum_{k=0}^{K} v_k t^k, \\ \lambda(t) = \sum_{k=0}^{K} \lambda_k t^k \end{cases} \tag{4.10}$$

## 5. NUMERICAL EXAMPLES

The three examples are solved by the proposed technique in this section.

**Example 5.1.** *Consider a system of nonlinear index-2 DAEs as follows:*

$$\begin{cases} u_1' = u_2^2 - 2u_1\lambda \\ u_2' = u_1u_2 + 2u_2\lambda \\ 0 = u_1^2 - u_2^2 + 1 \end{cases} \tag{5.1}$$

with initial values $u_1(0) = 0$, $u_2(0) = 1$, and exact solutions as follows : $u_1(t) = tan(t)$, $u_2(t) = sec(t)$, $\lambda(t) = 0$.

Whereas $M(u) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, $f(u) = \begin{pmatrix} u_2^2 \\ u_1u_2 \end{pmatrix}$, $g(u) = u_1^2 - u_2^2 + 1$, and the Jacobian of the function $g(u)$ in (5.1) is

$G(u) = \big(2u_1, -2u_2\big)$,

the full row rank is $r = 1$. The MsDTM is applied to the interval $[0, T] = [0, 6]$ in the order of approximation $K = 6$, and the subdivisions of the entire interval study are $N = 200$. The results obtained in Figures 1-6. Figures 1, 3, and 5 show the solution components $u_1$, $u_2$ and $\lambda$, whereas Figures 2, 4, 6 and Tables 5.1, and 5.2 show the errors of the components $u_1$, $u_2$ and $\lambda$.

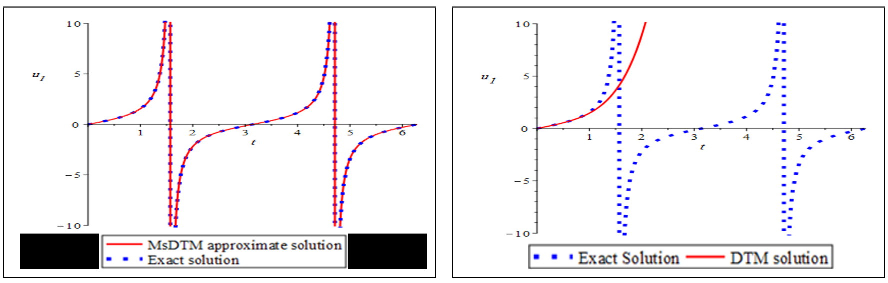


*Figure 1: Approximate solution of MsDTM, DTM and Exact Solution of $u_1$ of* (5.1)

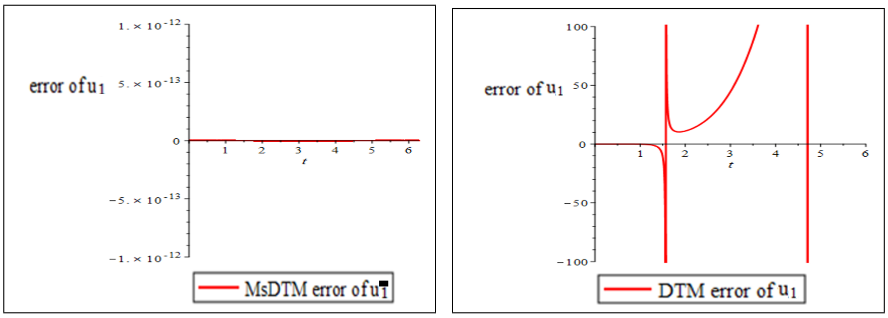


*Figure 2: MsDTM Error and DTM Error of $u_1$ of* (5.1)

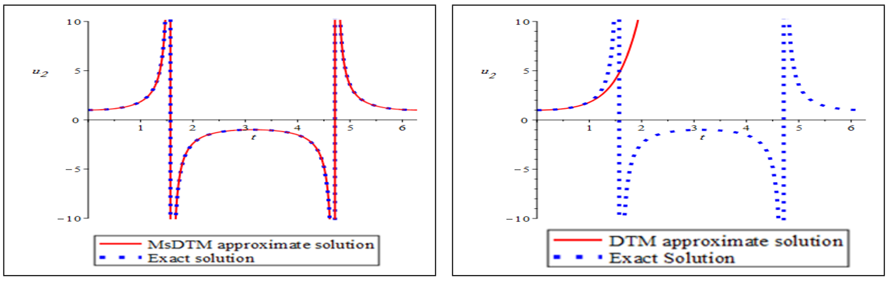


*Figure 3: Approximate solution of MsDTM, DTM and Exact Solution of $u_2$ of* (5.1)

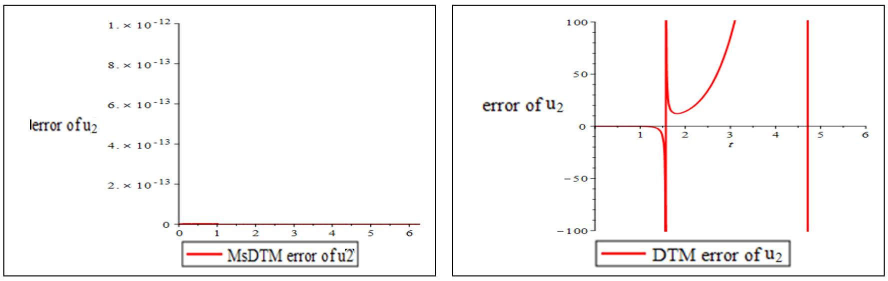


*Figure 4: MsDTM Error and DTM Error of $u_2$ of* (5.1)

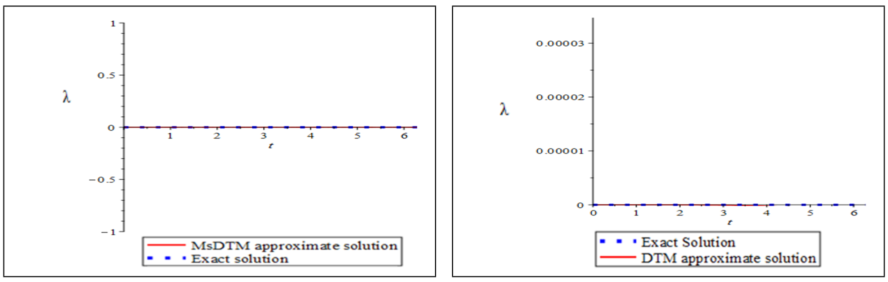


*Figure 5: Approximate solution of MsDTM, DTM and Exact Solution of $\lambda$ of* (5.1)

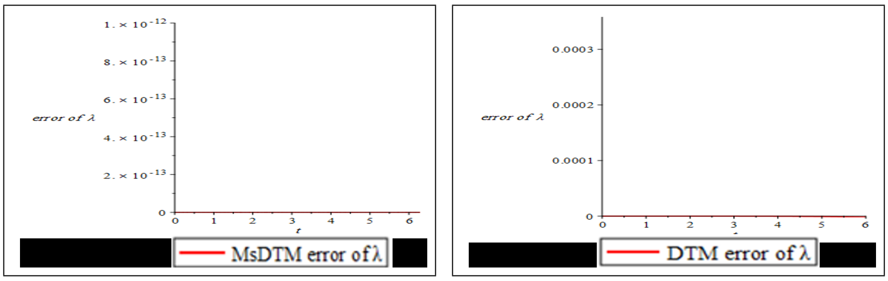


*Figure 6: MsDTM Error and DTM Error of $\lambda$ of* (5.1)

Absolute error of components $u_1$ and $u_2$ obtained for system (5.1) using MsDTM with step $h = \frac{T}{N} = 0.03$, $K = 6$ and $N = 200$ and $Eu_1$, $Eu_2$, $E\lambda$ are exact solutions of the components $u_1$, $u_2$ and $\lambda$ consecutively.

*Table 5.1: Absolute value error of MsDTM and Exact solution of $u$.*

| $t$ | $u_1$ | $Eu_1$ | $\|u_1 - Eu_1\|$ | $u_2$ | $Eu_2$ | $\|u_2 - Eu_2\|$ |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 1 | 1 | 0 |
| 0.2 | 0.2027093334 | 0.2027093334 | 0 | 1.020338755 | 1.020338755 | 0 |
| 0.4 | 0.4226986666 | 0.4226986666 | 0 | 1.085680355 | 1.085680355 | 0 |
| 0.6 | 0.6823680000 | 0.6823680000 | 0 | 1.427542755 | 1.427542755 | 0 |
| 0.8 | 1.014357333 | 1.014357333 | 0 | 1.793055555 | 1.793055555 | 0 |
| 1.0 | 1.466666666 | 1.466666666 | 0 | 2.404979200 | 2.404979200 | 0 |
| 1.2 | 2.107776000 | 2.107776000 | 0 | 3.418252356 | 3.418252356 | 0 |
| 1.4 | 3.031765334 | 3.031765334 | 0 | 5.066736355 | 5.066736355 | 0 |
| 1.6 | 4.363434666 | 4.363434666 | 0 | 7.688591199 | 7.688591200 | $1.0E - 9$ |
| 1.8 | 6.263423999 | 6.263424000 | $1.0E - 9$ | 11.75555555 | 11.75555556 | $1.0E - 8$ |
| 2.0 | 8.933333332 | 8.933333334 | $2E - 9$ | 17.90613075 | 17.90613076 | $1.0E - 8$ |

*Table 5.2: Absolute value error of MsDTM and Exact solution of $\lambda$.*

| $t$ | $\lambda$ | $E\lambda$ | $\|\lambda - E\lambda\|$ |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0.2 | $5.960005023E - 13$ | 0 | $5.960005023E - 13$ |
| 0.4 | $3.072004091E - 12$ | 0 | $3.072004091E - 12$ |
| 0.6 | $8.2801422E - 13$ | 0 | $8.2801422E - 13$ |
| 0.8 | $-2.96956492E - 11$ | 0 | $2.96956492E - 11$ |
| 1.0 | $-1.379999281E - 10$ | 0 | $1.379999281E - 10$ |
| 1.2 | $-4.055038685E - 10$ | 0 | $4.055038685E - 10$ |
| 1.4 | $-9.590277771E - 10$ | 0 | $9.590277771E - 10$ |
| 1.6 | $-1.974271644E - 9$ | 0 | $1.974271644E - 9$ |
| 1.8 | $-3.686795456E - 9$ | 0 | $3.686795456E - 9$ |
| 2.0 | $-6.399999196E - 9$ | 0 | $6.399999196E - 9$ |

The numerical results show that the approximate solutions of components $u_1$, $u_2$, and $\lambda$ in extremely close agreement with the exact solutions of the same components.

**Example 5.2.** *Consider a system of nonlinear index-2 DAEs as follows:*

$$\begin{cases} u_1' = u_1 + u_1^3 u_2 - 2u_1 u_2 \lambda \\ u_2' = u_1^4 u_2 - 2u_2 - u_1^2 \lambda \\ 0 = u_1^2 u_2 - 1 \end{cases} \tag{5.2}$$

with initial values $u_1(0) = 1$, $u_2(0) = 1$, and exact solutions as follows :
$u_1(t) = exp(t)$, $u_2(t) = exp(-2t)$, $\lambda(t) = exp(2t)$.
Whereas $M(u) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, $f(u) = \begin{pmatrix} u_1 + u_1^3 u_2 \\ u_1^4 - 2u_2 \end{pmatrix}$, $g(u) = u_1^2 u_2 - 1$, and the Jacobian of the equation is
$G(u) = \left(2u_1 u_2, u_1^2,\right)$ the full row rank is $r = 1$. The DTM is applied over the interval $[0, T] = [0, 2]$ with $K = 16$ and $N = 200$. The results obtained in Figures 7-12. Figures 7, 9, and 11 show the solution components $u_1$, $u_2$ and $\lambda$, whereas Figures 8, 10, and 12 show the errors of the components $u_1$, $u_2$ and $\lambda$.

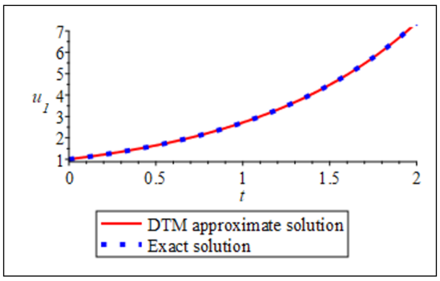


*Figure 7: Approximate Solution of DTM and Exact Solution of* $u_1$ *of* (5.2)

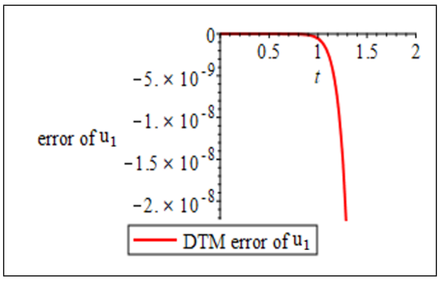


*Figure 8: DTM Error of* $u_1$ *of* (5.2)

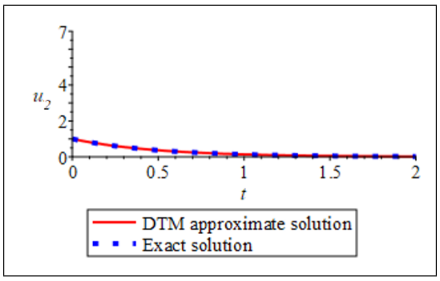


*Figure 9: Approximate Solution of DTM and Exact Solution of $u_2$ of* (5.2)

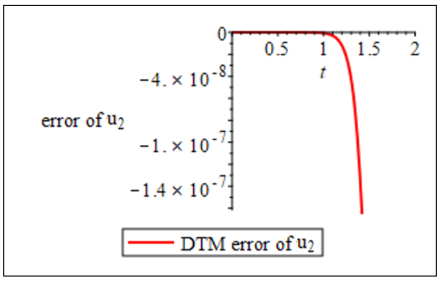


*Figure 10: DTM Error of $u_2$ of* (5.2)

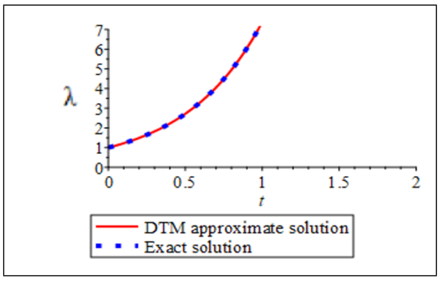


*Figure 11: Approximate Solution of DTM and Exact Solution of $\lambda$ of* (5.2)

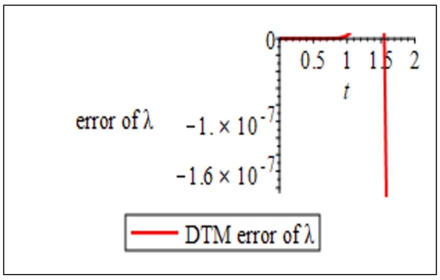


*Figure 12: DTM Error of $\lambda$ of* (5.2)

Although MsDTM is not used, the results are accurate and efficient because there is high agreement between the DTM's approximation and exact solutions.

**Example 5.3.** *Consider a system of nonlinear index-2 DAEs as follows:*

$$\begin{cases} u_1' = u_2 - 2u_1u_2^2 - 2u_1\lambda \\ u_2' = -2u_1^3 - u1 - 2u_2\lambda \\ 0 = u_1^2 + u_2^2 - 1 \end{cases} \tag{5.3}$$

with initial values $u_1(0) = 0$, $u_2(0) = 1$, and exact solutions as follows : $u_1(t) = sin(t)$, $u_2(t) = cos(t)$, $\lambda(t) = -cos^2(t)$.
Whereas $M(u) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, $f(u) = \begin{pmatrix} u_2 - 2u_1u_2^2 \\ -2u_1^3 - u_1 \end{pmatrix}$, $g(u) = u_1^2 + u_2^2 - 1$, and $G(u) = \left(2u_1^2, 2u_2^2\right)$ is the Jacobian of the function $g(u)$in (5.3), the full row rank is $r = 1$, the study interval is $[0, T] = [0, 3]$, the order of derivative is $K = 6$, and the divisions is $N = 300$. Figures 13, 15, and 17 show the solutions of components $u_1$, $u_2$ and $\lambda$. The errors of the components $u_1$, $u_2$ and $\lambda$ are presented in figures 14, 16, 18 and Tables 5.3 and 5.4. It is clear from the numerical results that there is a good agreement between the approximate and exact solutions of $u_1$, $u_2$ and $\lambda$.

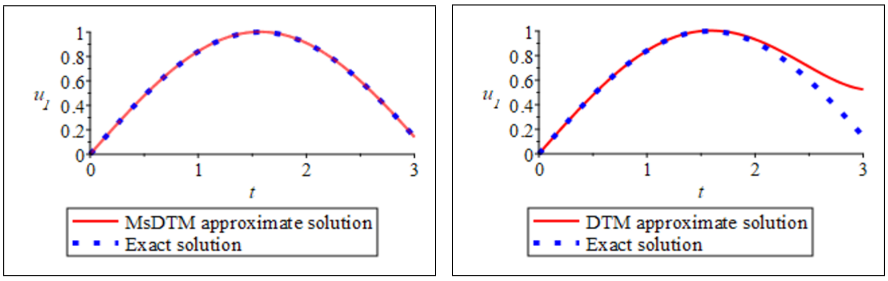


*Figure 13: Approximate Solution of MsDTM, DTM and Exact Solution of $u_1$ of* (5.3)

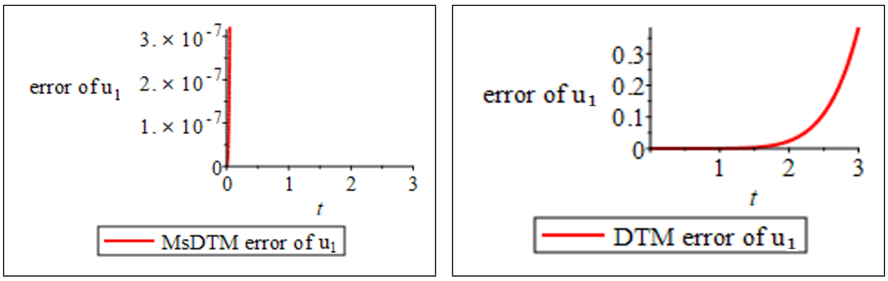


*Figure 14: MsDTM Error and DTM Error of $u_1$ of* (5.3)

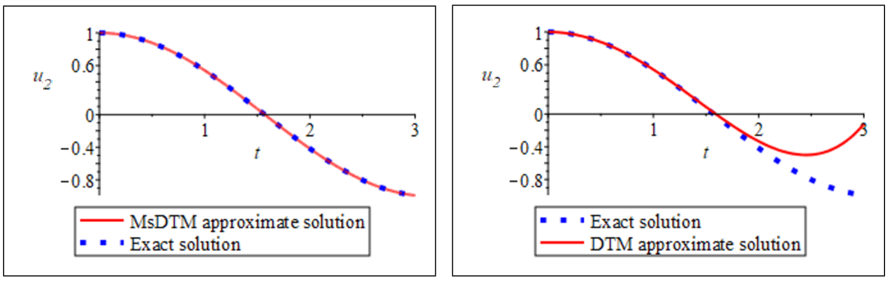


*Figure 15: Approximate Solution of MsDTM, DTM and Exact Solution of $u_2$ of* (5.3)

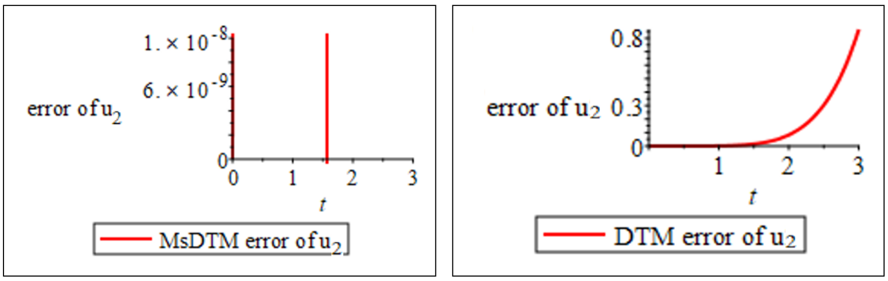


*Figure 16: MsDTM Error and DTM Error of $u_2$ of* (5.3)

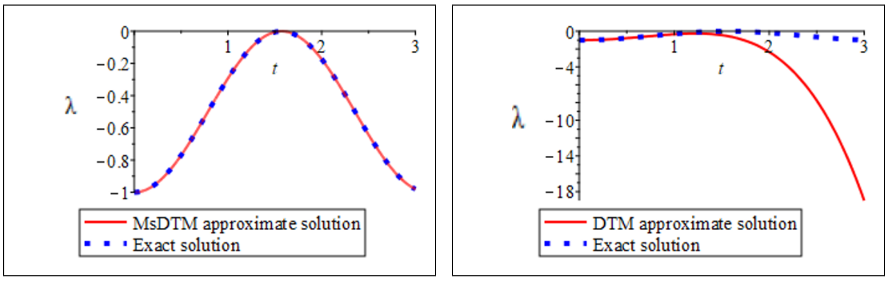


*Figure 17: Approximate Solution of MsDTM, DTM and Exact Solution of $\lambda$ of* (5.3)

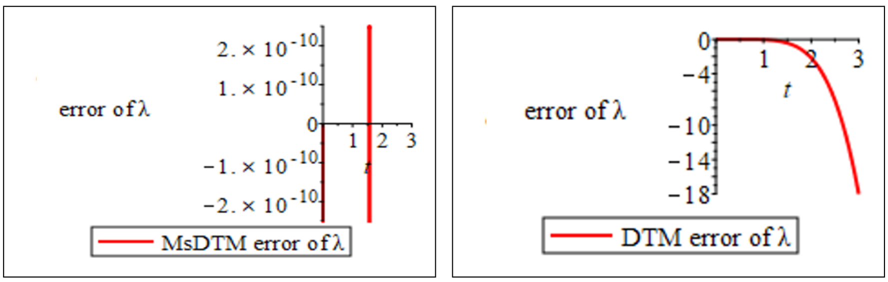


*Figure 18: MsDTM Error and DTM Error of $\lambda$ of* (5.3)

The absolute error of the components $u_1$ and $u_2$ obtained for the system (5.3) using MsDTM with step $h = \frac{T}{N} = 0.01$, $K = 6$, $N = 300$ and $Eu_1$, $Eu_2$, $E\lambda$ are exact solutions of the components $u_1$, $u_2$ and $\lambda$ consecutively.

*Table 5.3: Absolute Value Error of MsDTM and Exact solution of $u$.*

| $t$ | $u_1$ | $Eu_1$ | $\lvert u_1 - Eu_1\rvert$ | $u_2$ | $Eu_2$ | $\lvert u_2 - Eu_2\rvert$ |
|---|---|---|---|---|---|---|
| 0 | 1 | 1 | 0 | 1 | 1 | 0 |
| 0.2 | 1.221402759 | 1.221402759 | 0 | 0.6703200461 | 0.6703200461 | 0 |
| 0.4 | 1.49182408 | 1.49182408 | 0 | 0.4493289640 | 0.4493289640 | 0 |
| 0.6 | 1.822118801 | 1.822118801 | 0 | 0.30119421210 | 0.30119421210 | 0 |
| 0.8 | 2.225540929 | 2.225540929 | 0 | 0.2018965183 | 0.2018965182 | $1.0E-10$ |
| 1.0 | 2.7182828182 | 2.7182828183 | $1.0E-9$ | 0.1353352831 | 0.1353352837 | $6.0E-10$ |
| 1.2 | 3.320116916 | 3.320116923 | $7.0E-9$ | 0.09071794220 | 0.09071796050 | $1.8E-8$ |
| 1.4 | 4.055199893 | 4.055199967 | $7.4E-8$ | 0.06080992824 | 0.06081016007 | $12.3E-7$ |
| 1.6 | 4.953031901 | 4.953032424 | $5.23E-7$ | 0.04076119257 | 0.04076312682 | $1.9E-6$ |
| 1.8 | 6.049644650 | 6.049647465 | $2.8E-6$ | 0.02731731270 | 0.02733042486 | $1.3E-5$ |
| 2.0 | 7.389043626 | 7.382056099 | $1.2E-5$ | 0.01828434604 | 0.01835508622 | $7.0E-5$ |

*Table 5.4: Absolute Value Error of MsDTM and Exact solution of $\lambda$.*

| $t$ | $\lambda$ | $E\lambda$ | $\lvert\lambda - E\lambda\rvert$ |
|---|---|---|---|
| 0 | 1 | 1 | 0 |
| 0.2 | 1.491824628 | 1.491824628 | 0 |
| 0.4 | 2.225540929 | 2.225540929 | 0 |
| 0.6 | 3.320116923 | 3.320116923 | 0 |
| 0.8 | 4.953032424 | 4.953032424 | 0 |
| 1.0 | 7.389056103 | 7.389056099 | $4.0E-9$ |
| 1.2 | 11.23017644 | 11.02317638 | $6.0E-8$ |
| 1.4 | 16.44464695 | 16.44446650 | $3.0E-7$ |
| 1.6 | 24.53252978 | 24.5325288 | $9.0E-7$ |
| 1.8 | 36.59822288 | 36.59822441 | $1.8E-6$ |
| 2.0 | 54.59804733 | 54.59808820 | $4.0E-5$ |

## 6. CONCLUSION

Although the MsDTM method is well established, this study introduces a novel approach to utilize it to solve systems of nonlinear DAE equations of index 2. Unlike traditional methods, this approach avoids the need for linearization, index reduction, or altering the inherent behavior of the solutions. To demonstrate its effectiveness, the proposed technique was applied to three distinct examples, each highlighting its ability to handle complex systems with precision. The numerical results consistently showed an excellent agreement between the approximate and exact solutions, underscoring the method's reliability and potential for broader application in solving similar differential algebraic systems.

## COMPETING INTERESTS

The authors declare that they have no conflicts of interest to report on the present study.

## AUTHORS CONTRIBUTIONS

All authors contributed equally to this work. All authors read and approved the final manuscript.

## REFERENCES


[1] J. ZHOU, *Differential transformation and its applications for electrical circuits*, Huazhong Univ. Press, Wuhan, China, 1986.

[2] A. ARIKAN and I. OZKOL, *Solution of fractional differential equations by using differential transform method*, Chaos, Solitons & Fractals, 34 (2007), pp. 1473–1481.

[3] B. BENHAMMOUDA and H. VAZQUEZ-LEAL, *A new multi-step technique with differential transform method for analytical solution of some nonlinear variable delay differential equations*, SpringerPlus, 5 (2016), Article 1723.

[4] M. TAKAMATSU and S. IWATA, *Index reduction for differential–algebraic equations by substitution method*, Linear Algebra Appl., 429 (2008), pp. 2268–2277.

[5] G. ABD and R. ZABOON, *Approximate solution of a reduced-type index-Hessenberg differential-algebraic control system*, J. Appl. Math., 2021 (2021), Article ID 3548902.

[6] L. XIE, C. ZHOU, and S. XU, *Solving the systems of equations of Lane-Emden type by differential transform method coupled with Adomian polynomials*, Mathematics, 7 (2019), Article 377.

[7] T. MEYER, P. LI, and B. SCHWEIZER, *Backward differentiation formula and Newmark-type index-2 and index-1 integration schemes for constrained mechanical systems*, J. Comput. Nonlinear Dyn., 15 (2020), 021006.

[8] Z. ODIBAT, C. BERTELLE, M. AZIZ-ALAOUI, and G. DUCHAMP, *A multi-step differential transform method and application to non-chaotic or chaotic systems*, Comput. Math. Appl., 59 (2010), pp. 1462–1472.

[9] E. EL-ZAHAR, *Applications of adaptive multi step differential transform method to singular perturbation problems arising in science and engineering*, Appl. Math. Inf. Sci., 9 (2015), pp. 223–232.

[10] S. WANG and Y. YU, *Application of multistage homotopy-perturbation method for the solutions of the chaotic fractional order systems*, Int. J. Nonlinear Sci., 13 (2012), pp. 3–14.

[11] S. MOTSA, P. DLAMINI, and M. KHUMALO, *A new multistage spectral relaxation method for solving chaotic initial value systems*, Nonlinear Dyn., 72 (2013), pp. 265–283.

[12] U. ASCHER and P. LIN, *Sequential regularization methods for nonlinear higher-index DAEs*, SIAM J. Sci. Comput., 18 (1997), pp. 160–181.

[13] B. BENHAMMOUDA, *A new numerical technique for index-3 DAEs arising from constrained multibody mechanical systems*, Discrete Dyn. Nat. Soc., 2022 (2022), Article ID 6534896.

[14] B. BENHAMMOUDA and H. VAZQUEZ-LEAL, *A new multi-step technique with differential transform method for analytical solution of some nonlinear variable delay differential equations*, SpringerPlus, 5 (2016), Article 1723.

[15] B. BENHAMMOUDA and H. VAZQUEZ-LEAL, *Analytical solution of a nonlinear index-three DAEs system modelling a slider-crank mechanism*, Discrete Dyn. Nat. Soc., 2015 (2015), Article ID 768148.