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$\aleph_0$-ALGEBRA AND ITS NOVEL APPLICATION IN EDGE DETECTION

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ABSTRACT. In this paper a new type of $\aleph_0$-algebra has been defined. With its help, the fuzzy cross subalgebra and the fuzzy η-relation on the $\aleph_0$-algebra are introduced and their respective properties are derived. Moreover, the fuzzy cross $\aleph_0$-ideal of the $\aleph_0$-algebra is defined with some theorems and intuitionistic fuzzy $\aleph_0$-ideals of the $\aleph_0$-algebra are introduced. This fuzzy algebra concept is applied in image processing to detect edges. This $\aleph_0$-algebra is a novelty in the field of research.

Key words and phrases: $\aleph_0$-algebra; sub algebra of $\aleph_0$-algebra; fuzzy cross sub algebra of $\aleph_0$-algebra; fuzzy cross $\aleph_0$-ideal of $\aleph_0$-algebra; edge detection.

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1. INTRODUCTION

The concept of fuzzy sets is introduced by Zadeh [20]. In the algebraic structures these ideas have been applied. In 1991, the concept of fuzzy sets to *BCK* -algebras is applied by Xi [19], which is introduced by Imai and Iséki ([8, 9]). Also they introduced the BCK-Subalgebra in 1966. BCK-algebra is generalized by many authors ([5, 6, 16, 18]). Again in 2016, J. Zhan derived more theories on fuzzy BCK-Subalgebras [21]. M. Akram introduced d-algebra [1]. K. H. Kim introduced fuzzy dot Subalgebra of d-algebra in 2009 and he developed fuzzy dot subalgebra for minimum values in d-algebra [12]. N. O. Alsherie introduced fuzz d-ideals in d-algebra [2]. K. T. Atanassov [3] is the first one to propose intuitionistic fuzzy.

The fuzzy cross-subalgebra of a $\aleph_0$-algebra is defined here as a generalisation of a fuzzy subalgebra of a $\aleph_0$-algebra with maximal values. Then we derived some basic properties related to fuzzy cross $\aleph_0$-subalgebras. Then, the fuzzy η-relation on the $\aleph_0$-algebra and the fuzzy η-product relation on the $\aleph_0$-algebra are introduced. Also fuzzy cross $\aleph_0$-ideals of the $\aleph_0$-algebra are defined and explained with some theorems. By referring ([7, 10, 11, 14]), we introduce the intuitionistic fuzzy $\aleph_0$-ideals of the $\aleph_0$-algebra and study some interesting properties as well as some relations to the intuitionistic fuzzy $\aleph_0$-algebra. From ([4, 13, 15, 17]), $\aleph_0$-algebra concept is applied to detect edges in image processing.

2. BASIC CONCEPTS

Definition 2.1. $\aleph_0$-*algebra*. A $\aleph_0$-algebra is an algebra (A, $*$, 0), where A is a non empty set of type (2, 0) satisfying the important following laws:

(1) $\alpha_0 * \alpha_0 = 1$, $\alpha_0 \neq 0 \in$ A
(2) $\alpha_0 * 0 = 0$, $\alpha_0 \neq 0 \in$ A
(3) $\alpha_0 * \beta_0$ = 1 and $\beta_0 * \alpha_0$ =1⇒α_0=β_0 for all $\alpha_0 \neq 0$, $\beta_0 \neq 0 \in$ A.

Example 2.1. *Let A = $\{0, 1, \alpha\}$ where 0 $< \alpha <$ 1. Consider the Cayley table 2.1. Clearly this table shows that, the system (A, $*$, 0) is an $\aleph_0$-algebra.*

*	0	1	α
0	0	1	0
1	0	1	α
α	0	0	1

Table 2.1:

Definition 2.2. *Subalgebra of* $\aleph_0$-*algebra*. A nonempty subset B of $\aleph_0$-algebra A is said to be a Subalgebra of A if $\alpha_0 * \beta_0 \in$ B, for all α_0, $\beta_0 \in$ B.

Example 2.2. *Consider a $\aleph_0$-algebra A = $\{0, 1, \alpha\}$ where 0 $< \alpha <$ 1 with the Cayley table 2.2. Clearly this table shows that B = $\{0, \alpha\}$ is a subalgebra of a $\aleph_0$-algebra A.*

Definition 2.3. *Fuzzy cross subalgebra of an* $\aleph_0$-*algebra*. A non empty fuzzy subset ν of A is said to be fuzzy cross subalgebra of $\aleph_0$-algebra A if ν (α_0*β_0) $\leq \nu(\alpha_0) \times \nu(\beta_0)$ for all α_0, $\beta_0 \in$ A and α_0 = $\beta_0 \neq 1 \in$ A.

*	0	1	α
0	0	0	0
1	0	1	1
α	0	1	1

Table 2.2:

Example 2.3. *Consider an $\aleph_0$-algebra A = {0, 1, 2} with the Cayley table 2.3 as follow:*

*	0	1	2
0	0	0	0
1	0	1	1
2	0	0	1

Table 2.3:

Define a fuzzy set ν in A by ν (0) = 1, ν (1) = 0.2, ν (2) = 1. Very clearly it is true that ν is fuzzy cross subalgebra of an $\aleph_0$-algebra.

3. SOME RESULTS ON FUZZY CROSS SUBALGEBRA OF $\aleph_0$-ALGEBRA

Theorem 3.1. *If τ is a fuzzy cross subalgebra of an $\aleph_0$-algebra A, then we have $\tau(1) \leq (\tau(s_0))^2$ for all $s_0 \neq 1 \in A$.*

Proof. Consider a $\neq$ 1$\in$ A, then

$$\tau(1) = \tau(s_0 * s_0) \leq \tau(s_0) \times \tau(s_0) = (\tau(s_0))^2.$$

Thus, the result follows. ∎

Theorem 3.2. *If τ and λ are fuzzy cross subalgebras of an $\aleph_0$-algebra A, then so is $\tau \cup \lambda$.*

Proof. Let $s_0, t_0 \in$ A. Then

$$\begin{aligned}(\tau \cup \lambda)(s_0 * t_0) &= max\{\tau(s_0 * t_0), \lambda(s_0 * t_0)\}\\ &\leq Max\{\tau(s_0) \times \ \tau(t_0), \lambda(s_0) \times \lambda(t_0)\}\\ &\leq (max\{\tau(s_0), \lambda(s_0)\}) \cdot (max\{\tau(t_0), \lambda(t_0)\})\\ &= ((\tau \cup \lambda)(s_0)) \times ((\tau \cup \lambda)(t_0)).\end{aligned}$$

Hence $\tau \cup \lambda$ is fuzzy cross subalgebra of a $\aleph_0$-algebra A. ∎

Theorem 3.3. *If λ and τ are fuzzy cross subalgebras of an $\aleph_0$-algebra A, then $\lambda \times \tau$ is fuzzy cross subalgebra of $\aleph_0$-algebra A×A.*

Proof. For any $p_0^1, p_0^2, q_0^1, q_0^2 \in$ A,

$$\begin{aligned}(\lambda \times \tau)((p_0^1, q_0^1) * (p_0^2, q_0^2)) &= (\lambda \times \tau)(p_0^1 * p_0^2, q_0^1 * q_0^2)\\ &= \lambda(s_0^1 * s_0^2) \times \tau(t_0^1 * t_0^2)\\ &\leq ((\lambda(s_0^1) \times \lambda(s_0^2)) \times ((\tau(t_0^1) \times \tau(t_0^2))\\ &= ((\lambda(s_0^1) \times \tau(t_0^1)) \times (\lambda(s_0^2) \times \tau(t_0^2))\\ &= (\lambda \times \tau)(s_0^1, t_0^1) \times (\lambda \times \tau)(s_0^2, t_0^2).\end{aligned}$$

Thus, the result follows. ∎

Definition 3.1. *Fuzzy η-relation $\aleph_0$-algebra.* The fuzzy subset τ_η of A $\times$ A of a fuzzy η -relation on $\aleph_0$-algebra A is given by $\tau_\eta(s_0, t_0) = \eta(s_0) \times \eta(t_0)$ for all $s-0, t_0 \in$ A.

Definition 3.2. *Fuzzy η-product relation on $\aleph_0$-algebra.* A fuzzy η -product relation of a fuzzy relation τ on $\aleph_0$-algebra A is defined as if $\tau(s_0, t_0) \leq \eta(s_0) \times \eta(t_0)$ for all $s_0, t_0 \in$ A.

Definition 3.3. *Left Fuzzy relation on $\aleph_0$-algebra.* A left fuzzy relation on Let η be a fuzzy relation on $\aleph_0$-algebra A. Then η is a left fuzzy relation if $\tau(s_0, t_0) = \eta(s_0)$ for all $s_0, t_0 \in$ A. It is very clear that a left fuzzy relation on η is a fuzzy η-product relation.

Theorem 3.4. *Let τ_η be the fuzzy η-relation on $\aleph_0$-algebra A, where η is a fuzzy subset of a $\aleph_0$-algebra A. If η is fuzzy cross subalgebra of $\aleph_0$-algebra A, then τ_η is fuzzy cross subalgebra of A$\times$A.*

Proof. Let us assume that η is a fuzzy cross subalgebra of $\aleph_0$-algebra A. Let $s_0^1, s_0^2, t_0^1, t_0^2 \in$A, then

$$\begin{aligned}
\tau_\eta((s_0^1, t_0^1) * (s_0^2, t_0^2)) &= \tau_\eta(s_0^1 * s_0^2, t_0^1 * t_0^2) \\
&= \eta(s_0^1 * s_0^2) \times \eta(t_0^1 * t_0^2 \\
&\leq (\eta(s_0^1) \times (s_0^2)) \times (\eta(t_0^1) \times \eta(t_0^2)) \\
&= (\eta(s_0^1) \times \eta(t_0^1)) \times (\eta(s_0^2) \times \eta(t_0^2)) \\
&= \tau_\eta(s_0^1, t_0^1) \times \tau_\eta(s_0^2, t_0^2).
\end{aligned}$$

and so τ_η is fuzzy cross subalgebra of A$\times$A. ∎

Theorem 3.5. *Let τ be a left fuzzy relation on a fuzzy subset η of a $\aleph_0$-algebra A. If ν is fuzzy cross subalgebra of A $\times$ A, then η is fuzzy cross subalgebra of a $\aleph_0$-algebra A.*

Proof. Suppose that a left fuzzy relation τ on η is a fuzzy cross subalgebra of A$\times$A. Then

$$\begin{aligned}
\eta(s_0^1 * s_0^2) &= \tau(s_0^1 * s_0^2, t_0^1 * t_0^2) \\
&= \tau((s_0^1, t_0^1) * (s_0^2, t_0^2)) \\
&\leq \tau(s_0^1, t_0^1) \times \tau(s_0^2, t_0^2) \\
&= \eta(s_0^1) \times \eta(s_0^2) \forall s_0^1, s_0^2, t_0^1, t_0^2 \in A.
\end{aligned}$$

Hence η is fuzzy cross sub algebra of an $\aleph_0$-algebra A. ∎

4. FUZZY CROSS $\aleph_0$-IDEAL OF $\aleph_0$-ALGEBRA

Definition 4.1. *$\aleph_0$-ideal of $\aleph_0$-algebra.* Let U be a subset of $\aleph_0$-algebra A, then U is called $\aleph_0$-ideal of A if it satisfies the important following laws:

(1) $1 \in$ U
(2) $\alpha_0 * \beta \in$ U and $\alpha_0 \in U \Rightarrow \beta_0 \in$ U
(3) $\alpha_0 \in$ U, $\beta_0 \in$ A $\Rightarrow \alpha_0 * \beta_0 \in$ U

Example 4.1. *Consider a $\aleph_0$-algebra A = $\{$0, 1, 2, 3$\}$ with the Cayley table 4.1. Clearly this table shows that U = $\{$0, 1, 2$\}$ is $\aleph_0$-ideal of $\aleph_0$-algebra.*

Definition 4.2. *Fuzzy $\aleph_0$-ideal of $\aleph_0$-algebras.* Let τ be a fuzzy subset of $\aleph_0$-algebra A. Then τ is said to be a fuzzy $\aleph_0$-ideal of A if it satisfies the following conditions for all α_0, $\beta_0 \in$ A:

(1) τ (1) $\leq \nu(\alpha_0)$
(2) $\tau(\alpha_0) \leq$ max$\{\tau(\alpha_0 * \beta_0), \tau(\beta_0)\}$
(3) $\tau(\alpha_0 * \beta_0) \leq$ max$\{\tau(\alpha_0), \nu(\beta_0)\}$

$*$	0	1	2	3
0	0	0	0	0
1	1	1	2	0
2	2	1	1	0
3	1	0	0	1

Table 4.1:

Example 4.2. *From the table 4.1, consider a fuzzy set τ in A by τ(0) = 0.6, τ (1) = 0.5, τ (2) = 0.8, τ (3)=0.9. Then obviously τ is a fuzzy $\aleph_0$-ideal of $\aleph_0$-algebra A.*

Theorem 4.1. *Let τ be a fuzzy subset of a non empty set E. Then τ is a fuzzy $\aleph_0$-ideal of E if and only if, for any x $\in$ [0, 1] such that $L(\tau_E(r_0), x) \neq \emptyset$, is a fuzzy $\aleph_0$-ideal of E, where $L(\tau_E(r_0), x) = \{r \in E \| \tau(r_0) \leq x\}$, which is called a lower-level subset of τ.*

Proof. Let us assume that τ is a fuzzy $\aleph_0$-ideal of E. Suppose that $r_0, s_0 \in$ L$(\tau_E(r_0), x)$ and r_0 $*s_0 \in$ L$(\tau_E(r_0), x)$. Then $\tau(r_0) \leq$ x, $\tau(s_0) \leq$ x and $\tau(r_0 * s_0) \leq$ x, hence max$\{\tau(r_0 * s_0), \tau(s_0)\}$ $\leq$ x.

Since τ is a fuzzy $\aleph_0$-ideal of E, $\tau(1) \leq \tau(r_0) \leq$ x, $\tau(r_0) \leq$ max$\{\tau(r_0 * s_0), \tau(s_0)\} \leq$ x and $\tau(r_0 * s_0) \leq$ max $\{\tau(r_0), \tau(s_0)\} \leq$ x. Hence L($\tau_E(r_0)$, x) is a fuzzy $\aleph_0$-ideal of E.

Conversely, assume L$(\tau_E(r_0), x)$ is a fuzzy $\aleph_0$-ideal of E, for any x $\in$ [0, 1] with L$(\tau_E(r_0), x) \neq$ $\emptyset$.

For any $r_0, s_0 \in$ E, let $\tau(r_0) = x_1$, $\tau(s_0) = x_2$, $\tau(r_0 * s_0) = x_3(x_i \in$[0, 1]). If we let x = max$\{x_1, x_2, x_3\}$, then $r_0, s_0 \in L(\tau_E(r_0), x) \neq \emptyset$ and $r_0 * s_0 \in$ L$(\tau_E(r_0), x)$.

Since L($\tau_E(r_0)$, x) is a fuzzy $\aleph_0$-ideal of E, we have $r_0 \in$ L($\tau_E(r_0)$, x), i.e., $\tau(r_0) \leq$ max$\{\tau(r_0 * s_0), \tau(s_0)\}$, $\tau(r_0 * s_0) \leq$ max$\{\tau(r_0), \tau(s_0)\}$ so that τ is a fuzzy $\aleph_0$-ideal of E. ∎

Definition 4.3. *Fuzzy Cross $\aleph_0$-ideals of $\aleph_0$-algebras*. Let τ be a fuzzy subset of $\aleph_0$-algebra A. Then τ is said to be a fuzzy cross $\aleph_0$-ideal of A if it satisfies the following conditions for all α_0, β_0, and α_0 = $\beta_0 \neq$ 1$\in$ A.

(1) $\tau(1) \leq \tau(\alpha_0)$
(2) $\tau(\alpha_0) \leq \tau(\alpha_0 * \beta_0) \times \tau(\beta_0)$
(3) $\tau(\alpha_0 * \beta_0) \leq \tau(\alpha_0) \times \tau(\beta_0)$

Example 4.3. *Let A = $\{$0, 1, α, β $\}$. Consider the following Cayley table. Clearly this table 4.2 shows that, the system (A, $*$, 0) is a $\aleph_0$-algebra. U = $\{$0, α, 1$\}$ is a $\aleph_0$-ideal.*

$*$	0	α	β	1
0	0	0	0	0
α	α	1	0	α
β	β	0	1	β
1	1	0	0	1

Table 4.2:

Let τ be a fuzzy set in A defined by $\tau(0) = 1$, $\tau(1) = 0.5$, $\tau(\alpha) = \tau(\beta) = 1$. Then clearly τ is a fuzzy cross subalgebra of a $\aleph_0$-algebra. Also it is fuzzy cross $\aleph_0$-ideal of $\aleph_0$-algebra A.

Theorem 4.2. *Every fuzzy cross $\aleph_0$-ideal of a $\aleph_0$-algebra A is a fuzzy cross subalgebra of A.*

Proof. By the definition of fuzzy cross $\aleph_0$-ideal of a $\aleph_0$-algebra A, it is clearly true that every fuzzy cross $\aleph_0$-ideal of a $\aleph_0$-algebra A is a fuzzy cross subalgebra of A. ∎

Remark 4.1. The converse of the Theorem 3.7 is not true.

Theorem 4.3. *If λ and τ are fuzzy cross $\aleph_0$-ideals of a $\aleph_0$-algebra A, then so is $\lambda \cup \tau$.*

Proof. Let α_0, $\beta_0 \in$ A. Then

$$\begin{aligned}(\lambda \cup \tau)(1) &= max\{\lambda(1), \tau(1)\}\\ &\leq max\{\lambda(\alpha_0), \tau(\alpha_0)\}\\ &= (\lambda \cup \tau)(\alpha_0).\end{aligned}$$

Also,

$$\begin{aligned}(\lambda \cup \tau)(\alpha_0) &= max\{\lambda(\alpha_0), \tau(\alpha_0)\}\\ &\leq max\{\lambda(\alpha_0 * \beta_0) \times \lambda(\beta_0), \tau(\alpha_0 * \beta_0) \times \tau(\beta_0)\}\\ &\leq (max\{\lambda(\alpha_0 * \beta_0), \tau(\alpha_0 * \beta_0)\}) \times (max\{\lambda(\alpha), \tau(\alpha_0)\})\\ &= ((\lambda \cup \tau)(\alpha_0 * \beta_0)) \times ((\lambda \cup \tau)(\alpha_0)).\end{aligned}$$

And,

$$\begin{aligned}(\lambda \cup \tau)(\alpha_0 * \beta_0) &= max\{\lambda(\alpha_0 * \beta_0), \tau(\alpha_0 * \beta_0)\}\\ &\leq max\{\lambda(\alpha_0) \times \lambda(\beta_0), \tau(\alpha_0) \times \tau(\beta_0)\}\\ &\leq (max\{\lambda(\alpha_0), \nu(\alpha_0)\}) \cdot (max\{\lambda(\beta_0), \tau(\beta_0)\})\\ &= ((\lambda \cup \tau)(\alpha_0)) \times ((\lambda \cup \tau)(\beta_0)).\end{aligned}$$

Hence $\lambda \cup \tau$ is a fuzzy cross $\aleph_0$-ideal of a $\aleph_0$-algebra A. ∎

Theorem 4.4. *If λ and τ are fuzzy cross $\aleph_0$-ideals of a $\aleph_0$-algebra A, then $\lambda \times \tau$ is a fuzzy cross $\aleph_0$-ideal of A×A.*

Proof. Let $\alpha_0, \beta_0 \in$A. Then

$$(\lambda \times \tau)(1, 1) = \lambda(1) \times \tau(1) \leq \lambda(\alpha_0) \times \tau(\beta_0).$$

For any $\alpha_0, \alpha_0^1, \beta_0, \beta_0^1 \in$ A, we have

$$\begin{aligned}(\lambda \times \tau)(\alpha_0, \beta_0) &= \lambda(\alpha_0) \times \tau(\beta_0)\\ &\leq (\lambda(\alpha_0 * \alpha_0^1) \times \lambda(\alpha_0^1)) \times (\tau(\beta_0 * \beta_0^1) \times \tau(\beta_0^1))\\ &= (\lambda(\alpha_0 * \alpha_0^1) \times \tau(\beta_0 * \beta_0^1)) \times (\lambda(\beta_0^1) \times \tau(\beta_0^1))\\ &= (\lambda \times \tau)(\alpha_0, \beta_0) \times (\lambda \times \tau)(\alpha_0^1, \beta_0^1)\\ &= (\lambda \times \tau)((\alpha_0, \beta_0) * (\alpha_0^1, \beta_0^1)) \times (\lambda \times \tau)(\alpha_0^1, \beta_0^1).\end{aligned}$$

Also,

$$\begin{aligned}(\lambda\times\tau)((\alpha_0,\beta_0)*(\alpha_0^1,\beta_0^1)) &= (\lambda\times\tau)((\alpha_0*\alpha_0^1),(\beta_0*\beta_0^1))\\ &= \lambda(\alpha_0*\alpha_0^1)\times\tau(\beta_0*\beta_0^1)\\ &\leq ((\lambda(\alpha_0)\times\lambda(\alpha_0^1))\times(\tau(\beta_0)\times\tau(\beta_0^1))\\ &= (\lambda(\alpha_0)\times\tau(\beta_0))\times(\lambda(\alpha_0^1)\times\tau(\beta_0^1))\\ &= (\lambda\times\tau)(\alpha_0,\beta_0)*(\lambda\times\tau)(\alpha_0^1,\beta_0^1).\end{aligned}$$

Hence $\lambda\times\nu$ is a fuzzy cross $\aleph_0$-ideal of A $\times$ A. ∎

Theorem 4.5. *Let η be a fuzzy subset of a $\aleph_0$-algebra A and τ_η be the fuzzy η-relation on $\aleph_0$-algebra A. Then η is a fuzzy cross $\aleph_0$-ideal of A if and only if τ_η is a fuzzy cross $\aleph_0$-ideal of A $\times$ A.*

Proof. Let η be a fuzzy cross $\aleph_0$-ideal of A. For any $\alpha_0,\beta_0\in$ A we have

$$\begin{aligned}\tau_\eta(1,1) &= \eta(1)\cdot\eta(1)\\ &\leq \eta(\alpha_0)\cdot\eta(\beta_0)\\ &= \tau_\eta(\alpha_0,\beta_0).\end{aligned}$$

Let $\alpha_0,\alpha_0^1,\beta_0,\beta_0^1\in$ A. Then

$$\begin{aligned}\tau_\eta((\alpha_0,\alpha_0^1)*(\beta_0,\beta_0^1))\cdot\tau_\eta(\beta_0,\beta_0^1) &= \tau_\eta(\alpha_0*\beta_0,\alpha_0^1*\beta_0^1)\times\tau_\eta(\beta_0,\beta_0^1)\\ &= (\eta(\alpha_0*\beta_0)\times\eta(\alpha_0^1*\beta_0^1))\times(\eta(\beta_0)\times\eta(\beta_0^1))\\ &= (\eta(\alpha_0*\beta_0)\times\eta(\beta_0))\times(\eta(\alpha_0^1*\beta_0^1)\times\eta(\beta_0^1))\\ &\geq \eta(\alpha_0)\times\eta(\alpha_0^1)\\ &= \tau_\eta(\alpha_0,\alpha_0^1)\end{aligned}$$

and,

$$\begin{aligned}\tau_\eta(\alpha_0,\alpha_0^1)\cdot\tau_\eta(\beta_0,\beta_0^1) &= (\eta(\alpha_0)\times\eta(\alpha_0^1))\times(\eta(\beta_0)\times\eta(\beta_0^1))\\ &= (\eta(\alpha_0)\times\eta(\beta_0))\times(\eta(\alpha_0^1)\times\eta(\beta_0^1))\\ &\geq \eta(\alpha_0*\beta_0)\times\eta(\alpha_0^1*\beta_0^1)\\ &= \tau_\eta(\alpha_0*\beta_0,\alpha_0^1*\beta_0^1)\\ &= \tau_\eta((\alpha_0^1,\alpha_0^1)*(\beta_0^1,\beta_0^1)).\end{aligned}$$

Thus τ_η is a fuzzy cross $\aleph_0$-ideal of A $\times$ A.
Conversely suppose that τ_η is a fuzzy cross $\aleph_0$-ideal of A $\times$ A.

$$\begin{aligned}(\eta(1))^2=\eta(1)\times\eta(1) &= \nu_\eta(1,1)\leq\tau_\eta(\alpha_0,\alpha_0)\\ &= \tau(\alpha_0)\times\tau(\alpha_0)=(\tau(\alpha_0))^2\end{aligned}$$

and so $\eta(1)\leq\eta(\alpha_0)$ for all $\alpha_0\in$A. Also we have

$$\begin{aligned}(\eta(\alpha_0))^2 &= \nu_\eta(\alpha_0,\alpha_0)\\ &\leq \tau_\eta((\alpha_0,\alpha_0)*(\beta_0,\beta_0))\times\nu_\eta(\beta_0,\beta_0)\\ &= \tau_\eta((\alpha_0*\beta_0),(\alpha_0*\beta_0))\times\tau_\eta(\beta_0,\beta_0)\\ &= \eta((\alpha_0*\beta_0)\cdot\eta(\beta_0))^2,\end{aligned}$$

which implies that $\eta(\alpha_0) \leq \eta(\alpha_0 * \beta_0) \times \eta(\beta_0)$ for all $\alpha_0, \beta_0 \in$A. Also we have

$$\begin{aligned} (\eta(\alpha_0 * \beta_0))^2 &= \tau_\eta(\alpha_0 * \beta_0, \alpha_0 * \beta_0) \\ &= \tau_\eta((\alpha_0, \alpha_0) * (\beta_0, \beta_0)) \\ &\leq \tau_\eta(\alpha_0, \alpha_0) \times \tau_\eta(\beta_0, \beta_0) \\ &= (\eta(\alpha_0) \times \eta(\beta_0))^2 \\ \eta(\alpha_0 * \beta_0) &\leq \eta(\alpha_0) \times \eta(\beta_0) \forall \alpha_0, \beta_0 \in A. \end{aligned}$$

Therefore η is a fuzzy cross $\aleph_0$-ideal of A. ∎

Theorem 4.6. *Let τ be a left fuzzy relation on a fuzzy subset η of a $\aleph_0$-algebra A. If τ is a fuzzy cross $\aleph_0$-ideal of A $\times$ A, then η is a fuzzy cross $\aleph_0$-ideal of a $\aleph_0$-algebra A.*

Proof. Let the fuzzy relation τ on η be a fuzzy cross $\aleph_0$-ideal of A $\times$A. Then η(1) = $\tau(1, \gamma) \forall \gamma \in$ A.

By putting γ=1,
$\eta(1) = \tau(1, 1) \leq \nu(\alpha_0, \beta_0) = \eta(\alpha_0)$, for all $\alpha_0 \in$ A.
For any $\alpha_0, \alpha_0^1, \beta_0, \beta_0^1 \in$ A

$$\begin{aligned} \eta(\alpha_0) &= \tau(\alpha_0, \beta_0) \\ &\leq \tau((\alpha_0, \beta_0) * (\alpha_0^1, \beta_0^1)) \times \tau(\alpha_0^1, \beta_0^1) \\ &= \tau((\alpha_0 * \alpha_0^1), (\beta_0 * \beta_0^1)) \times \tau(\alpha_0^1, \beta_0^1) \\ &= \eta(\alpha_0 * \alpha_0^1) \times \eta(\alpha_0^1) \end{aligned}$$

Also,

$$\begin{aligned} \eta(\alpha_1 * \alpha_0^1) &= \tau(\alpha_0 * \alpha_0^1, \beta_0 * \beta_0^1) \\ &= \tau((\alpha_0, \beta_0) * (\alpha_0^1, \beta_0^1)) \\ &\leq \tau(\alpha_0, \beta_0) \times \tau(\alpha_0^1, \beta_0^1) \\ &= \eta(\alpha_0) \times \eta(\alpha_0^1). \end{aligned}$$

Thus η is a fuzzy cross $\aleph_0$-ideal of a $\aleph_0$-algebra A. ∎

5. INTUITIONISTIC FUZZY $\aleph_0$-IDEAL OF $\aleph_0$-ALGEBRA

Here, the idea of intuitionistic fuzzy $\aleph_0$-ideal of $\aleph_0$-algebra is introduced and we investigate several interesting properties, and study some relation on intuitionistic fuzzy $\aleph_0$-algebra.

Definition 5.1. Let X be an $\aleph_0$-algebra. An intuitionistic fuzzy set E = $< r_0, \gamma_E, \delta_E >$ is called an intuitionistic fuzzy $\aleph_0$-algebra if it satisfies

(1) $\gamma_E(r_0 * s_0) \leq Max\{\gamma_E(r_0), \gamma_E(s_0)\}$
(2) $\delta_E(r_0 * s_0) \geq Min\{\delta_E(r_0), \delta_E(s_0)\}$

Example 5.1. *Consider an $\aleph_0$-algebra E = {0, 1, α} with the Cayley table 5.1. Here*

*	0	1	α
0	0	0	0
1	0	1	1
α	0	1	1

Table 5.1:

$$\gamma_E(x) = \begin{cases} 0.2 & if\ x = 1 \\ 0.3 & if\ x = 0, 2 \end{cases}, \gamma_E(x) = \begin{cases} 0.4 & if\ x = 1 \\ 0.3 & if\ x = 0, 2 \end{cases}.$$

Then E is an intuitionistic fuzzy $\aleph_0$-algebra.

Definition 5.2. An intuitionistic fuzzy $\aleph_0$-ideal of $\aleph_0$-algebra X is defined in X as follows:

(1) $\gamma_E(r_0) \leq Max\{\gamma_E(r_0 * s_0), \gamma_E(s_0)\}$
(2) $\delta_E(r_0) \geq Min\{\delta_E(r_0 * s_0), \delta_E(s_0)\}$
(3) $\gamma_E(r_0 * s_0) \leq \gamma_E(r_0)$
(4) $\delta_E(r_0 * s_0) \geq \delta_E(r_0)$ for all $r_0, s_0 \in$ X

Example 5.2. *Consider an $\aleph_0$ algebra E = $\{$0, 1, $\alpha\}$ where 0 $< \alpha <$ 1 with the Cayley table 5.2. Here*

*	0	1	α
0	0	1	0
1	0	1	α
α	0	0	1

Table 5.2:

$$\gamma_E(x) = \begin{cases} 0.5 & if\ x = 1 \\ 0.3 & if\ x = 0, \alpha \end{cases}, \gamma_E(x) = \begin{cases} 0.4 & if\ x = 1 \\ 0.6 & if\ x = 0, \alpha \end{cases}$$

Then E is an intuitionistic fuzzy $\aleph_0$-algebra.

Theorem 5.1. *Let E = $<r_0,\ \gamma_E,\ \delta_E>$ be an intuitionistic fuzzy $\aleph_0$-ideal of $\aleph_0$-algebra X. Then $\gamma_E(1) \leq \gamma_E(r)$ and $\delta_E(1) \geq \delta_E(r_0)$ for all $r_0 \neq 0 \in X$.*

Proof. Since $\gamma_E(r_0 * s_0) \leq \gamma_E(r_0)$ and $\delta_E(r_0 * s) \geq \delta_E(r_0)$ for all r $\in$ X, then
$\gamma_E(r_0 * r_0) \leq \gamma_E(r_0)$ and $\delta_E(r_0 * r_0) \geq \delta_E(r_0)$ for all $r_0 \neq 0 \in$ X
Hence $\gamma_E(1) \leq \gamma_E(r_0)$ and also $\delta_E(1) \geq \delta_E(r_0)$ for all $r_0 \neq 0 \in$ X. ∎

Theorem 5.2. *Any intuitionistic fuzzy $\aleph_0$-ideal is intuitionistic fuzzy $\aleph_0$-algebra X.*

Proof. Let E = $< r_0, \gamma_E, \delta_E >$ be any intuitionistic fuzzy $\aleph_0$-ideal. Then $\gamma_E(r_0 * s_0) \leq \gamma_E(r_0) \leq Max\{\gamma_E(r_0 * s_0), \gamma_E(s_0)\} \leq Max\{\gamma_E(r_0), \gamma_E(s_0)\}$.

And also $\delta_E(r_0 * s_0) \geq \delta_E(r_0) \geq$ Min$\{\delta_E(r_0 * s_0), \delta_E(s_0)\} \geq Min\{\delta_E(r_0 * s_0), \delta_E(s_0)\}$ $\forall r_0, s_0 \in$ X. ∎

Theorem 5.3. *Let E = $< r_0, \gamma_E, \delta_E >$ be an intuitionistic fuzzy set. E is an intuitionistic fuzzy $\aleph_0$-ideal of $\aleph_0$-algebra X if and only if the fuzzy set γ_E and $\overline{\delta_E}$ are intuitionistic fuzzy $\aleph_0$-ideal.*

Proof. Assume that E = $< r_0, \gamma_E, \delta_E >$ be an intuitionistic fuzzy $\aleph_0$-ideal of $\aleph_0$-algebra X. Clearly γ_E is a fuzzy $\aleph_0$-ideal. Here, $\overline{\delta_E}$(1) = 1 - $\delta_E(1) \leq 1 - \delta_E(r_0) = \overline{\delta_E}(r_0)$.

Now $\overline{\delta_E}(r_0)$= 1 - $\delta_E(r_0) \leq 1 - Min\{\delta_E(r_0 * s_0), \delta_E(s_0)\} = Max\{1 - \delta_E(r_0 * s_0), 1 - \delta_E(s_0)\}$.

Hence $\overline{\delta_E}(r_0) \leq Max\{\overline{\delta_E}(r_0 * s_0), \overline{\delta_E}(r_0)\}$.

Again $\overline{\delta_E}(r_0 * s_0) = 1 - \delta_E(r_0 * s_0) \leq 1 - \delta_E(s_0) = \overline{\delta_E}(s_0)$.

i.e., $\overline{\delta_E}(r_0) \leq \overline{\delta_E}(s_0)$ and then $\overline{\delta_E}$ is a fuzzy $\aleph_0$-ideal of X.

Conversely, assume the fuzzy set γ_E and $\overline{\delta_E}$ are intuitionistic fuzzy $\aleph_0$-ideal of $\aleph_0$-algebra X.

For any $r_0, s_0 \in$ X, $\gamma_E(r_0) \leq Max\{\gamma_E(r_0 * s_0), \gamma_E(s_0)\}$ 1-$\delta_E(r_0) = \overline{\delta_E}(r_0) \leq Max\{\overline{\delta_E}(r_0 * s_0), \overline{\delta_E}(r_0)\} = Max\{1 - \delta_E(r_0 * s_0), 1 - \delta_E(s_0)\} = 1 - Min\{\delta_E(r_0 * s_0), \delta_E(s_0)\}$.

Then, $\delta_E(r_0) \geq Min\{\delta_E(r_0 * s_0), \delta_E(s_0)\}$.

Also $\gamma_E(r_0 * s_0) \leq \gamma_E(r_0)$ and $\overline{\delta_E}(r_0 * s_0) \leq \overline{\delta_E}(s_0)$. Then 1 - $\delta_E(r_0 * s_0) \leq 1 - \delta_E(s_0)$ and hence

$\delta_E(r_0 * s_0) \geq \delta_E(s_0)$. Thus E= $< r_0, \gamma_E, \delta_E >$ is an intuitionistic fuzzy $\aleph_0$-ideal of $\aleph_0$-algebra. ∎

Theorem 5.4. *Let E = $< r_0, \gamma_E, \delta_E >$ be an intuitionistic fuzzy set. Then E is a fuzzy $\aleph_0$-ideal if and only if, for any $x_0, y_0 \in$ [0, 1] such that L($\gamma_E(r_0), x_0$) $\neq \emptyset$ and U($\delta_E(r), y_0$) $\neq \emptyset$ are $\aleph_0$-ideals, where L($\gamma_E(r_0), x_0$) = $\{r_0 \in E \| \gamma(r_0) \leq x_0\}$ and U($\delta_E(r_0), y_0$) = $\{r_0 \in E \| \delta(r_0) \geq y_0\}$.*

Proof. Assume that E is a fuzzy $\aleph_0$-ideal of E. Suppose that $x_0, y_0 \in$ E and $s_0 \in$ L($\gamma_E(r_0), x_0$) and $r_0 * s_0 \in$ L($\gamma_E(r_0), x_0$). Then $\gamma(s_0) \leq x_0$ and $\gamma(r_0 * s_0) \leq x_0$.

Since γ is a fuzzy $\aleph_0$ -ideal of E, $\gamma(r_0) \leq$ max $\{\gamma(r_0 * s_0)$, $\gamma(s_0) \leq x_0$ and $\gamma(r_0 * s_0) \leq$ max $\{\gamma(r_0), \gamma(s_0)\ \} \leq x_0$, i.e., $r_0 \in$ L($\gamma_E(r_0), x_0$). Hence L($\gamma_E(r_0), x_0$) is a $\aleph_0$-ideal of E.

Again, if $x_0, y_0 \in$ E and $s_0 \in$ U($\delta_E(r_0), y_0$) and $r_0 * s_0 \in$ U($\delta_E(r_0), y_0$), then $\delta(s_0) \geq y_0$ and $\delta(r_0 * s_0) \geq y_0$.

Since τ is a fuzzy $\aleph_0$-ideal, $\delta(r_0) \geq$ min $\{\delta(r_0 * s_0), \delta(s_0)\ \} \geq y_0$ and $\delta(r_0 * s_0) \geq$ min $\{\delta(r_0), \delta(s_0)\ \} \geq y_0$, i.e.,$r_0 \in$ U($\delta_E(r_0), y_0$). Hence U($\delta_E(r_0), y_0$) is a $\aleph_0$-ideal.

Conversely, assume L($\gamma_E(r_0), x_0$), U($\delta_E(r_0), y_0$) are fuzzy $\aleph_0$-ideals, for any $x_0 \in$ [0, 1] with L($\gamma_E(r_0), x_0$) $\neq \emptyset$ and U($\delta_E(r_0), y_0$) $\neq \emptyset$.

For any $r_0, s_0 \in$ E, let $\gamma(r_0) = x_0^1$, $\gamma(s_0) = x_0^2$, $\gamma(r_0 * s_0) = x_0^3$ ($x_0^i \in$[0, 1]). If we let x_0 = max $\{x_0^1, x_0^2, x_0^3\ \}$, then $r_0, s_0 \in$ L($\gamma_E(r_0), x_0$) $\neq \emptyset$ and $r_0 * s_0 \in$ L($\gamma_E(r_0), x_0$).

Since L($\gamma_E(r_0), x_0$) is a fuzzy $\aleph_0$-ideal, we have $r_0 \in$ L($\gamma_E(r_0), x_0$), i.e., $\gamma(r_0) \leq$ max $\{\tau(r_0$ $*$ s), $\tau(s_0)\ \}$, $\gamma(r_0 * s_0) \leq$ max$\{\gamma(r_0), \gamma(s_0)\ \}$.

Also for any $r_0, s_0 \in$ E, let $\gamma(r_0) = y_0^1$, $\gamma(s_0) = y_0^2$, $\gamma(r_0 * s_0) = y_0^3$ ($y_0^i \in$[0, 1]). If we let y_0 = min $\{y_0^1, y_0^2, y_0^3\ \}$, then $r_0, s_0 \in$ U($\delta_E(r_0), y_0$)$\neq \emptyset$ and $r_0 * s_0 \in$ U($\delta_E(r_0), y_0$).

Since U($\delta_E(r_0), y_0$) is a fuzzy $\aleph_0$-ideal, we have $r_0 \in$ U($\delta_E(r_0), y_0$), i.e., $\delta(r_0) \geq$ min $\{\delta(r_0$ $* s_0), \delta(s_0)\ \}$, $\delta(r_0 * s_0) \geq$ max $\{\delta(r_0), \delta(s_0)\ \}$ so that E is a fuzzy $\aleph_0$-ideal. ∎

6. IMAGE PROCESSING ON $\aleph_0$-ALGEBRA

In recent years, the field of computer vision has experienced significant growth. Computer vision is concerned with the development of systems that can interpret the content of natural scenes. Computer vision systems start with detective work and finding some options within the input image. One of the main advantages of feature extraction is that it significantly reduces the amount of data required to represent an image in order to understand its content.

To obtain the appropriate purpose of a feature using similarity matching, a template window is used. This window is moved element by element over a larger search window around a computable corresponding purpose, and the similarity between the two regions is measured at each position. The position of the most effective match is determined by the maximum or minimum value of the resulting measurements. Normalized cross correlation is a well-known technique for determining the degree of similarity between two regions.

6.1. **EDGE DETECTION.** Edge detection is a essential device in image processing, particularly for function detection and extraction, with the intention of figuring out factors in a virtual photograph in which the brightness of the image adjustments abruptly, and detecting discontinuities. Edge detection reduces the quantity of statistics in an image whilst keeping the structural functions for similarly image processing. Edge in a grayscale photograph is a neighborhood function that separates areas inside a community in which the grayscale is greater or much less uniform and has special values on both aspect of the threshold. It is tough to locate edges in a loud image due to the fact each the threshold and the noise include excessive frequency content, ensuing in a blurred and distorted image.

The FFAST edge detector is a suitable edge detector, which extracts the edge of the image.

The working rules of F corner detection must satisfy the $\aleph_0$-algebra, which is Δ_x = {{ (k, l) & (k, l+1) & (k+1, l+1)} are whites and {(k-1, l) & (k, l-1) & (k-1, l-1) & (k-1, l+1) & (k+1, l-1), (k+1, l)} are blacks}(see Figure 1)

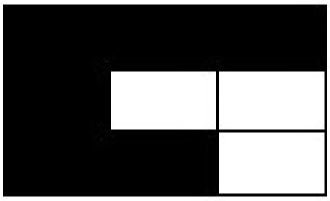

Figure 1: Δ_x

Δ_y = {{(k-1, l), (k, l) & (k, l+1) & (k+1, l+1)} are whites and {(k-1, l-1) & (k-1, l+1) &(k, l-1) & (k+1, l-1) &(k+1, l+1)} are blacks}(see Figure 2)

y.jpg

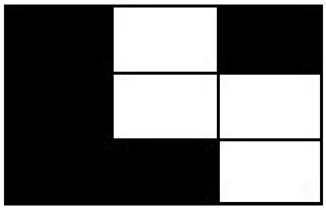

Figure 2: Δ_y

In the proposed F edge detection, instead of only considering a limited configuration space, in order to create a more efficient solution, a thorough configuration space is examined. The F edge detector operates in the following manner:

Algorithm 1. (1) *Using a good edge detector such as Canny, extract the edge contours from the input image.*
(2) *Fill in any small gaps in the edge contours. Mark the gap as a T-corner when it forms a T=1.96* $\widehat{\sigma}_s$ *-junction.*
(3) *Calculate the curvature of the edge contours at a large scale.*
(4) *The edge points are defined as the mini - maxi principles (Fuzzy Concept) of absolute curvature that are greater than a certain threshold value.*
(5) *To improve localization, track the corners through multiple lower scales.*

6.2. **EXPERIMENTAL STUDY.** The F edge detection algorithm was implemented in MATLAB Tool. The performance of the F edge is tested with images of different types file. The processing for detection of edges in images summarized in Figure 3.

7. CONCLUSION

In this paper we have introduced a new algebra and its corresponding fuzzy subalgebra. Fuzzy ideals and intuitionistic fuzzy ideals are defined and explained with some important results. Finally, this concept is applied in image processing to detect the edges. In future, this study can be extended to do more research on $\aleph_0$- algebra.

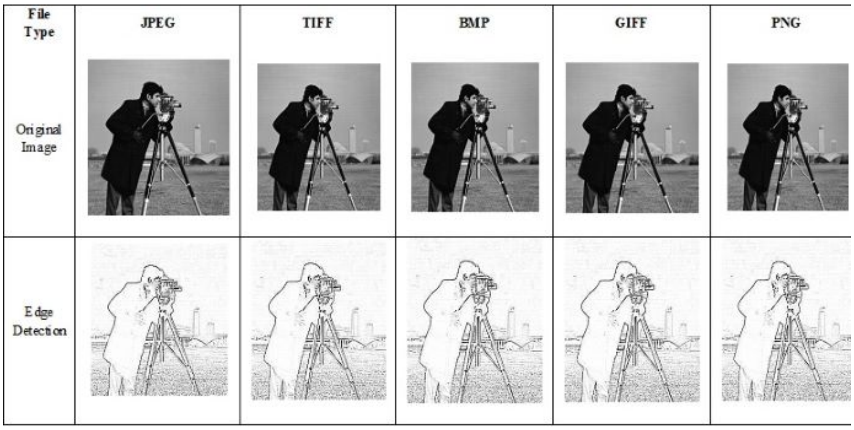

Figure 3: Types of files in edge detection

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DECLARATIONS

Conflict of interest/Competing interests. The authors declare that they have no competing interests.

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Authors' contributions. All authors contributed equally to this work. The authors declare that they have read and approved the final manuscript.

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