Aust. J. Math. Anal. Appl.
Vol. **21** (2024), No. 2, Art. 12, 10 pp.
AJMAA

PORTFOLIO OPTIMIZATION OF SHARIA AND NON-SHARIA STOCKS USING SINGLE INDEX MODEL (CASE STUDY: JAKARTA SHARIA INDEX AND KOMPAS 100 INDEX)

KWARDINIYA ANDAWANINGTYAS, MUHAMMAD LUTHFI, MARJONO,
ENDANG WAHYU HANDAMARI, UMU S'ADAH, EVI ARDIYANI

Received 22 July, 2023; accepted 24 July, 2024; published 7 October, 2024.

DEPARTMENT OF MATHEMATICS, BRAWIJAYA UNIVERSITY, MALANG, INDONESIA.,
DEPARTMENT OF MATHEMATICS, IPB UNIVERSITY, BOGOR, INDONESIA.
dina_math@ub.ac.id
muhammadluthfi@student.ub.ac.id
marjono@ub.ac.id
ewahyu-math@ub.ac.id
u.saadah@ub.ac.id
ardiyanievi@apps.ipb.ac.id

ABSTRACT. Stocks are instruments with high returns but also have increased risks. One way to overcome this risk is to form a stock portfolio. This study observed 30 sharia stocks listed on the Jakarta Islamic Index (JII) and 28 non-shariah stocks listed on the Kompas 100 Index from March 2020 to September 2022. The data used is the daily closing price of stocks, the number of stock dividends, and the daily closing price of the Jakarta Composite Index (JCI) from 3rd March 2020, to 31st August 2022. In addition, interest rate of Bank Indonesia Certificate (SBI) is used as risk-free rate. This study aims to optimize the sharia and non-sharia stocks portfolio using the Single Index Model (SIM), which will then be evaluated using the Sharpe, Treynor, and Jensen ratio. The result is that the optimal portfolio of sharia stocks have better performance than the optimal portfolio of non-sharia stocks based on the Treynor ratio. Meanwhile, the optimal portfolio of non-sharia stocks have better performance than the optimal portfolio of sharia stocks based on the Sharpe and Jensen ratio.

Key words and phrases: Portfolio; Risks; Sharia; Single Index Model; Stocks.

2010 *Mathematics Subject Classification.* Primary: 91G910, 62P05.

ISSN (electronic): 1449-5910

1. INTRODUCTION

Investment is an activity of suspending the use of funds, for now, to get future benefits [1]. One of the investment instruments is stocks. Stocks are proof or signs of capital ownership in a company [2]. Every investor wants to get high returns on investment, but there are investment risks that investors must consider. One way to control investment risk is to create a stock portfolio. The stock portfolio is a combination of various stocks owned by investors [3]. Furthermore, the optimal portfolio consists of several stocks that can produce the highest return and lowest risk combination [4].

In previous studies, [5] applied the Single Index Model (SIM) of portfolio construction and evaluate the model's performance on the stocks from Chittagong Stock Exchange (CSE). [6] made the optimal portfolio using the SIM with five companies from Nifty Index. [7] analyzed ten popular stocks in America with SIM, then find the optimal portfolio and minimal risk under five different constraints of Goal Programming. [8] minimized risk and maximized sharp ratio as constraint to get optimal portfolio from SIM of ten stocks and SPX Index. [9] analyzed ten stocks form SnP 500 index with constrained optimization and SIM analysis. [10] assessed and ranked the performance of various mutual fund with SIM. [11] used SIM to establish optimization inputs for calculating portfolios while considering existing regulations and risk aversion. [12] researched ten stocks in the US using SIM and five constraints of Goal Programming, then compared to the Markowitz model's portfolio. [13] researched 15 stocks in BSE Limited using the SIM and produced six selected stocks to form an optimal portfolio. [14] created an optimal portfolio of the seven best stocks out of 30 stocks in BSE Sensex using the SIM. [15] researched 15 stocks in NSE India and concluded that there were five stocks that formed the optimal portfolio. [16] determined the ideal portfolio from 50 Nifty stocks using SIM.

This research aims to optimize sharia and non-sharia stock portfolios using the SIM. The SIM method determines which stocks are eligible to be included in the optimal portfolio. Then it will be evaluated using the Sharpe, Treynor, and Jensen ratio.

2. METHODS

2.1. Portfolio Construction.

In this study, we use the SIM to form an optimal portfolio. The SIM is a portfolio analysis developed by William Sharpe in 1963. This model develops Markowitz's portfolio theory in calculating returns and risks using market returns as common factor. But before implementing the SIM, we need to do basic statistical calculations [2].

(1) Calculating the Stock Return and Market Return
Stock return is the rate of return obtained through a number of investments in stocks. According to Elton et al. [2], the stock return can be calculated using the following formula.

$$R_{i(t)} = \frac{P_t - P_{t-1} + D_t}{P_{t-1}}, \tag{2.1}$$

where $R_{i(t)}$ is the stock return i in the period t, P_t is the stock price in the period t, and D_t is the dividend in the period t. The market index is represented by the Jakarta Composite Index (JCI). The market return can be calculated using the following formula [2].

$$R_{M(t)} = \frac{JCI_t - JCI_{t-1}}{JCI_{t-1}}, \tag{2.2}$$

where $R_{M(t)}$ is the market return in the period t, JCI_t is the market price in the period t.

(2) Calculating the Stock Expected Return and Market Expected Return
Based on Elton et al. [2], we can estemate the profit form investing in stock using th following formula.

$$E[R_i] = \frac{\sum_{t=1}^{p} R_{i(t)}}{p}\,, \tag{2.3}$$

where $E[R_i]$ is the stock expected return i and p is number of observation periods. In the same way, the market expected return can be calculated using the following formula [2].

$$E[R_M] = \frac{\sum_{t=1}^{p} R_{M(t)}}{p}\,,$$

where $E[R_M]$ is the market expected return.

(3) Calculating the Stock Variance and Market Variance
The variance helps determine the spread of an asset's price from its average price. Therefore, the variance is used as an indicator of the volatility of price movements [2]. The stock variance can be calculated using the following formula [2].

$$\sigma_i^2 = \frac{\sum_{i=1}^{p}(R_{i(t)} - E[R_i])^2}{p}\,, \tag{2.4}$$

where σ_i^2 is the stock variance i. In the same way, the market expected return can be calculated using the following formula [2].

$$\sigma_M^2 = \frac{\sum_{i=1}^{p}(R_{M(t)} - E[R_M])^2}{p}\,, \tag{2.5}$$

where σ_M^2 is the market variance.

(4) Calculating the Covariance of Stock and Market Returns
The covariance of stock and market returns is used to determine the relationship between stock price movements and market index. A positive covariance means that when the market index moves up, stock prices tend to move up too, likewise for vice versa. The covariance of stock and market returns is expressed by the following equation [2].

$$\sigma_{iM} = \frac{\sum_{i=1}^{p}(R_{i(t)} - E[R_i]) \cdot (R_{M(t)} - E[R_M])}{p}\,, \tag{2.6}$$

where $\sigma_i M$ is the covariance of stock and market returns.

(5) Establish a Single Index Model
According to Elton et al. [2], the SIM measures return and risk in the form of a simple linear regression. Stock return as the y-axis or the dependent variable and market returns as the x-axis or the independent variable. The SIM equation is stated as follows.

$$R_{i(t)} = \alpha_i + \beta_i R_{M(t)} + \epsilon_{i(t)}\,, \tag{2.7}$$

where $R_{i(t)}$ is the stock return i, α_i is alpha i, β_i is beta i, $\mathrm{R}_{M(t)}$ is the market return, and $\epsilon_{i(t)}$ is varriance error residual.

(6) Calculating the Beta
Beta is a measurement of the volatility between a stock's return and market returns. The calculation of beta is stated by the following equation [2].

$$\beta_i = \frac{\sigma_{iM}}{\sigma_M^2}\,, \tag{2.8}$$

where β_i is beta i.

(7) Calculating the Alpha
Alpha is a measure of changes in stock returns that are not affected by changes in market returns. Alpha calculation is expressed by the following equation [2].

$$\alpha_i = E[R_i] - \beta_i \cdot E[R_M]\,, \tag{2.9}$$

where α_i is alpha i.

(8) Calculating Variance Error Residual
Variance error residual is the stock price movement variance that is not related to the market index movement. The variance error residual value of a stock represents the value of the stock's unsystematic risk. The calculation of the residual error variance is expressed by the following equation [2].

$$\sigma_{\epsilon_i}^2 = \sigma_i^2 - (\beta_i^2 \cdot \sigma_M^2)\,, \tag{2.10}$$

where $\sigma_{\epsilon_i}^2$ is variance error residual as unsystematic risk.

(9) Calculating Excess Return to Beta
Excess Return to Beta (ERB) is the excess value of stock returns relative to beta. The ERB value is needed to determine the stocks that make up the optimal portfolio. The following equation expresses the ERB value [2].

$$ERB_i = \frac{E[R_i] - R_f}{\beta_i}\,, \tag{2.11}$$

where ERB_i is ERB of i and R_F is risk-free rate. We use the interest rate of Bank Indonesia Certificate (SBI) as risk-free rate.

(10) Calculating Cut-off point
The cut-off point determines whether a stock can be included in the portfolio and determines the high and low limits of a stock's ERB value. Therefore, stocks with an ERB value greater than or equal to the cut-off point value will be selected for inclusion in the optimal portfolio. The cut-off point value is determined using the following equation [2].

$$C_i = \frac{\sigma_M^2 \sum_{i=1}^{q} (\beta_i \dfrac{E[R_i] - R_F}{\sigma_{e_i}^2})}{1 + \sigma_M^2 \sum_{i=1}^{q} (\frac{\beta_i^2}{\sigma_{e_i}^2})}\,, \tag{2.12}$$

where C is cut-off point.

(11) Calculating Investment Fund Allocation
After determining which stocks are included in the optimal portfolio based on ERB and cut-off point, then we calculate a percentage of stock weight or called the portion of investment fund allocation. The following equation calculates the portion of the investment fund allocation [2].

$$W_i = \frac{Z_i}{\sum_{i=1}^{q} Z_i}\,, \tag{2.13}$$

$$Z_i = \frac{\beta_i}{\sigma_{e_i}^2}(ERB_i - C^*)\,,$$

where W_i is weighted i for portion of the fund allocation in i.

2.2. **Portfolio Evaluation.**

Portfolio performance can't only look at the rate of return but also needs to pay attention to other factors such as volatility, risk, and expected portfolio returns [2]. In this study, we use the Sharpe, Treynor, and Jensen ratio to evaluate portfolio performance.

1. Sharpe ratio
Sharpe ratio, also known as the reward to variability ratio, was discovered by William Sharpe in 1966 [2]. The Sharpe ratio is used to measure the risk premium for each unit of risk in a portfolio. The higher the Sharpe ratio value of a portfolio compared to other portfolios, the better the performance of the portfolio. Sharpe ratio calculation is expressed by the following equation [2].

$$S_P = \frac{E[R_P] - R_F}{\sigma_P}\,, \tag{2.14}$$

where S_P is Sharpe ratio and σ_P is portfolio risk.

2. Treynor ratio
Treynor ratio or commonly known as the reward to volatility ratio was discovered by Jack Treynor in 1965 [2]. The Treynor ratio assumes that the portfolio is well diversified so that the risk considered relevant is systematic risk (measured by beta). The higher the Treynor ratio value of a portfolio compared to other portfolios, the better the performance of the portfolio. Treynor ratio calculation is expressed by the following equation [2].

$$T_P = \frac{E[R_P] - R_F}{\beta_P}\,, \tag{2.15}$$

where T_P is Treynor ratio and β_P is beta portfolio.

3. Jensen ratio
Jensen ratio was discovered by Michael Jensen in 1968 [2]. The Jensen ratio shows the difference between the expected portfolio return and the expected rate if the portfolio is on the capital market line. If the Jensen ratio is positive, then the portfolio return is greater than the expected return. The calculation of the Jensen ratio is expressed by the following equation [2].

$$J_p = E[R_P] - [R_F + (E[R_M] - R_F)\beta_P]\,, \tag{2.16}$$

where J_P is Jensen ratio and $E[R_P]$ is expected portfolio return.

3. RESULTS AND DISCUSSSION

The results have been obtained and will be discussed in this section.

3.1. Data.

We researched 30 sharia stocks that were consistently listed on Jakarta Islamic Index (JII) and 28 non-sharia stocks that were consistently listed on the Kompas 100 Index from March 2020 to September 2022. The data used is the daily closing price of stocks, the number of stock dividends, and the daily closing price of the Jakarta Composite Index (JCI) from 3rd March 2020, to 31st August 2022. In addition, interest rate of Bank Indonesia Certificate (SBI) is used as risk-free rate.

3.2. Portfolio Optimization Using Single Index Model.

After collecting the data, we calculate the ERB values for sharia and non-sharia stocks. The following are results from calculating the ERB for sharia and non-sharia stocks based on equation (2.11).

Table 3.1: The calculation result of ERB sharia stocks

Stocks	$E[R_i]$	$\sigma^2_{\epsilon_i}$	β_i	ERB
ADRO	0.0028	0.0008	1.4534	0.0006
ANTM	0.0028	0.0016	0.1540	0.0064
BRPT	0.0005	0.0015	0.1998	-0.0068
CPIN	0.0005	0.0007	-0.0964	0.0137
ERAA	0.0013	0.0010	0.0007	-0.7746
EXCL	0.0006	0.0010	0.1269	-0.0098
ICBP	0.0000	0.0004	-0.0281	0.0667
INCO	0.0021	0.0011	0.3613	0.0006
INDF	0.0003	0.0004	0.0023	-0.6639
INTP	0.0000	0.0009	-0.0105	0.1785
ITMG	0.0031	0.0010	0.0722	0.0168
JPFA	0.0007	0.0009	0.0585	-0.0192
KLBF	0.0009	0.0006	-0.0388	0.0250
MNCN	-0.0002	0.0008	0.2084	-0.0097
PGAS	0.0014	0.0010	0.0209	-0.0225
PTBA	0.0020	0.0009	-0.0244	-0.0074
SCMA	0.0007	0.0013	-0.0242	0.0487
TLKM	0.0009	0.0005	0.0925	-0.0100
TPIA	0.0007	0.0006	0.1207	-0.0099
UNTR	0.0018	0.0008	-0.1788	0.0005
UNVR	-0.0003	0.0006	-0.0507	0.0418
WIKA	-0.0002	0.0013	-0.0016	1.2269

Table 3.2: The calculation result of ERB non-sharia stocks

Stocks	$E[R_i]$	$\sigma^2_{\epsilon_i}$	β_i	ERB
ASII	0.0008	0.0006	-0.0910	0.0118
BBCA	0.0008	0.0004	0.0267	-0.0407
BBNI	0.0008	0.0007	0.0150	-0.0711
BBRI	0.0006	0.0007	0.0415	-0.0308
BBTN	0.0003	0.0009	0.0231	-0.0654
BMRI	0.0009	0.0006	-0.0373	0.0266
DOID	0.0023	0.0019	0.1830	0.0026
ESSA	0.0044	0.0023	0.0677	0.0375
GGRM	-0.0008	0.0006	0.0005	-5.8298
HMSP	-0.0004	0.0006	0.0649	-0.0351
INDY	0.0034	0.0018	0.0141	0.1099
JSMR	-0.0001	0.0007	0.0503	-0.0391
MEDC	0.0016	0.0014	0.3615	-0.0006
PNLF	0.0013	0.0010	0.00187	-0.0304
TBIG	0.0022	0.0010	-0.0885	-0.0037
TOWR	0.0011	0.0005	0.0534	-0.0139
WKST	-0.0002	0.0014	0.1406	-0.0145

After finding the ERB value, the next step is calculating the cut-off point value. To calculate the cut-off point value, we first select stocks with a positive ERB value, then sort them from the largest to the smallest ERB value. The following are results from calculating the cut-off point of Sharia and non-Sharia stocks based on equation (2.12).

Table 3.3: The calculation of cut-off point sharia stocks

Stocks	$\frac{(E[R_i]-R_F)\beta_i}{\sigma^2_{\epsilon_i}}$	$\frac{\beta_i^2}{\sigma^2_{\epsilon_i}}$	C_i
SCMA	0,022436	0,460686	0,000002
KLBF	0,065995	2,637048	0,000007
ITMG	0,089911	5,338359	0,000014
CPIN	0,172996	12,639988	0,000028
ANTM	0,096498	15,076516	0,000035
ADRO	1,725387	2771,815148	0,000140
INCO	0,068659	113,740258	0,000144
UNTR	0,019396	40,403344	0,000144
BRPT	-0,186239	27,380455	0,000132
PTBA	-0,005163	0,698071	0,000132
EXCL	-0,165167	16,850184	0,000121
TPIA	-0,246591	24,973338	0,000105
TLKM	-0,179548	17,975253	0,000094
JPFA	-0,070286	3,655313	0,000089
PGAS	-0,009513	0,423305	0,000089
INDF	-0,008176	0,012315	0,000088
ERAA	-0,000354	0,000457	0,000088

Based on the ERB value of sharia stocks in Table 3.1 and the cut-off point value of sharia stocks in Table 3.3, stocks with an ERB value greater than or equal to the cut-off point value are SCMA, KLBF, ITMG, CPIN, ANTM, ADRO, INCO, and UNTR.

Table 3.4: The calculation of cut-off point non-sharia stocks

Stocks	$\frac{(E[R_i]-R_F)\beta_i}{\sigma^2_{\epsilon_i}}$	$\frac{\beta_i^2}{\sigma^2_{\epsilon_i}}$	C_i
INDY	0,012492	0,113677	0,000002
ESSA	0,073953	1,973419	0,000014
BMRI	0,057292	2,153951	0,000023
ASII	0,162614	13,835299	0,000048
DOID	0,046428	17,750243	0,000055
MEDC	-0,056511	90,321617	0,000046
TBIG	-0,030145	8,202412	0,000041
TOWR	-0,073577	5,277363	0,000030
PNLF	-0,011124	0,365613	0,000028
BBRI	-0,080381	2,608305	0,000016
BBCA	-0,079576	1,954527	0,000003
BBTN	-0,037176	0,568797	-0,000002
BBNI	-0,022506	0,316415	-0,000006

Based on the ERB values of non-sharia stocks in Table 3.2 and the cut-off point values of non-sharia stocks in Table 3.4, stocks with ERB values greater than or equal to the cut-off point values are INDY, ESSA, BMRI, ASII, and DOID.

3.3. **Investment Fund Allocation.**

In the next stage, we will determine the percentage of investment fund allocation to the portfolio formed based on the Single Index Model method. Following are the results of fund allocation of sharia and non-sharia stock portfolios based on equation (2.13).

Table 3.5: The result of fund allocation for sharia stocks portfolio

Stocks	Z_i	W_i
CPIN	1.776518	0.240952
KLBF	1.692806	0.229598
ITMG	1.235335	0.167551
SCMA	0.923912	0.125312
ADRO	0.911568	0.123638
ANTM	0.612382	0.083059
INCO	0.144544	0.019605
UNTR	0.075835	0.010286

Table 3.6: The result of fund allocation for non-sharia stocks portfolio

Stocks	Z_i	W_i
ASII	1.779293867	0.321648106
BMRI	1.531282855	0.276814437
ESSA	1.09010284	0.197061048
INDY	0.882800263	0.159586361
DOID	0.248322887	0.044890048

3.4. **Portfolio Performance.**

After determining the optimal portfolio-forming stocks based on the Single Index Model method and the percentage of fund allocation, we will evaluate the portfolio's performance. Following are the results of assessing the performance of sharia and non-sharia stock portfolios using the Sharpe, Treynor, and Jensen ratio.

Table 3.7: The result of portfolio performance

Optimum portfolio	Sharpe ratio	Treynor ratio	Jensen ratio
Sharia stocks	-0.00963010	-0.00164565	-0.00005451
Non-sharia stocks	0.00436700	-0.00957299	0.00012995

Based on Table 3.7, the value of the Sharpe ratio and Jensen ratio of the optimal portfolio of non-sharia stocks is greater than the optimal portfolio of sharia stocks. It means that the performance of the optimal portfolio of non-sharia stocks is better than the optimal portfolio of sharia stocks based on the Sharpe and Jensen ratio. Meanwhile, the Treynor ratio value of the optimal portfolio of sharia stocks is greater than the optimal portfolio of non-sharia stocks. It means that the performance of the optimal portfolio of sharia stocks is better than the optimal portfolio of non-sharia stocks based on the Treynor ratio.

4. CONCLUSIONS

The result of using the Single Index Model to form an optimal portfolio is that the optimal portfolio of sharia stocks consists of SCMA, KLBF, ITMG, CPIN, ANTM, ADRO, INCO, and UNTR. On the other hand, the optimal portfolio of non-sharia stocks consists of INDY, ESSA, BMRI, ASII, and DOID. The Sharpe ratio and Jensen ratio of the non-sharia stocks portfolio exceed those of the sharia stocks portfolio, indicating better performance of the non-sharia stocks portfolio based on these metrics. Meanwhile, the Treynor ratio of the sharia stocks portfolio surpasses that of the non-sharia stocks portfolio, indicating a better performance of the sharia stocks portfolio based on the Treynor ratio. For further research, we recommend using the Multiple Index Model because this method utilizes factors other than market indices, such as the interest rate factor in portfolio calculations to form an optimal portfolio of stocks.

Acknowledgments: We would like to express our gratitude to Pusat Publikasi Ilmiah dan Ketahanan Jurnal Universitas Brawijaya.

REFERENCES

[1] J. ZWEIG, *The Intelligent Investor - Revised Edition*, Harper Collins, (2023).

[2] E. J ELTON, M. J. GRUBER, S. J BROWN, and W. N GOETZMANN, *Modern Portfolio Theory and Investment Analysis*, Jogn Wiley & Sons, (2014).

[3] H. SHIMIZU and T. SHIOHAMA, Multifactor portfolio construction by factor risk parity strtegies: an empirical comparison of global stock markets, *Asia-Pasific Financial Markets*, **26** (2019), pp. 453-477.

[4] A. OYENUBI, Diversificarion measures and the optimal number of stocks in a portfolio : an information theoretic explanation, *Computational Economics*, **54** (2019), pp. 1443-1471.

[5] I. MAHMUD, Optimal portfolio construction using sharpe's single-index model : evidence from chittagong stock exchange, *International Journal of Commerce and Finance*, **6** (2022), pp. 127-142.

[6] S. GAUTAMI, B. NAIK, and N. REDDY, A study on construction of optimal portfolio for selected stocks using Sharpe's Single Index Model, *Neuro Quantology*, **20** (2022), pp.46-58.

[7] B. ZHAO, Modern risk portfolio optimization: Assets allocation in stock using Single Index Model under five constrains, *International Conference on Management Science and Industrial Economy Development*, (2022), pp. 360-266.

[8] B. HU and J. LI, Comparison of the Markowitx and single index model in optimal portfolio, *International Conference on Management Science and Industrial Economy Development*, (2022), pp. 312-319.

[9] Z. A. ZHANG, Study of portfolio performance under certain restraint comparison: Markowitz model and Single Index Model on S&P 500, *International Conference on Social Sciences and Economic Development*, (2022), pp. 1930-1938.

[10] S. BHALERAO and M. O. B. CHAVARE, A study of using the sharp index model to construct an optimal liquid fund portfolio, *International Conference on Innovation in Management and Information Technology*, (2022), pp 141-148.

[11] T. ZHU, Index model portfolio construction, *International Conference on Management Science and Industrial Economy Development*,S. BHALERAO and M. O. B. CHAVARE, A study of using the sharp index model to construct an optimal liquid fund portfolio, pp 326-336.

[12] X. LI and J. WANG, Markowirz and index model comparison using different stocks, *International Conference on Ecnomonics, Smart Finance and Conremporary Trade*, (2022), pp. 808-817.

[13] K. J. SARVAMANGALA and G. S. REDDY, Construction of optimal portfolio using Sharpe's single model index, *International Journal of Education and Technology*, **4** (2022), pp 242-247.

[14] M. B. C. S. GURU and C. BAGRECHA, Building an optimal portfolio using Sharpe's single index model, *International Journal of Health Sciences*, **6** (2022), pp. 11567-11581.

[15] D. S. YOGESH and V. AYRE, A study on Indian capital market of building an optimal equity portfolio using sharpe single index model, *International Journal of Research Publication and Reviews*, **3** (2022), pp. 562-567.

[16] K. REDDY, S. N. LAKSHMI, S. THILAGA, and M. BASHA, Construction of an optimal portfolio using the single index model: an empirical study of pre and post Covid-19, *Journal of Pharmaceutical Negative Results*, **14** (2023), pp. 406-417.