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A GENERALIZATION OF A PARTIAL b-METRIC AND FIXED POINT THEOREMS

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ABSTRACT. The purpose of this paper is to introduce the concept of a Partial α, β b-metric as a generalization of a partial b-metric and prove theorems for some contractive type mapping.

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1. INTRODUCTION

The advancements in technology have lead researchers to extend and improve the concept of a metric space to accomodate the various applications like, modular metric, complex-valued metric , b-metric spaces and partial metric spaces. The concept of partial metric space was proposed by Matthews in 1992.

Matthews, introduced the notion of a partial metric space as a denotational semantics of dataflow network. The most important difference of a partial metric rather than a standard metric is the existing possibility of a non-zero self distance, [2]. The author further showed that the Banach contraction principle is valid in partial metric spaces. In 1993, Bakhtin, introduced the concept of b-metric space and proved the Banach contraction principle in the b-metric space,[1]. Shukla, introduced the partial b-metric and generalization of many results related to fixed point theories have been studied in these spaces, [3]. O'Neill generalized the concept of a partial metric space further by admitting negative distances. The partial metric defined by are dualistic partial metric,[5]. In a generalization by Heckmann, omitted the small self distance axiom and called the partial metric a weak partial metric,[6].

In 2014, Satish introduced the concept of a partial b-metric space and proved the fixed point theorem of the Banach contraction principle and Kannan type mapping in partial metric spaces, [4]. In the paper we shall focus on the very interesting generalization of metric spaces namely, partial metric spaces.

Definition 1.1. Let X be a non-empty set. A function $d : X \times X \to [0, \infty)$ is a partial b-metric on X [3] , if there exists a real number $\alpha \geq 1$ such that the following conditions hold for all $x, y, z \in X$:

(i) $d(x, x) = d(x, y) = d(y, y) \iff x = y$
(ii) $d(x, x) \leq d(x, y)$
(iii) $d(x, y) = d(y, x)$
(iv) $d(x, y) \leq \alpha\, [d(x, z) + d(z, y)] - d(z, z)$

The pair (X, d) is a called a partial b-metric space.

Example 1.1. *Let $X = [0, \infty)$, $p > 1$ a constant such that $d(x, y) = [\max_{x,y\in X} \{x, y\}]^p + |x - y|^p$ for all $x, y, z \in X$. Then (X, d) is a partial b-metric space.*

Definition 1.2. Let X be a non-empty set. A function $\rho : X \times X \to [0, \infty)$ is a generalized α, β partial b-metric on X if there exists real numbers $\alpha, \beta \geq 1$ such that the following conditions hold for all $x, y, z \in X$:

(i) $x = y \iff \rho(x, x) = \rho(x, y) = \rho(y, y)$
(ii) $\rho(x, x) \leq \rho(x, y)$
(iii) $\rho(x, y) = \rho(y, x)$
(iv) $\rho(x, y) \leq \alpha\rho(x, z) + \beta\rho(z, y) - \rho(z, z)$

The pair (X, ρ) is a called an α, β partial b-metric space. In an α, β b-metric space (X, ρ), if $\rho(x, y) = 0$ for $x, y \in X$ then $x = y$ but the converse may not be true. Every partial b-metric is an α, β partial b-metric with $\alpha = \beta$.

The following example justifies the generalization presented in definition 1.2.

Example 1.2. *Let $X = (1, 3)$ and let $\rho : X \times X \to [0, \infty)$ be a function defined by*

$$\rho(x, y) = e^{|x-y|} + \tfrac{1}{2},$$

To show that the example is a generalized α, β partial b-metric, we verify properties (i)-(iv) of definition 1.2.

Property (i): for $x, y \in X$
If $x = y$, we get

$$1 + \tfrac{1}{2} = e^{|x-y|} + \tfrac{1}{2}$$

since $e^0 = 1$.
If $\frac{3}{2} = e^{|x-y|} + \frac{1}{2}$ then $e^{|x-y|} = 1$ which implies $x = y$.

Property (ii): For $x, y \in X$
$\rho(x, x) = \frac{3}{2} \leq e^{|x-y|} + \frac{1}{2}$ *since $e^x \geq 1$ for all $x \geq 0$.*

Property (iii): For $x, y \in X$, since $e^{|x-y|} = e^{|y-x|}$ it follows that $\rho(x, y) = \rho(y, x)$
Property (iv): For $x, y, z \in X$,

$$\begin{aligned}
\rho(x, y) &\leq e^{|x-z|+|z-y|} \\
&= e^{\frac{1}{3}|x-z|+\frac{2}{3}|z-y|} e^{\frac{2}{3}|x-z|+\frac{1}{3}|z-y|} + \tfrac{1}{2} \\
&\leq \sup_{x,y,z\in X} e^{\frac{1}{3}|x-z|+\frac{2}{3}|z-y|} \left(\tfrac{2}{3}e^{|x-z|} + \tfrac{1}{3}e^{|z-y|}\right) + \tfrac{1}{2} \\
&\leq \tfrac{2}{3}e^2 e^{|x-z|} + \tfrac{1}{3}e^2 e^{|z-y|} + \tfrac{1}{2} \\
&< \tfrac{2}{3}e^2 e^{|x-z|} + \tfrac{1}{3}e^2 e^{|z-y|} + \tfrac{2}{3}e^2 \tfrac{1}{2} + \tfrac{1}{3}e^2 \tfrac{1}{2} - \tfrac{3}{2}
\end{aligned}$$

since $4 < e^2$, it follows that

$$\rho(x, y) \leq \alpha\rho(x, z) + \beta\rho(z, y) - \rho(z, z),$$

where $\alpha = \frac{2}{3}e^2 \geq 1$ and $\beta = \frac{1}{3}e^2 \geq 1$. We conclude that (X, ρ) is an α, β partial b-metric space.

One introduces a topology τ on a generalized α, β partial b-metric space (X, ρ) whose base is the family of open balls $B_\rho(x, \epsilon)$ with centre $x \in X$ and radius $\epsilon > 0$ is given by $B_\rho(x, \epsilon) = \{y \in X : \rho(x, y) < \epsilon + \rho(x, x)\}$ and $\overline{B_\rho(x, \epsilon)} = \{y \in X : \rho(x, y) \leq \epsilon + \rho(x, x)\}$

Definition 1.3. Let (X, ρ) be an α, β partial b-metric space, and let $\{x_n\}$ be a sequence in X and $x \in X$. Then:

(i) The sequence $\{x_n\}$ converges to $x \in X$, if $\lim_{n\to\infty} \rho(x_n, x) = \rho(x, x)$.
(ii) The sequence $\{x_n\}$ is a Cauchy in (X, ρ), if $\lim_{n,m\to\infty} \rho(x_n, x_m)$ exists and is finite.
(iii) The α, β partial b- metric space (X, ρ) is complete, if every Cauchy sequence $\{x_n\}$ in X there exists $x \in X$ such that $\lim_{n,m\to\infty} \rho(x_n, x_m) = \lim_{n\to\infty} \rho(x_n, x) = \rho(x, x)$.

2. FIXED POINT THEOREM FOR GENERALIZED α, β PARTIAL b-METRIC SPACES

The following theorem is an analog to the Banach contraction principle in α, β partial b-metric space.

Theorem 2.1. *Let (X, ρ) be a complete α, β partial b-metric space and $T : X \to X$ a mapping such that*

$$\rho(Tx, Ty) \leq \lambda\rho(x, y) \tag{2.1}$$

for all $x, y \in X$, where $\lambda \in [0, 1)$, then T has a unique fixed point.

Proof. We begin the proof by showing that if fixed points of T exists, then it is unique. Let $x^*, x^{**} \in X$ be distinct fixed points of T. It follows from (2.1) that

$$\rho(x^*, x^{**}) = \rho(Tx^*, Tx^{**}) \leq \lambda\rho(x^*, x^{**}) < \rho(x^*, x^{**})$$

a contradication. Therefore, we must have that $\rho(x^*, x^{**}) = 0$. Furthermore, if x^* is a fixed point and $\rho(x^*, x^*) > 0$ then from (2.1) we get

$$\rho(x^*, x^*) = \rho(Tx^*, Tx^*) \leq \lambda\rho(x^*, x^*) < \rho(x^*, x^*)$$

a contradiction, thus $\rho(x^*, x^*) = 0$.

For the existence of a fixed point for $\lambda \in [0, 1)$, we choose $n_0 \in \mathbb{N}$ such that $\lambda^{n_0} < \frac{\epsilon}{2(\alpha+\beta)}$ for $0 < \epsilon < 1$. Let $(T^{n_0})^k x_0 = x_k$ for all $k \in \mathbb{N}$, where $x_0 \in X$ is arbitrary.
For $x, y \in X$

$$\rho((T^{n_0})\, x, (T^{n_0})\, y) \leq \lambda^{n_0}\rho(x, y)$$

For any $k \in \mathbb{N}$, we get

$$\rho(x_{k+1}, x_k) \leq \lambda^{n_0}\rho(x_k, x_{k-1}) \tag{2.2}$$

$$\leq (\lambda^{n_0})^k \rho(x_1, x_0) \to 0 \tag{2.3}$$

as $k \to \infty$. It follows that we can choose $x_m \in \mathbb{N}$ such that $\rho(x_{m+1}, x_m) < \frac{\epsilon}{2(\alpha+\beta)}$. We shall now show that $T^{n_0}\left(\overline{B_\rho(x_m, \frac{\epsilon}{2})}\right) \subset \overline{B_\rho(x_m, \frac{\epsilon}{2})}$.

Since $x_m \in B_\rho(x_m, \frac{\epsilon}{2})$ it follows that $B_\rho(x_m, \frac{\epsilon}{2}) \neq \emptyset$. Let $z \in B_\rho(x_m, \frac{\epsilon}{2})$ be arbitrary, then we get

$$\begin{aligned}\rho(T^{n_0}z, T^{n_0}x_m) &\leq \lambda^{n_0}\rho(z, x_m)\\ &\leq \tfrac{\epsilon}{2(\alpha+\beta)}\left[\tfrac{\epsilon}{2} + \rho(x_m, x_m)\right]\\ &< \tfrac{\epsilon}{2(\alpha+\beta)}\left[1 + \rho(x_m, x_m)\right]\end{aligned}$$

Therefore,

$$\begin{aligned}\rho(T^{n_0}z, x_m) &\leq \alpha\rho(T^{n_0}z, T^{n_0}x_m) + \beta\rho(T^{n_0}x_m, x_m) - \rho(T^{n_0}x_m, T^{n_0}x_m)\\ &\leq \alpha\tfrac{\epsilon}{2(\alpha+\beta)}\left[1 + \rho(x_m, x_m)\right] + \beta\tfrac{\epsilon}{2(\alpha+\beta)}\\ &= \tfrac{\epsilon}{2}\left(\tfrac{\alpha}{\alpha+\beta} + \tfrac{\beta}{\alpha+\beta}\right) + \tfrac{\alpha\epsilon}{2(\alpha+\beta)}\rho(x_m, x_m)\\ &< \tfrac{\epsilon}{2} + \rho(x_m, x_m)\end{aligned}$$

thus $T^{n_0}z \in \overline{B_\rho(x_m, \frac{\epsilon}{2})}$.

For $x_m \in \overline{B_\rho(x_m, \frac{\epsilon}{2})}$ it follows that $T^{n_0}x_m \in \overline{B_\rho(x_m, \frac{\epsilon}{2})}$ and repeating the process we get $(T^{n_0})^n x_m \in \overline{B_\rho(x_m, \frac{\epsilon}{2})}$ for all $n \in \mathbb{N}$. Thus we obtain $\rho(x_n, x_k) < \frac{\epsilon}{2} + \rho(x_m, x_m)$ for all $n, k > m$ and the sequence $\{x_n\}$ is a Cauchy sequence since $\rho(x_m, x_m) \leq \rho(x_{m+1}, x_m) < \frac{\epsilon}{2(\alpha+\beta)} < \frac{\epsilon}{2}$. By the completeness of X there exists $x^* \in X$ such that $\lim_{n\to\infty} \rho(x_n, x^*) = \rho(x^*, x^*)$. We shall show that x^* is a fixed point of T. For $n \in \mathbb{N}$ we get

$$\begin{aligned}\rho(x^*, Tx^*) &\le \alpha\rho(x^*, x_{n+1}) + \beta\rho(x_{n+1}, Tx^*) - \rho(x_{n+1}, x_{n+1})\\ &< \alpha\rho(x^*, x_{n+1}) + \beta\rho(Tx_n, Tx^*)\\ &\le \alpha\rho(x^*, x_{n+1}) + \beta\lambda\rho(x_n, x^*)\end{aligned}$$

Thus we obtain that $\rho(x^*, Tx^*) = 0$. Thus x^* is a fixed point of T.
∎

Theorem 2.2. *Let (X, ρ) be a complete α, β partial b-metric space and let $T : X \to X$ be a mapping such that*

$$\rho(Tx, Ty) \le \lambda\left[\rho(x, Tx) + \rho(y, Ty)\right] \tag{2.4}$$

for every $x, y \in X$, where $\lambda \in [0, \frac{1}{2})$, $\lambda \neq \frac{1}{\beta}$. Then T has a unique fixed point in X.

Proof. For the existence of a fixed point, let $x_0 \in X$, the sequence $\{x_n\}$ generated by the formula $x_n = Tx_{n-1} = T^n x_0$. Without loss of generality we may assume that $\rho(x_n, x_{n+1}) > 0$ for all $n \in \mathbb{N}$, otherwise x_n is a fixed point of T for at least one $n \ge 0$. For any $n \in \mathbb{N}$ it follows that

$$\begin{aligned}\rho(x_n, x_{n+1}) &= \rho(Tx_{n-1}, Tx_n)\\ &\le \lambda\left[\rho(x_{n-1}, Tx_{n-1}) + \rho(x_n, Tx_n)\right]\\ &= \lambda\left[\rho(x_{n-1}, x_n) + \rho(x_n, x_{n+1})\right]\end{aligned}$$

therefore it follows that

$$\rho(x_n, x_{n+1}) \le \left(\tfrac{\lambda}{1-\lambda}\right)\rho(x_{n-1}, x_n) \tag{2.5}$$

where $\frac{\lambda}{1-\lambda} < 1$ since $\lambda < \frac{1}{2}$. On repeating the process we obtain

$$\rho(x_n, x_{n+1}) \le \left(\tfrac{\lambda}{1-\lambda}\right)^n \rho(x_0, x_1). \tag{2.6}$$

therefore $\lim_{n\to\infty} \rho(x_n, x_{n+1}) = 0$. We shall now show that the sequence $\{x_n\}$ is a Cauchy sequence. For $n, m \in \mathbb{N}$

$$\begin{aligned}\rho(x_n, x_m) &= \rho(Tx_{n-1}, Tx_{m-1})\\ &\le \lambda[\rho(x_{n-1}, Tx_{n-1}) + \rho(x_{m-1}, Tx_{m-1})]\\ &= \lambda[\rho(x_{n-1}, x_n) + \rho(x_{m-1}, x_m)] \qquad (2.7)\end{aligned}$$

since $\lim_{n\to\infty} \rho(x_n, x_{n+1}) = 0$. For every $\epsilon > 0$ there exists $n_0 \in \mathbb{N}$ such that $\rho(x_{n-1}, x_n) < \frac{\epsilon}{2}$ and $\rho(x_{m-1}, x_m) < \frac{\epsilon}{2}$ for all $n, m \ge n_0$. It follows from inequality (2.7) that

$$\rho(x_n, x_m) < \tfrac{\epsilon}{2} + \tfrac{\epsilon}{2} = \epsilon \tag{2.8}$$

for all $n, m \ge n_0$. Thus $\{x_n\}$ is a Cauchy sequence and $\lim_{n,m\to\infty} \rho(x_n, x_m) = 0$. By the completeness of the X there exists $x^* \in X$ such that $\lim_{n\to\infty} \rho(x_n, x^*) = \lim_{n,m\to\infty} \rho(x_n, x_m) = \rho(x^*, x^*) = 0$. We shall show that x^* is a fixed point of T. For $n \in \mathbb{N}$

$$\begin{aligned}\rho(x^*, Tx^*) &\le \alpha\rho(x^*, x_{n+1}) + \beta\rho(x_{n+1}, Tx^*) - \rho(x_{n+1}, x_{n+1})\\ &\le \alpha\rho(x^*, x_{n+1}) + \beta\rho(Tx_n, Tx^*)\\ &\le \alpha\rho(x^*, x_{n+1}) + \beta[\lambda\rho(x_n, Tx_n) + \rho(x^*, Tx^*)]\end{aligned}$$

$$\begin{aligned}\rho(x^*, Tx^*)[1-\beta\lambda] &\leq \alpha\rho(x^*, x_{n+1}) + \beta\lambda\rho(x_n, Tx_n)\\ \rho(x^*, Tx^*) &\leq \tfrac{\alpha}{1-\beta\lambda}\rho(x^*, x_{n+1}) + \tfrac{\beta\lambda}{1-\beta\lambda}\rho(x_n, Tx_n)\end{aligned}$$

therefore it follows that $\rho(x^*, Tx^*) = 0$. Thus x^* is a fixed point of T.

We shall show that if x^* is a fixed point of T then $\rho(x^*, x^*) = 0$ From (2.4) it follows that

$$\begin{aligned}\rho(x^*, x^*) &= \rho(Tx^*, Tx^*)\\ &\leq \lambda[\rho(x^*, Tx^*) + \rho(x^*, Tx^*)]\\ &= 2\lambda\rho(x^*, x^*)\\ &< \rho(x^*, x^*)\end{aligned}$$

a contradiction. Therefore it follows that $\rho(x^*, x^*) = 0$.

Suppose that $x^*, x^{**} \in X$ are distinct fixed points of T then $x^* = Tx^*$, $x^{**} = Tx^{**}$ and

$$\begin{aligned}\rho(x^*, x^{**}) &= \rho(Tx^*, Tx^{**})\\ &\leq \lambda[\rho(x^*, Tx^*) + \rho(x^{**}, Tx^{**})]\\ &= \lambda[\rho(x^*, x^*) + \rho(x^{**}, x^{**})]\end{aligned}$$

It follows that $\rho(x^*, x^{**}) = 0$, i.e., $x^* = x^{**}$. ∎

Theorem 2.3. *Let (X, ρ) be a complete α, β partial b-metric space and let $T : X \to X$ be a mapping such that*

$$\rho(Tx, Ty) \leq \lambda \max_{x,y\in X} \{\rho(x, y), \rho(x, Tx), \rho(y, Ty)\}, \tag{2.9}$$

where $\lambda \in [0, \frac{1}{\beta})$, for all $x, y \in X$. Then T has a unique fixed point.

Proof. We begin by showing that, if fixed points of T exists, then it is unique. let x^*, x^{**} be two distinct fixed points of T then $Tx^* = x^*$ and $Tx^{**} = x^{**}$. It follows that from (2.9) that

$$\begin{aligned}\rho(x^*, x^{**}) &= \rho(Tx^*, Tx^{**})\\ &\leq \lambda \max_{x^*,x^{**}\in X} \{\rho(x^*, x^{**}), \rho(x^*, Tx^*), \rho(x^{**}, Tx^{**})\}\\ &\leq \lambda \max_{x^*,x^{**}\in X} \{\rho(x^*, x^{**}), \rho(x^*, x^*), \rho(x^{**}, x^{**})\}\end{aligned}$$

From definition 1.2,property [(ii)]

$$\begin{aligned}\rho(x^*, x^{**}) &\leq \lambda\rho(x^*, x^{**})\\ &< \rho(x^*, x^{**})\end{aligned}$$

a contradiction, thus $\rho(x^*, x^{**}) = 0$. For the existence of the fixed point, let $x_0 \in X$ be arbitrary and define a sequence by $x_n = Tx_{n-1}$ for all $n \in \mathbb{N}$. From (2.9) we get

$$\begin{aligned}\rho(x_{n+1}, x_n) &= \rho(Tx_n, Tx_{n-1}) \\ &\leq \lambda \max_{n\in\mathbb{N}} \{\rho(x_n, x_{n-1}), \rho(x_n, Tx_n), \rho(x_{n-1}, Tx_{n-1})\} \\ &= \lambda \max_{n\in\mathbb{N}} \{\rho(x_n, x_{n-1}), \rho(x_n, x_{n+1}), \rho(x_{n-1}, x_n)\} \\ &= \lambda \max_{n\in\mathbb{N}} \{\rho(x_n, x_{n-1}), \rho(x_n, x_{n+1})\}\end{aligned}$$

If $\max_{n\in\mathbb{N}} \{\rho(x_n, x_{n-1}), \rho(x_n, x_{n+1})\} = \rho(x_n, x_{n+1})$ then we obtain that $\rho(x_{n+1}, x_n) \leq \lambda\rho(x_{n+1}, x_n) < \rho(x_{n+1}, x_n)$, a contradiction thus we get that $\max_{n\in\mathbb{N}} \{\rho(x_n, x_{n-1}), \rho(x_n, x_{n+1})\} = \rho(x_n, x_{n-1})$ and we obtain that

$$\rho(x_{n+1}, x_n) \leq \rho(x_n, x_{n-1}).$$

Repeating the process we get $\rho(x_{n+1}, x_n) \leq \lambda^n \rho(x_1, x_0)$ for all $n \geq 0$. For $m, n \in \mathbb{N}$,we have

$$\begin{aligned}&\rho(x_n, x_{n+m}) \\ &\leq \alpha\rho(x_n, x_{n+1}) + \beta\rho(x_{n+1}, x_{n+m}) - \rho(x_{n+1}, x_{n+1}) \\ &\leq \alpha\rho(x_n, x_{n+1}) + \beta\alpha\rho(x_{n+1}, x_{n+2}) + \beta^2\rho(x_{n+2}, x_{n+m}) - \beta\rho(x_{n+2}, x_{n+2}) \\ &\leq \alpha\rho(x_n, x_{n+1}) + \alpha\beta\rho(x_{n+1}, x_{n+2}) + \cdots + \alpha\beta^{m-2}\rho(x_{n+m-2}, x_{n+m-1}) \\ &+ \beta^{m-1}\rho(x_{n+m-1}, x_{n+m}) \\ &\leq \alpha\lambda^n\rho(x_0, x_1) + \alpha\beta\lambda^{n+1}\rho(x_0, x_1) + \cdots + \alpha\beta^{m-2}\lambda^{n+m-2}\rho(x_0, x_1) \\ &+ \beta^{m-1}\lambda^{n+m-1}\rho(x_0, x_1) \\ &= \alpha\lambda^n\rho(x_0, x_1)\left[1 + \beta\lambda + \cdots + \beta^{m-2}\lambda^{m-2}\right] + \beta^{m-1}\lambda^{n+m-1}\rho(x_0, x_1) \\ &= \lambda^n\left[\alpha\frac{1 - \beta^{m-1}\lambda^{m-1}}{1 - \beta\lambda} + \beta^{m-1}\lambda^{m-1}\right]\rho(x_0, x_1) \\ &= \frac{\lambda^n}{(1 - \beta\lambda)}\left[\alpha - \beta^{m-1}\lambda^{m-1}(\alpha + \beta\lambda - 1)\right]\rho(x_0, x_1) \\ &\leq \lambda^n\frac{\alpha}{(1 - \beta\lambda)}\rho(x_0, x_1).\end{aligned}$$

Since $\lambda \in [0, \frac{1}{\beta})$ and $\beta \geq 1$ implies that $\rho(x_n, x_{n+m}) \to 0$ as $n \to \infty$. It follows that the sequence $\{x_n\}$ is a Cauchy sequence in (X, ρ). Since (X, ρ) is complete there exists $x^* \in X$ such that

$$\lim_{n\to\infty} \rho(x_n, x^*) = \rho(x^*, x^*) = 0. \tag{2.10}$$

We now show that x^* is a fixed point of the mapping T. For $n \in \mathbb{N}$

$$\begin{aligned}\rho(x^*, Tx^*) &\leq \alpha\rho(x^*, x_{n+1}) + \beta\rho(x_{n+1}, Tx^*) - \rho(x_{n+1}, x_{n+1}) \\ &\leq \alpha\rho(x^*, x_{n+1}) + \beta\rho(Tx_n, Tx^*) \\ &\leq \alpha\rho(x^*, x_{n+1}) + \beta\lambda\rho(x_n, x^*) \qquad (2.11)\end{aligned}$$

Using (2.10) and taking $n \to \infty$, we obtain $\rho(x^*, Tx^*) = 0$. Thus x^* is a fixed point of T.
∎

3. CONCLUSION

In this paper, we have presented an α, β partial b-metric and proved some fixed point results for this new class of generalized metric. The generalization may bring a wider applications of fixed point results. We have shown that for partial b-metric spaces the contractive mappings that have fixed points have fixed points in the generalized α, β partial b-metric space.

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