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CORRIGENDUM FOR MULTISTAGE ANALYTICAL APPROXIMATE SOLUTION OF QUASI-LINEAR DIFFERENTIAL-ALGEBRAIC SYSTEM OF INDEX 2

IBRAHIM M. ALBAK, FARAH AINI ABDULLAH*, AND ZARITA ZAINUDDIN

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SCHOOL OF MATHEMATICAL SCIENCES, UNIVERSITI SAINS MALAYSIA, 11800 USM, PENANG, MALAYSIA,
ibra13975@gmail.com
farahaini@usm.my
zarita@usm.my

ABSTRACT. This article is a corrigendum to AJMAA Volume 18, Issue 2, Article 13, PDF Link.

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The Fourth Section of the paper deals with "Adomian Polynomials and DTM". In this section, Definition 4.1 provides the definition which is as given below:

The proof of Theorem 1 is given in [19]. The following definitions and theorems are new, and their proofs follow directly from Theorem 1 and the properties of power series.

Definition 4.1 Let $f \in R^n$ be an analytical function with$f\left(u\right)=\left(f_1\left(u\right),\ \ldots,\ f_n\left(u\right)\right)^{\ T}$ and let $f_{t,k}$ be the $k-$th AP corresponding to the component $f_i\left(u\right)$. Then $f_k\ =\left(f_{1,\ k}\ ,\ \ldots,f_{n,\ k}\right)$ is called the vector of the kth AP corresponding to $f(u)$.

should read

The proof of Theorem 1 is given in [19]. The following definitions, theorems and their proofs follow directly from Theorem 1 and the properties of power series.

Definition 4.1 [20] Let $f \in R^n$ be an analytical function with$f\left(u\right)=\left(f_1\left(u\right),\ \ldots,\ f_n\left(u\right)\right)^{\ T}$ and let $f_{t,k}$ be the $k-$th AP corresponding to the component $f_i\left(u\right)$. Then $f_k\ =\left(f_{1,\ k}\ ,\ \ldots,f_{n,\ k}\right)$ is called the vector of the kth AP corresponding to $f(u)$.

Similarly, in Definition 4.2, page 5, should be replaced as follows:

Definition 4.2 Let $A\in R^{nxn}$ be analytical with $A\ \left(u\right)=\left(A_{ij}\ \left(u\right)\right)$ and let $A_{ij,k}$ be the kth AP corresponding the entry $A_{ij}\left(u\right)$. Then

$$A_k=\left(A_{ij,1},...,A_{ij,k}\right)^T$$

corresponding to $A(u)$.

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$$A_k=\left(A_{ij,1},...,A_{ij,k}\right)^T$$

corresponding to $A(u)$.

In Theorem 2, page 4, it should be replaced as follows:

Theorem 2. Let $f(u), g(u), h\left(u\right)\in R^n$ and $A\left(u\right), H\left(u\right)\in R^{nxn}$ be analytical, then the vectors and matrices of the kth AP corresponding to $f\left(u\right),\ g\left(u\right),\ h\left(u\right),\ A\left(u\right)$ and $H\ \left(u\right)$ satisfy:

(1) $h\left(u\right)=f\ \left(u\right)+\alpha g\left(u\right)$ then $h_k=f_k+\alpha g_k;\ \alpha$ is scalar,

(2) $h\left(u\right)=f\ \left(u\right)^T,$ then $h_k=f_{k.}^T,$

(3) $h\left(u\right)=A\ \left(u\right)f\left(u\right)$ then $h_k=\sum_{t=0}^{k}A_{k-i}\ f_{i.}$

(4) $h\left(u\right)=A(u)^T$ then $H_k=A_{k.}^T$

(5) $h\left(u\right)=A^{-1}\left(u\right)f\left(u\right)$ then $h_kA_0^{-1}(f_k-\sum_{t=0}^{k-1}A_{k-i-1}\ f_{i.})$

should read

Theorem 2. [20] Let $f(u), g(u), h(u) \in R^n$ and $A(u), H(u) \in R^{nxn}$ be analytical, then the vectors and matrices of the kth AP corresponding to $f(u)$, $g(u)$, $h(u)$, $A(u)$ and $H\ (u)$ satisfy:

(1) $h(u) = f\ (u) + \alpha g(u)$ then $h_k = f_k + \alpha g_k;\ \alpha$ is scalar,

(2) $h(u) = f\ (u)^T$, then $h_k = f_{k.}^T$,

(3) $h(u) = A\ (u) f(u)$ then $h_k = \sum_{t=0}^{k} A_{k-i}\ f_{i.}$

(4) $h(u) = A(u)^T$ then $H_k = A_{k.}^T$

(5) $h(u) = A^{-1}(u) f(u)$ then $h_k A_0^{-1}(f_k - \sum_{t=0}^{k-1} A_{k-i-1}\ f_{i.})$

The Fifth Section of the paper deals with "Quasilinear DAE of Index-2". In this section, it should be replaced as follows:

The DAE of index-2 is given by

$$u'(x) = f(x) - G^T,\ 0 = g(x), \tag{5.1}$$

where $f:\ R^n \times R^n \to R^n,\ g: R^n \to R^n$ and $G = g'(u(x))$. The Jacobian of g and GG^T is a nonsingular matrix and full row rank.

By using the results of Theorem 2 and applying the DTM, the following series are obtained

$$\sum_{k=1}^{\infty} k_{n_k} t^{k^{-1}} = \sum_{k=0}^{\infty} (G_{K_{-L}} \lambda_L);\ \sum_{k=0}^{\infty} g_k(x) t^k = 0. \tag{5.2}$$

Equating like terms in Eq. (5.1), the algebraic recursive system is then created, given by

$$\begin{aligned} k u_k(x) &= f_{k-1}\ - \sum_{i=0}^{k-1} G_{K_{-1_{-l}}}^T \lambda_l, \\ G_0 u_k(x) &= s_k, \\ s_k &= -g_k(x) + G_0 u_k(x). \end{aligned} \tag{5.3}$$

Should read

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$$u'(x) = f(x) - G^T,\ 0 = g(x), \tag{5.1}$$

where $f:\ R^n \times R^n \to R^n,\ g: R^n \to R^n$ and $G = g'(u(x))$. The Jacobian of g and GG^T is a nonsingular matrix and full row rank.

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An addition of reference

- B. BENHAMMOUDA, Approximate analytical solution of index-2 DAEs arising from constrained multibody systems, *Asian Research Journal of Mathematics*, **8**, (2018), pp. 1–15;

to the list of **References** as below

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