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SOME CRITERIA FOR SUBSPACE-HYPERCYCLICITY OF C_0-SEMIGROUPS

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ABSTRACT. We research subspace-hypercyclic C_0-semigroups in this paper. We present various types of subspace-hypercyclicity criteria for C_0-semigroups. Some of them are stronger than the criteria introduced before. Also, we state that if a C_0-semigroup $(T_t)_{t\geq 0}$ satisfies in any of them, then $(T_t \oplus T_t)_{t\geq 0}$ is subspace-hypercyclic.

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1. INTRODUCTION

Let $T : X \to X$ be a bounded and linear operator or briefly an operator, where X is a separable and infinite-dimensional Banach space. The orbit of $x \in X$ under T is denoted by $orb(T, x)$ and it is defined as $\{x, Tx, ..., T^n x, ...\}$. If for an element $x \in X$, $\overline{orb(T, x)} = X$, then T is said a hypercyclic operator and x is called a hypercyclic vector for T. Hypercyclicity is an important concept in dynamical systems. One can see [9] as an excellent reference for hypercyclicity and related topics.

Strongly continuous semigroups of operators or C_0-semigroups are exciting structures. We name a family $(T_t)_{t\geq 0}$ of operators on a Banach space X, strongly continuous semigroup of operators or C_0-semigroup, if $T_0 = I$ and for all $s, t \geq 0$ and for all $x \in X$,

$$T_{t+s} = T_t T_s \quad \text{and} \quad lim_{s\to t} T_s x = T_t x.$$

Desh, Schappacher and Webb in [7] defined hypercyclicity for C_0-semigroups of operators as follows.

Definition 1.1. A C_0-semigroup $(T_t)_{t\geq 0}$ on a Banach space X is said a hypercyclic C_0-semigroup if there exists $x \in X$ such that

$$\overline{orb((T_t)_{t\geq 0}, x)} = \overline{\{T_t x : t \geq 0\}} = X.$$

Hypercyclicity in C_0-semigroups can be considered as the continuous case instead of discrete case that one sees in hypercyclicity of an operator.

It is established that hypercyclic C_0-semigroups do not exist on finite-dimensional spaces but infinite-dimensional and separable Banach spaces support hypercyclic C_0-semigroups([9]).

One can find more interesting matters about hypercyclic C_0-semigroups in [2] and [3].

In 2011, subspace-hypercyclic operators defined in [12]. An operator T on a Banach space X is called M-hypercyclic if $x \in X$ can be found such that

$$\overline{orb(T, x) \cap M} = M,$$

where M is a closed and nonzero subspace of X. By this definition, they extend the concept of hypercyclicity. Bamerni, Kadets and Kilicman by showing that hypercyclic operators are subspace-hypercyclic proved that subspace-hypercyclic operators like to hypercyclic operators, can be found on any infinite-dimensional and separable Banach space([1]). One can find interesting results and theorems in [13] and [14] for this subject.

The concept of subspace-hypercyclicity was extended for C_0-semigroups in [15] and authors investigated them.

Definition 1.2. ([15]) Let $(T_t)_{t\geq 0}$ be a C_0-semigroup on a Banach space X. Consider M is a closed and nonzero subspace of X. If there exists $x \in X$ such that

$$\overline{orb((T_t)_{t\geq 0}, x) \cap M} = M,$$

then we say $(T_t)_{t\geq 0}$ is an M-hypercyclic C_0-semigroup. In this case, x is called an M-hypercyclic vector for T.

It is clear by definition that hypercyclic C_0-semigroups are subspace hypercyclic, since it is sufficient to consider $M := X$. If $(T_t)_{t\geq 0}$ is a hypercyclic C_0-semigroup on a space X, then $(I \oplus T_t)_{t\geq 0}$ is an example of a subspace-hypercyclic C_0-semigroup with respect to subspace $\{0\} \oplus X$ that is not hypercyclic. Also, authors in [15] stated a criteria for subspace-hypercyclic C_0-semigroups.

In section 2, we recall some preliminaries and results in subspace-hypercyclic C_0-semigroups. We improve some results that exist before. Also, we prove that if one member of $(T_t)_{t\geq 0}$ have somewhere dense orbit, then $(T_t)_{t\geq 0}$ is M-hypercyclic.

In section 3, we state various new versions of subspace-hypercyclicity criteria for subspace-hypercyclicity of C_0-semigroups that some of them are stronger than previous criteria. Some of them based on eigenvectors and backward orbits. Also, by the idea of outer and inner supercyclicity criteria, we establish two criteria for subspace-hypercyclicity of C_0-semigroups.

2. SOME PROPERTIES OF SUBSPACE-HYPERCYCLIC SEMIGROUPS

Authors in [15, Proposition 2.4] proved that if there exists $s > 0$ such that T_s is an M-hypercyclic operator, then $(T_t)_{t\geq 0}$ is M-hypercyclic. It is not hard to see that if T^n is an M-hypercyclic operator, then T is M-hypercyclic too, since $orb(T^n, x) \subseteq orb(T, x)$. By this fact, we can improve their result as follows.

Theorem 2.1. *Suppose that $(T_t)_{t\geq 0}$ is a C_0-semigroup on a Banach space X. If there exist $s > 0$ and natural number p such that T_s^p is subspace-hypercyclic with respect to a closed and nonzero subspace M of X, then $(T_t)_{t\geq 0}$ is M-hypercyclic.*

Call to mind that if $\overline{A}$ contains a nonempty open set, we say that A is a somewhere dense set. Costakis and Peris determined in [6] that if a C_0-semigroup $(T_t)_{t\geq 0}$ has a somewhere dense orbit, then it is hypercyclic. Jiemenez-Munguina, Martinez-Avendano and Peris, by making an example in [10] showed that there is somewhere dense orbit $orb(T, x)$ of an operator T such that $orb(T, x) \cap M$ is somewhere dense in the closed subspace M but it is not dense in M. Now, this question becomes appear:

Question. If a C_0-semigroup $(T_t)_{t\geq 0}$ has a somewhere dense orbit in M, can we conclude that $(T_t)_{t\geq 0}$ is an M-hypercyclic C_0-semigroup?

In the next theorem, we give a partial answer to this question.

Theorem 2.2. *Assume that X is a Banach space and $(T_t)_{t\geq 0}$ is a C_0-semigroup on it. If there is some $t_0 > 0$ and some $x \in X$ such that $orb(T_{t_0}, x)$ is somewhere dense in X, then $(T_t)_{t\geq 0}$ is a subspace-hypercyclic C_0-semigroup.*

Proof. By hypothesis, there is $t_0 > 0$ such that T_{t_0} has a somewhere dense orbit $orb(T_{t_0}, x)$. Therefore, $orb(T_{t_0}, x)$ is dense in X [9, Theorem 6.5]. So, there exists a closed and nontrivial subspace M of X such that $\overline{orb(T_{t_0}, x) \cap M} = M$[1, Theorem 2.1]. Hence, T_{t_0} is M-hypercyclic. Now by Theorem 2.1, $(T_t)_{t\geq 0}$ is an M-hypercyclic C_0-semigroup. ∎

We say that $(T_t)_{t\geq 0}$ is satisfied in M-hypercyclicity criteria if there exists an increasing sequence (t_n) of positive real numbers such that $T_{t_n}(M) \subseteq M$ and there exist dense subsets Y and Z of M such that for any $y \in Y$, $T_{t_n}(y) \to 0$ and for any $z \in Z$ there exists (z_n) in M such that $z_n \to 0$ and $T_{t_n}(z_n) \to z$([16]).

Now, we mention a lemma from [16].

Lemma 2.3. *Assume that $(T_t)_{t\geq 0}$ is a C_0-semigroup on a Banach space X. Let M be a closed and nonzero subspace of X. If for any nonempty relatively open sets $U, V \subseteq M$, there is $s \geq 0$ such that $T_s^{-1}(U) \cap V \neq \phi$, and $T_s(M) \subseteq M$, then $(T_t)_{t\geq 0}$ is M-hypercyclic.*

By using Lemma 2.3, it is established in [15] that if $(T_t)_{t\geq 0}$ satisfies in the M-hypercyclicity criteria, then $(T_t)_{t\geq 0}$ is M-hypercyclic.

Tajmouati, El Bakkali and Toukmati also state another subspace-hypercyclicity criteria in [15] as follows.

Theorem 2.4. *Consider $(T_t)_{t\geq 0}$ is a C_0-semigroup on a Banach space X. Let $X_0 \subseteq M$ and $Y_0 \subseteq M$ be dense subsets of M. Consider (t_n) be an increasing sequence of positive real numbers tending to infinity. If $T_{t_n}(M) \subseteq M$ and there exist operators $S_{t_n} : Y_0 \to M$ such that:*

(i) *for any* $x \in X_0$, $T_{t_n}x \to 0$,
(ii) *for any* $y \in Y_0$, $S_{t_n}y \to 0$,
(iii) *for any* $y \in Y_0$, $T_{t_n}S_{t_n}y \to y$,

then $(T_t)_{t\geq 0}$ *is an* M*-hypercyclic* C_0*-semigroup.*

We can immediately conclude a corollary from these subspace-hypercyclicity criteria.

Corollary 2.5. *If a* C_0*-semigroup* $(T_t)_{t\geq 0}$ *satisfies in* M*-hypercyclicity criteria, then* $(T_t \oplus T_t)_{t\geq 0}$ *is* $M \oplus M$*-hypercyclic* C_0*-semigroup.*

Proof. If $(T_t)_{t\geq 0}$ satisfies in the M-hypercyclicity criteria for C_0-semigroups, then it is not hard to see that $(T_t \oplus T_t)_{t\geq 0}$ satisfies in this criteria with respect to $M \oplus M$. Hence, by what said before $(T_t \oplus T_t)_{t\geq 0}$ is $M \oplus M$-hypercyclic C_0-semigroup.
∎

3. SOME CRITERIA FOR SUBSPACE-HYPERCYCLICITY OF C_0-SEMIGROUPS

In this section, we determine various versions of criteria for subspace-hypercyclicity of C_0-semigroups. First, we state a subspace-hypercyclicity criteria that is stronger than Theorem 2.4. In fact, in the next criteria, condition (i) is weaker than the condition (i) of Theorem 2.4.

Theorem 3.1. *Suppose that* $(T_t)_{t\geq 0}$ *is a* C_0*-semigroup on a Banach space* X*. Let* $X_0 \subseteq M$ *and* $Y_0 \subseteq M$ *be dense subsets of* M*. Consider* (t_n) *be an increasing sequence of positive real numbers tending to infinity. Suppose that* $T_{t_n}(M) \subseteq M$ *and there exist operators* $S_{t_n} : Y_0 \to M$ *such that:*

(i) *for any* $x \in X_0$*, there exists a subsequence* (t_{n_k}) *of* (t_n), $T_{t_{n_k}}x \to 0$,
(ii) *for any* $y \in Y_0$, $S_{t_n}y \to 0$,
(iii) *for any* $y \in Y_0$, $T_{t_n}S_{t_n}y \to y$.

Then $(T_t)_{t\geq 0}$ *is an* M*-hypercyclic* C_0*-semigroup.*

Proof. Let U and V be two relatively open nonempty sets in M. By hypothesis, X_0 and Y_0 are dense in M. So, there is $y \in U \cap Y_0$ and there is $x \in V \cap X_0$. Suppose that $u_k := S_{t_{n_k}}y$. Hence, $u_k \in M$ and by (ii), $u_k \to 0$. Therefore $x + u_k \to x$. So, there is N_1 such that for any $k \geq N_1$,

$$x + u_k \in V. \tag{3.1}$$

On the other hand,

$$\begin{aligned} &T_{t_{n_k}}(x + u_k) = T_{t_{n_k}}(x) + T_{t_{n_k}}(u_k) \\ &= T_{t_{n_k}}(x) + T_{t_{n_k}}(S_{t_{n_k}}y) \\ &\to y. \end{aligned}$$

So, there is N_2 such that for any $k \geq N_2$,

$$T_{t_{n_k}}(x + u_k) \in U. \tag{3.2}$$

Now, if we consider $N := \max\{N_1, N_2\}$, then by (3.1) and (3.2) for any $k \geq N$,

$$T^{-1}_{t_{n_k}}(U) \cap V \neq \phi. \tag{3.3}$$

Since $T_{t_{n_k}}(M) \subseteq M$, by (3.3) and Lemma 2.3, we can conclude that $(T_t)_{t\geq 0}$ is M-hypercyclic.
∎

Feldman, Miller and Miller stated outer and inner supercyclicity criteria in [8] for operators. By the idea of their theorems, we unify outer and inner subspace-hypercyclicity criteria for C_0-semigroups.

Theorem 3.2. *(Outer subspace-hypercyclicity criteria) Let $(T_t)_{t\geq 0}$ be a C_0-semigroup on a Banach space X. Suppose that there exists an increasing sequence (t_n) of positive real numbers such that $t_n \to \infty$ and $T_{t_n}(M) \subseteq M$, for any $n \in \mathbb{N}$. Consider Y_0 is a dense subset of M. If there exist operators $S_{y,t_n} : Y_0 \to M$ such that:*

(i) *for any $y \in Y_0$, $S_{t_n} y \to 0$,*
(ii) *for any $y \in Y_0$, $(T_{t_n} S_{t_n})(y) \to y$,*
(iii) *for any $y \in Y_0$, there exists a dense subset X_y of M such that $T_{t_n} x \to 0$ for any $x \in X_y$,*

then $(T_t)_{t\geq 0}$ is M-hypercyclic.

Proof. We prove that for every arbitrary relatively open and nonempty sets U and V of M, there exists a positive number N such that $T_{t_n}^{-1}(U) \cap V \neq \phi$ for any $n \geq N$. For this, first note that by hypothesis, Y_0 is dense in M. So, there exists $y \in U \cap Y_0$. We define $u_n := S_{t_n} y$. Hence, (u_n) is a sequence in M and $u_n \to 0$.

Another by hypothesis, correspondence to y, we can find a dense subset X_y of M. Therefore, we can find an element x such that $x \in V \cap X_y$. Hence,

$$x + u_n \to x, T_{t_n} x \to 0 \quad \text{and} \quad T_{t_n}(x + u_n) \to y. \tag{3.4}$$

Using (3.4), we can find the positive number that we claim at the beginning of the proof. Now Lemma 2.3 completes the proof. ∎

In the next theorem, we state the inner subspace-hypercyclicity criteria as follows.

Theorem 3.3. *(Inner subspace-hypercyclicity criteria) Consider $(T_t)_{t\geq 0}$ is a C_0-semigroup on a Banach space X. Presume that there exists an increasing sequence (t_n) of positive real numbers such that $t_n \to \infty$ and $T_{t_n}(M) \subseteq M$, for any $n \in \mathbb{N}$, where M is a closed and nonzero subspace of X. Let Y_0 be a dense subset of M. If for any $y \in Y_0$ there exists a dense subset X_y of M and operators $S_{y,t_n} : X_y \to M$ such that:*

(i) *$S_{y,t_n} x \to 0$ for every $x \in X_y$,*
(ii) *$(T_{t_n} S_{y,t_n})(x) \to x$, for every $x \in X_y$,*
(iii) *$T_{t_n} y \to 0$ for any $y \in Y_0$,*

then $(T_t)_{t\geq 0}$ is M-hypercyclic.

Proof. Let U and V be open subsets of M. By hypothesis, we can find $y \in V \cap Y_0$ and a dense subset X_y of M such that it satisfies in the conditions of the theorem. So, we can consider there is an element x such that $x \in U \cap X_y$. If we define $u_n := S_{y,t_n} x$ then,

$$u_n \to 0 \quad \text{and} \quad y + u_n \to y. \tag{3.5}$$

Also,

$$T_{t_n}(y + u_n) = T_{t_n}(y) + T_{t_n}(S_{y,t_n} x) \to x. \tag{3.6}$$

Now, by (3.5) and (3.6) and similar to proof of Theorem 3.2, $(T_t)_{t\geq 0}$ is M-hypercyclic.
∎

Remark 3.1. If a C_0-semigroup $(T_t)_{t\geq 0}$ satisfies in the conditions of outer subspace-hypercyclicity criteria, then it is satisfying in the conditions of inner subspace-hypercyclicity criteria too, since it is sufficient to consider $X_y := Y_0$ for any $y \in Y_0$.

The following criteria is presented in the format of Godefroy-Shapiro Criteria for hypercyclicity [9, Theorem 3.1] and we determine in it that a suitable set of eigenvectors can lead us to subspace-hypercyclicity of a C_0-semigroup.

Theorem 3.4. *Let $(T_t)_{t\geq 0}$ be a C_0-semigroup on a Banach space X. Let M be a closed and nonzero subspace of X. Suppose that λ and μ be two scalars such that $|\lambda| < 1$ and $|\mu| > 1$. Consider there exists an increasing sequence (t_n) of positive real numbers that tending to infinity such that $T_{t_n}(M) \subseteq M$. If*

$$Y = span\{y \in M : T_{t_n}(y) = \lambda^n y \quad \text{for any } n \in \mathbb{N}\}$$

and

$$Z = span\{z \in M : T_{t_n}(z) = \mu^n z \quad \text{for any } n \in \mathbb{N}\}$$

are dense in M, then $(T_t)_{t\geq 0}$ is an M-hypercyclic C_0-semigroup.

Proof. Let U and V be two relatively open nonempty sets in M. Since Y and Z are dense in M, there exist $u \in U \cap Z$ and $v \in V \cap Y$. Hence, we can write

$$u = \Sigma_{k=1}^{p} \alpha_k z_k \quad \text{and} \quad v = \Sigma_{k=1}^{p} \beta_k y_k,$$

where, $z_1, z_2, ..., z_p \in Z$ and $y_1, y_2, ..., y_p \in Y$. Therefore,

$$\begin{aligned} T_{t_n} v &= \Sigma_{k=1}^{p} \beta_k T_{t_n}(y_k) \\ &= \Sigma_{k=1}^{p} \beta_k \lambda^n y_k \\ &\to 0. \end{aligned}$$

If we define $x_n = \Sigma_{k=1}^{p} \frac{\alpha_k}{\mu^n} z_k$, then $x_n \to 0$ and

$$\begin{aligned} T_{t_n} x_n &= \Sigma_{k=1}^{p} \frac{\alpha_k}{\mu^n} T_{t_n}(z_k) \\ &= \Sigma_{k=1}^{p} \frac{\alpha_k}{\mu^n} \mu^n z_k \\ &= \Sigma_{k=1}^{p} \alpha_k z_k \\ &= u. \end{aligned}$$

Hence, $v + x_n \to v$ and $T_{t_n}(v + x_n) \to u$. So, there is N large enough such that for any $n \geq N$, $v + x_n \in T_{t_n}^{-1}(U) \cap V$. But $T_{t_n}(M) \subseteq M$. Therefore $(T_t)_{t\geq 0}$ is M-hypercyclic by Lemma 2.3. ∎

Le in [11] stated a subspace-hypercyclicity criterion for operators. We determine a subspace-hypercyclicity criteria for C_0-semigroups similar to his criteria.

Theorem 3.5. *Suppose that $(T_t)_{t\geq 0}$ is a C_0-semigroup on a Banach space X. Let M be a closed and nonzero subspace of X. Consider $Y_0 \subseteq M$ be a set such that $\overline{Y_0} = M$. If there is $X_0 \subseteq M$ and if there is a strictly increasing sequence (t_n) of positive real numbers tending to infinity such that $T_{t_n}(M) \subseteq M$ and:*

(i) *for any $x \in X_0$, $T_{t_n}(x) \to 0$,*
(ii) *for any $y \in Y_0$, there is a sequence (x_n) in X_0, where $x_n \to 0$ and*

$$T_{t_n}(x_n) \to 0,$$

(iii) *$X_0 \subseteq \cap_{n=1}^{\infty} T_{t_n}^{-1}(M)$,*

then $(T_t)_{t\geq 0}$ is an M-hypercyclic C_0-semigroup.

Proof. The proof of this criteria is similar to the proof of the criteria that Le stated in [11, Theorem 2.5]. ∎

Note that the conditions of Theorem 3.5 are weaker than conditions in subspace-hypercyclicity criterion for C_0-semigroups.

Definition 3.1. Let $(T_t)_{t\geq 0}$ be a C_0-semigroup on a Banach space X. Consider $x \in X$. A family $(x_t)_{t\geq 0}$ of elements of X is named the backward orbit x under $(T_t)_{t\geq 0}$ if

$$x_0 = x, \quad T_t x_s = x_{s-t} \text{ for any } s \geq t \geq 0.$$

Conejero and Peris stated a hypercyclicity criteria based on backward orbits in [4]. Now, by the idea of their criterion, we establish such criteria for subspace-hypercyclicity of C_0-semigroups.

Theorem 3.6. *Let $(T_t)_{t\geq 0}$ be a C_0-semigroup on a Banach space X. Let M be a closed and nonzero subspace of X. Suppose that there are dense subsets X_0 and Y_0 of M. Let (t_n) be an increasing sequence of positive real numbers tending to infinity such that $T_{t_n}(M) \subseteq M$ and:*

(i) *for any $y \in Y_0$, $T_{t_n}(y) \to 0$,*
(ii) *for any $x \in X_0$, there exists a backward orbit $(x_t)_{t\geq 0}$ in M such that $x_{t_n} \to 0$.*

Then, $(T_t)_{t\geq 0}$ is an M-hypercyclic C_0-semigroup.

Proof. We show that $(T_t)_{t\geq 0}$ satisfies in conditions of M-hypercyclicity criteria and hence, it is M-hypercyclic.

By hypothesis, for any member y of the dense subset Y_0, we have $T_{t_n}(y) \to 0$. Now, consider x be a member of dense subset X_0. By (ii), there exists a backward orbit $(x_t)_{t\geq 0}$ such that $x_{t_n} \to 0$. If we consider $u_n := x_{t_n}$, then $u_n \to 0$ and

$$T_{t_n} u_n = T_{t_n} x_{t_n} = x_{t_n - t_n} = x_0 = x.$$

So, for any $x \in X_0$, there exists (u_n) in M such that $u_n \to 0$ and $T_{t_n} u_n \to x$. Hence, $(T_t)_{t\geq 0}$ satisfies in conditions of M-hypercyclicity criteria.

∎

Remark 3.2. If the C_0-semigroup $(T_t)_{t\geq 0}$ satisfies the condition of one of the theorems that are mentioned in this section, then $(T_t \oplus T_t)_{t\geq 0}$ also satisfies in these theorems respectively. So, Corollary 2.5 can be restated for these theorems.

4. CONCLUSION

Studying mathematical structures and stating criteria for them to have a particular property, is remarkable for researchers in mathematics. In this paper, we investigated the subspace-hypercyclicity of C_0-semigroups that are exciting structures in dynamical systems.

We proved that a C_0-semigroups $(T_t)_{t\geq 0}$ on a Banach space X is hypercyclic if one of T_t's have a somewhere dense orbit in X. We stated various types of criteria for subspace-hypercyclicity, for which if $(T_t)_{t\geq 0}$ satisfying them, then $(T_t \oplus T_t)_{t\geq 0}$ is subspace-hypercyclic.

By the idea of criteria in [8] for supercyclicity of operators, we presented outer and inner subspace-hypercyclicity criteria for C_0-semigroups in Theorem 3.2 and Theorem 3.3. These criteria are based on the existence of some dense subsets and some operators with dense ranges with special properties. They extend our instrument to recognize subspace-hypercyclic semigroups.

Now, this question raises in mind that can we state some other criteria for subspace-hypercyclicity of C_0-semigroups using some other conditions said in [8]?

Excitingly, as shown in Theorem 3.4, eigenvectors of the operators of a C_0-semigroup can help us to recognize that the semigroup is subspace-hypercyclic or not.

In Theorem 3.5, we state a criterion for subspace-hypercyclicity of C_0-semigroups by the criteria stated in [11] for subspace-hypercyclicity of operators. Hence, this question arises that which other conditions in [11] or [12] can be develop for C_0-semigroups?

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