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CONVERGENCE SPEED OF SOME RANDOM IMPLICIT-KIRK-TYPE ITERATIONS FOR CONTRACTIVE-TYPE RANDOM OPERATORS

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ABSTRACT. The main aim of this paper is to introduce a stochastic version of multistep type iterative scheme called a modified random implicit-Kirk multistep iterative scheme and prove strong convergence and stability results for a class of generalized contractive-type random operators. The rate of convergence of the random iterative schemes are also examined through an example. The results show that our new random implicit kirk multistep scheme perform better than other implicit iterative schemes in terms of convergence and thus have good potentials for further applications in equilibrium problems in computer science, physics and economics.

Key words and phrases: Random implicit-Kirk multistep iterative schemes; Generalized random contractive-type operators; Separable Banach spaces; Strong convergence; Stability results.

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1. INTRODUCTION

Probabilistic functional analysis is an aspect of mathematics that deals with probabilistic models to solve uncertainties and ambiguities that exist in real world problems. Random nonlinear analysis is a vital area of probabilistic functional analysis that deals with various classes of random operator equations and related problems and solutions. The development of various random methods have transformed the financial markets. Random fixed theorems are well known stochastic generalizations of classical fixed point theorems and are usually needed in the theory of random equations, random matrices, random differential equations and different classes of random operators emanating in physical systems [[22]]. The origin and various generalizations of random fixed point theorems exist in literature, for a complete survey see ([[9]], [[18]] and several related references therein). Several interesting papers have been written on the convergence and stability of different random iterative schemes for various random operators, chief among them are: ([[3]], [[4]], [[6]], [[10] - [12]], [[14]], [[18]], [[19]], [[21]], [[22]] and [[24]]). For example, in 2011, Zhang et al. [[24]] proved almost sure T- stability and convergence of Ishikawa-type and Mann-type random algorithm for ϕ- weakly contractive type random operators in a separable Banach space. Recently, in 2015, Okeke and Kim [[22]] proved strong convergence and summable $T-$stability of the random Picard-Mann hybrid iterative process for a generalized class of random operators in separable Banach spaces.

Definition 1.1. *Let $(E, \|.\|)$ be a normed linear space and D a non-empty, convex, closed subset of E and $T : D \to D$ be a selfmap of D, let $x_0 \in D$, the sequence $\{x_n\}_{n=1}^{\infty}$ defined by*

$$x_{n+1} = \sum_{i=0}^{k} \alpha_i T^i x_n, \quad n \geq 0, \tag{1.1}$$

where k is a fixed integer with $k \geq 1$, $\alpha_i \geq 0$ for each i and $\sum_{i=0}^{k} \alpha_i = 1$, is called Kirk iterative scheme.

Various authors have written inspiring papers on Kirk-type iterative schemes, worthy to mention are the following: the explicit Kirk-Mann [[23]], explicit Kirk-Ishikawa [[23]], explicit Kirk-Noor [[2]] and explicit Kirk-multistep [[2]] iterative schemes.

In 2014, Akewe et al. [[2]] proposed an explicit Kirk-multistep iterative schemes and proved strong convergence and stability results for contractive-like operators in a normed linear space, they also gave useful numerical examples to back up their schemes. Chugh et al. [[13]], introduced an implicit iterative scheme and observed that implicit iterations have an advantage over explicit iterations for nonlinear problems as they provide better approximation of fixed points and are widely used in many applications when explicit iterations are inefficient. The authors [[13]], discovered that approximation of fixed points in computer oriented programs by using implicit iterations can reduce the computational cost of the fixed point problems, they went further to consider a new implicit iteration and study its strong convergence, stability, and data dependence, they also proved through numerical examples that newly introduced iteration has better convergence rate than well known implicit Mann iteration as well as implicit Ishikawa iteration and implicit iterations converge faster as compared to corresponding explicit iterative schemes for a single map T.

Definition 1.2 ([13]). *Let E be a Banach space. For a self map $T : E \to E$ there exists a real number $\delta \in [0, 1)$ and a monotone increasing function $\varphi : \mathbb{R}^+ \to \mathbb{R}^+$ such that $\varphi(0) = 0$ and for every $x, y \in Y$, we have*

$$\|Tx - Ty\| \leq \delta\|x - y\| + \varphi(\|x - Tx\|). \tag{1.2}$$

2. METHODS/EXPERIMENTAL

The purpose of this study is in three folds: firstly, to develop a modified random implicit-Kirk multistep iteration and secondly, use the scheme to prove strong convergence and T- stability for generalized random contractive-type operators. Finally, demonstrate our convergence results with an example to get better rate of convergence.

We shall need the following definitions, lemmas, and iterative schemes in proving our results:

Definition 2.1. *Let (Ω, Σ) be a measurable space and C be a nonempty closed convex subset of a separable Banach space E. A function $T : \Omega \to C$ is said to be measurable if $T^{-1}(\mathcal{B} \cap C) \in \Sigma$ for each Borel set $\mathcal{B}$ of E. A function $T : \Omega \times C \to C$ is called a random operator if $T(\cdot, x) : \Omega \to C$ is measurable for every $x \in C$. A measurable function $p : \Omega \to C$ is called a random fixed point for the operator $T : \Omega \times C \to C$ if $T(\omega, p(\omega)) = p(\omega)$.*

Definition 2.2. *Let (Ω, ξ, μ) be a complete probability measure space and E be a nonempty subset of a separable Banach space X. A random operator $T : \Omega \times E \to E$ is said to be generalized random φ-contractive-like operator if there exists a continuous and nondecreasing function $\varphi : \mathbb{R}^+ \to \mathbb{R}^+$ with $\varphi(t) > 0 \; \forall \; t \in (0, \infty)$ and $\varphi(0) = 0$ such that for each $p(\omega) \in F(T)$, $x, y \in E$ and $\omega \in \Omega$, we have*

$$\|T(\omega, x(\omega)) - T(\omega, y(\omega))\| \leq \delta(\omega)\|x(\omega) - y(\omega)\| + \varphi(\|x(\omega) - T(\omega, x(\omega))\|), \tag{2.1}$$

where $0 \leq \delta(\omega) < 1$.

Definition 2.3 ([8]). *Let $\{a_n\}_{n=0}^{\infty}$ and $\{b_n\}_{n=0}^{\infty}$ be two nonnegative real sequences which converge to a and b respectively. Let $J = lim_{n\to\infty} \frac{|a_n - a|}{|b_n - b|}$,*

i. *if $J = 0$, then $\{a_n\}_{n=0}^{\infty}$ converges to a faster than $\{b_n\}_{n=0}^{\infty}$ converges to b;*
ii. *if $0 < J < \infty$, then both $\{a_n\}_{n=0}^{\infty}$ and $\{b_n\}_{n=0}^{\infty}$ have the same convergence rate;*
iii. *if $J = \infty$, then $\{b_n\}_{n=0}^{\infty}$ converges to b faster than $\{a_n\}_{n=0}^{\infty}$ converges to a.*

Lemma 2.4 ([7]). *If δ is a real number such that $0 \leq \delta < 1$ and $\{e_n\}_{n=0}^{\infty}$ is a sequence of positive numbers such that $\lim_{n\to\infty} e_n = 0$, then for any sequence of positive numbers $\{u_n\}_{n=0}^{\infty}$ satisfying $u_{n+1} \leq \delta u_n + e_n$, $n \in \mathbb{N}$. Then we have $\lim_{n\to\infty} u_n = 0$.*

Lemma 2.5 ([23]). *Let $(X, \|.\|)$ be a normed linear space and $T : X \to X$ be a selfmap of X satisfying (1.2). Let $\varphi : \mathbb{R}^+ \to \mathbb{R}^+$ be a subadditive, monotone increasing function such that $\varphi(0) = 0$, $\varphi(Lu) = L\varphi(u)$, $L \geq 0$, $u \in \mathbb{R}^+$. Then, for all $i \in \mathbb{N}$, $L \geq 0$, $\delta \in [0, 1)$, and for all $x, y \in X$,*

$$\|T^i x - T^i y\| \leq \delta^i \|x - y\| + \sum_{j=1}^{i} \binom{i}{j} \delta^{i-j} \varphi^j(\|x - Tx\|). \tag{2.2}$$

Definition 2.6. *Let (Ω, Σ, μ) be a complete probability measure space and E be a nonempty subset of a separable Banach space X. Let $T : \Omega \times E \to E$ be a random operator. Let $F(T) = \{p(\omega) \in E : T(\omega, p(\omega) = p(\omega), \omega \in \Omega\}$ be the set of random fixed points of T. The random implicit Kirk-multistep iteration is defined thus:*

$$
(2.3) \quad \begin{cases} x_0(w) \in E, \\ x_{n+1}(\omega) = \alpha_{n,0} x_n^{(1)}(\omega) + \sum_{i=1}^{q_1} \alpha_{n,i} T^i x_{n+1}(\omega), \ \sum_{i=0}^{q_1} \alpha_{n,i} = 1, \\ x_n^{(l)}(\omega) = \beta_{n,0}^l x_n^{(l+1)}(\omega) + \sum_{i=1}^{q_{l+1}} \beta_{n,i}^l T^i x_n^{(l)}(\omega), \sum_{i=0}^{q_{l+1}} \beta_{n,i}^{(l)} = 1, l = 1, 2, ..., k-2, \\ x_n^{(k-1)}(\omega) = \beta_{n,0}^{(k-1)} x_n(\omega) + \sum_{i=1}^{q_k} \beta_{n,i}^{(k-1)} T^i x_n^{(k-1)}(\omega), \ \sum_{i=0}^{q_k} \beta_{n,i}^{(k-1)} = 1, \end{cases}
$$

$k \geq 2,\ n = 1, 2, ...,$ where $q_1, q_2, q_3, \cdots, q_k$ are random fixed integers with $q_1 \geq q_2 \geq q_3 \geq \cdots \geq q_k$, $\{\alpha_{n,i}\}_{n=1}^{\infty}$ and $\{\beta_{n,i}^{(l)}\}_{n=1}^{\infty}$ are sequences in [0,1] satisfying $\alpha_{n,i} \geq 0$, $\alpha_{n,0} \neq 0$, $\beta_{n,i} \geq 0$ and $\beta_{n,0}^{(l)} \neq 0$ for each l with $\sum_{n=1}^{\infty}(1 - \alpha_{n,0}) = \infty$.

The random implicit Kirk-Noor iteration is defined thus:

$$
(2.4) \quad \begin{cases} x_0(w) \in E, \\ x_{n+1}(\omega) = \alpha_{n,0} x_n^{(1)}(\omega) + \sum_{i=1}^{q_1} \alpha_{n,i} T^i x_{n+1}(\omega), \quad \sum_{i=0}^{q_1} \alpha_{n,i} = 1, \\ x_n^{(1)}(\omega) = \beta_{n,0}^{(1)} x_n^{(2)}(\omega) + \sum_{i=1}^{q_2} \beta_{n,i}^{(1)} T^i x_n^{(1)}(\omega), \ \sum_{i=0}^{q_2} \beta_{n,i}^{(1)} = 1, \\ x_n^{(2)}(\omega) = \beta_{n,0}^{(2)} x_n(\omega) + \sum_{i=1}^{q_3} \beta_{n,i}^{(2)} T^i x_n^{(2)}(\omega), \ \sum_{i=0}^{q_3} \beta_{n,i}^{(2)} = 1, \end{cases}
$$

where q_1, q_2, q_3 are random fixed integers with $q_1 \geq q_2 \geq q_3$, $\{\alpha_{n,i}\}_{n=1}^{\infty}$,$\{\beta_{n,i}^{(1)}\}_{n=1}^{\infty}$ and $\{\beta_{n,i}^{(2)}\}_{n=1}^{\infty}$ are sequences in [0, 1] satisfying $\alpha_{n,i} \geq 0$, $\alpha_{n,0} \neq 0$, $\beta_{n,i}^{(1)} \geq 0$, $\beta_{n,0}^{(1)} \neq 0$, $\beta_{n,i}^{(2)} \geq 0$ and $\beta_{n,0}^{(2)} \neq 0$ and $\sum_{n=1}^{\infty}(1 - \alpha_{n,0}) = \infty$.

The random implicit Kirk-Ishikawa iteration is defined thus:

$$
(2.5) \quad \begin{cases} x_0(w) \in E, \\ x_{n+1}(\omega) = \alpha_{n,0} x_n^{(1)}(\omega) + \sum_{i=1}^{q_1} \alpha_{n,i} T^i x_{n+1}(\omega), \quad \sum_{i=0}^{q_1} \alpha_{n,i} = 1, \\ x_n^{(1)}(\omega) = \beta_{n,0}^{(1)} x_n(\omega) + \sum_{i=1}^{q_2} \beta_{n,i}^{(1)} T^i x_n^{(1)}(\omega), \ \sum_{i=0}^{q_2} \beta_{n,i}^{(1)} = 1, \end{cases}
$$

where q_1, q_2 are random fixed integers with $q_1 \geq q_2$, $\{\alpha_{n,i}\}_{n=1}^{\infty}$ and $\{\beta_{n,i}^{(1)}\}_{n=1}^{\infty}$ are sequences in [0, 1] satisfying $\alpha_{n,i} \geq 0$, $\alpha_{n,0} \neq 0$, $\beta_{n,i}^{(1)} \geq 0$, $\beta_{n,0}^{(1)} \neq 0$ and $\sum_{n=1}^{\infty}(1 - \alpha_{n,0}) = \infty$.

The random implicit Kirk-Mann iteration is defined thus:

$$
(2.6) \quad \begin{cases} x_0(w) \in E, \\ x_{n+1}(\omega) = \alpha_{n,0} x_n(\omega) + \sum_{i=1}^{q_1} \alpha_{n,i} T^i x_{n+1}(\omega), \quad \sum_{i=0}^{q_1} \alpha_{n,i} = 1, \end{cases}
$$

where q_1 is a random fixed integer, $\{\alpha_{n,i}\}_{n=1}^{\infty}$ is a sequence in [0, 1] satisfying $\alpha_{n,i} \geq 0$, $\alpha_{n,0} \neq 0$ and $\sum_{n=1}^{\infty}(1 - \alpha_{n,0}) = \infty$.

The following remark gives the relationship between the random implicit Kirk-multistep iterative scheme (2.3) and the other random implicit Kirk-type [(2.4), (2.5) and (2.6)] iterations.

Remark 2.7. *The random implicit Kirk-multistep iteration (2.4) is an important generalization of random implicit Kirk-Noor (2.4), random implicit Kirk-Ishikawa (2.5) and implicit Kirk-Mann (2.6) iterations because one can recover (2.4), (2.5) and (2.6) from (2.3). Infact, if k=3 in (2.3), we get random implicit Kirk-Noor iteration (2.4). If k=2 in (2.3), we get random implicit Kirk-Ishikawa iteration (2.5) and if k=2 and $q_2 = 0$ in (2.3), we get random implicit Kirk-Mann iteration (2.6).*

Lemma 2.8. *Let $(X, \|.\|)$ be a normed linear space and $T : X \to X$ be a self random operators of X satisfying (2.1). Let $\varphi : \mathbb{R}^+ \to \mathbb{R}^+$ be a sublinear, monotone increasing function such that $\varphi(0) = 0$ and $\varphi(u) = (1-\delta)u$ for all $0 \leq \delta < 1,\ u \in \mathbb{R}^+$. Then for every $i \in \mathbb{N}$ and $x, y \in Y$, we have*

$$\|T^i(\omega, x) - T^i(\omega, y)\| \leq \delta^i \|x(\omega) - y(\omega)\| + \sum_{j=1}^{i} \tbinom{i}{j} \delta^{i-j} \varphi^j(\|x(\omega) - T(\omega, x)\|). \tag{2.7}$$

Proof. We start the proof by showing that if φ is subadditive then each of the φ^j of φ is subadditive. Since we assume that φ is subadditive, then $\varphi(x(\omega) + y(\omega)) \leq \varphi(x(\omega)) + \varphi(y(\omega))$, for every $x, y \in \mathbb{R}^+$. Thus, the subadditivity of φ^2 yields the following:
$\varphi^2(x(\omega) + y(\omega)) = \varphi(\varphi(x(\omega) + y(\omega))) \leq \varphi(\varphi(x(\omega))) + \varphi(\varphi(y(\omega)))$. Similarly, the subadditivity of φ^3 yields the following:
$\varphi^3(x(\omega)+y(\omega)) = \varphi(\varphi^2(x(\omega)+y(\omega))) \leq \varphi(\varphi^2(x(\omega)))+\varphi(\varphi^2(y(\omega))) = \varphi^3(x(\omega))+\varphi^3(y(\omega))$.
Therefore, in general, φ^n ($n = 1, 2, 3, \dots$) is subadditive, and it can be written as: $\varphi^n(x(\omega) + y(\omega)) \leq \varphi(\varphi^{n-1}(x(\omega))) + \varphi(\varphi^{n-1}(y(\omega))) = \varphi^n(x(\omega)) + \varphi^n(y(\omega))$.
The remaining part of the proof of Lemma 2.8, will be done by mathematical induction on i as follows:
Let $i = 1$, then contractive condition (2.7) becomes

$$\begin{aligned} \|T(\omega, x) - T(\omega, y)\| &\leq \delta\|x(\omega) - y(\omega)\| + \sum_{j=1}^{1} \tbinom{1}{j} \delta^{1-j} \varphi^j(\|x(\omega) - T(\omega, x)\|). && (2.8)\\ &= \delta\|x(\omega) - y(\omega)\| + \varphi(\|x(\omega) - T(\omega, x)\|). && (2.9) \end{aligned}$$

Next, suppose $i = n$, where $n \in \mathbb{N}$, then (2.7) becomes

$$\|T^n(\omega, x) - T^n(\omega, y)\| \leq \delta^n \|x(\omega) - y(\omega)\| + \sum_{j=1}^{n} \tbinom{n}{j} \delta^{n-j} \varphi^j(\|x(\omega) - T(\omega, x)\|). \tag{2.10}$$

We now show that the statement is true for $i = n + 1$.

$$\begin{aligned} \|T^{n+1}(\omega, x) - T^{n+1}(\omega, y)\| &= \|T^n(T(\omega, x)) - T^n(T(\omega, y))\| \\ &\leq \delta^n \|T(\omega, x) - T(\omega, y)\| \\ &\quad + \sum_{j=1}^{n} \tbinom{n}{j} \delta^{n-j} \varphi^j(\|T(\omega, x) - T^2(\omega, x)\|) \\ &\leq \delta^n(\delta\|x(\omega) - y(\omega)\| + \varphi(\|x(\omega) - T(\omega, x)\|)) \\ &\quad + \sum_{j=1}^{n} \tbinom{n}{j} \delta^{n-j} \varphi^j(\delta\|x(\omega) - T(\omega, x)\| \\ &\quad + \varphi(\|x(\omega) - T(\omega, x)\|)) \end{aligned}$$

$$
\begin{aligned}
&\leq \delta^{n+1}\|x(\omega)-y(\omega)\|+\delta^{n}\varphi(\|x(\omega)-T(\omega,x)\|)\\
&\quad+\sum_{j=1}^{n}\tbinom{n}{j}\delta^{n+1-j}\varphi^{j}(\|x(\omega)-T(\omega,x)\|)\\
&\quad+\sum_{j=1}^{n}\tbinom{n}{j}\delta^{n-j}\varphi^{j+1}(\|x(\omega)-T(\omega,x)\|)\\
&= \delta^{n+1}\|x(\omega)-y(\omega)\|+\tbinom{n+1}{1}\delta^{n}\varphi(\|x(\omega)-T(\omega,x)\|)\\
&\quad+\tbinom{n+2}{2}\delta^{n-1}\varphi^{2}(\|x(\omega)-T(\omega,x)\|)\\
&\quad+\tbinom{n+1}{3}\delta^{n-2}\varphi^{3}(\|x(\omega)-T(\omega,x)\|)+\cdots+\tbinom{n+1}{n-1}\delta^{2}\varphi^{n-1}(\|x(\omega)-T(\omega,x)\|)\\
&\quad+\tbinom{n+1}{n}\delta\varphi^{n}(\|x(\omega)-T(\omega,x)\|)+\tbinom{n+1}{n+1}\varphi^{n+1}(\|x(\omega)-T(\omega,x)\|). \qquad (2.11)
\end{aligned}
$$

∎

3. Results and Discussion

3.1. Convergence Results in Separable Banach Spaces.

Theorem 3.1. Let $(E, \|.\|)$ be a separable Banach space and $T : \Omega \times E \to E$ be a continuous generalized random φ-contractive-like operator with a random fixed point $p(\omega) \in F(T)$ which satisfies (2.7) for each $x, y \in E$, $0 \leq \delta(\omega) < 1$ and $\varphi : \mathbb{R}^+ \to \mathbb{R}^+$ a continuous and nondecreasing function with $\varphi(t) > 0\ \forall\ t \in (0, \infty)$ and $\varphi(0) = 0$. Let $\{x_n\}_{n=0}^{\infty}$ be the random implicit Kirk- multistep iteration defined by (2.3). Then

i. T defined by (2.7) has a unique random fixed point $p(\omega)$.
ii. the random implicit Kirk-multistep iteration converges strongly to $p(\omega)$ of T.

Proof. (i) We shall first establish that the mapping T satisfying the generalized random φ-contractive-like condition (2.7) has a unique random fixed point $p(\omega)$.
Suppose there exist $p_1(\omega), p_2(\omega) \in F_T$, and that $p_1(\omega) \neq p_2(\omega)$,
with $\|p_1(\omega) - p_2(\omega)\| > 0$, then

$$
\begin{aligned}
0 &< \|p_1(\omega)-p_2(\omega)\| = \|T^i(\omega,p_1(\omega))-T^i(\omega,p_2(\omega))\|\\
&\leq \sum_{j=0}^{i}\tbinom{i}{j}\delta^{i-j}\varphi^{j}(\|p_1(\omega)-T(\omega,p_1(\omega))\|)+\delta^{i}\|p_1(\omega)-p_2(\omega)\|\\
&= \sum_{j=0}^{i}\tbinom{i}{j}\delta^{i-j}\varphi^{j}(0)+\delta^{i}\|p_1(\omega)-p_2(\omega)\|. \qquad (3.1)
\end{aligned}
$$

Thus, $(1-\delta^i)\|p_1(\omega)-p_2(\omega)\| \leq 0$. Since $\delta \in [0, 1)$, then $1-\delta^i > 0$ and $\|p_1(\omega)-p_2(\omega)\| \leq 0$. Since norm is nonnegative we have that $\|p_1(\omega) - p_2(\omega)\| = 0$. That is, $p_1(\omega) = p_2(\omega) = p(\omega)$ (say). Thus, T has a unique random fixed point $p(\omega)$.

Next, we shall establish that $lim_{n\to\infty} x_n(\omega) = p(\omega)$. That is, we show that the random implicit Kirk-multistep iterative scheme converges strongly to $p(\omega)$ of T.

Using generalized $\varphi-$ contractive-like condition (2.7) in (2.3), we have

$$\begin{aligned}
\|x_{n+1}(\omega)-p(\omega)\| &\leq \alpha_{n,0}\|x_n^1(\omega)-p(\omega)\|+\sum_{i=1}^{q_1}\alpha_{n,i}\|T^i(\omega,x_{n+1}(\omega))-T^i(\omega,p(\omega))\| \\
&\leq \alpha_{n,0}\|x_n^{(1)}(\omega)-p(\omega)\|+(\sum_{i=1}^{q_1}\alpha_{n,i}\delta^i)\|x_{n+1}(\omega)-p(\omega)\| \\
&\quad +\sum_{i=1}^{q_1}\alpha_{n,i}(\sum_{j=0}^{i}\tbinom{i}{j}\delta^{i-j}\varphi^j(\|p(\omega)-T^i(\omega,p(\omega))\|)) \\
&\leq \frac{\alpha_{n,0}}{1-\sum_{i=1}^{q_1}\alpha_{n,i}\delta^i}\|x_n^{(1)}(\omega)-p(\omega)\|. \qquad (3.2)
\end{aligned}$$

We note that $\beta_{n,i}^l$ are measurable sequences in [0,1] for each l and q_1, q_l are fixed integer (for each l) for $n=1,2,3,\ldots$ and $1\leq l\leq k-2$. Similarly, using the generalized φ-contractive-like condition (2.7) in (2.3), we have the following

$$\|x_n^{(1)}(\omega)-p(\omega)\| \leq \frac{\beta_{n,0}^1}{1-\sum_{i=1}^{q_2}\beta_{n,i}^1\delta^i}\|x_n^{(2)}(\omega)-p(\omega)\|. \qquad (3.3)$$

$$\|x_n^{(2)}(\omega)-p(\omega)\| \leq \frac{\beta_{n,0}^2}{1-\sum_{i=1}^{q_3}\beta_{n,i}^2\delta^i}\|x_n^{(3)}(\omega)-p(\omega)\|. \qquad (3.4)$$

$$\vdots$$

$$\|x_n^{(k-2)}(\omega)-p(\omega)\| \leq \frac{\beta_{n,0}^{k-2}}{1-\sum_{i=1}^{q_{k-1}}\beta_{n,i}^{k-2}\delta^i}\|x_n^{(k-1)}(\omega)-p(\omega)\|. \qquad (3.5)$$

Finally, using the generalized φ-contractive-like condition (2.7) in (2.3) for (k-1), we have

$$\|x_n^{(k-1)}(\omega)-p(\omega)\| \leq \frac{\beta_{n,0}^{k-1}}{1-\sum_{i=1}^{q_k}\beta_{n,i}^{k-1}\delta^i}\|x_n(\omega)-p(\omega)\|. \qquad (3.6)$$

Substituting (3.3), (3.4), (3.5) and (3.6) respectively in (3.2) and simplifying, we obtain

$$\begin{aligned}
\|x_{n+1}(\omega)-p(\omega)\| &\leq [\frac{\alpha_{n,0}}{1-\sum_{i=1}^{q_1}\alpha_{n,i}\delta^i}][\frac{\beta_{n,0}^1}{1-\sum_{i=1}^{q_2}\beta_{n,i}^1\delta^i}][\frac{\beta_{n,0}^2}{1-\sum_{i=1}^{q_3}\beta_{n,i}^2\delta^i}]\ldots \\
&\quad [\frac{\beta_{n,0}^{k-2}}{1-\sum_{i=1}^{q_{k-1}}\beta_{n,i}^{k-2}\delta^i}][\frac{\beta_{n,0}^{k-1}}{1-\sum_{i=1}^{q_k}\beta_{n,i}^{k-1}\delta^i}]\|x_n(\omega)-p(\omega)\|. \qquad (3.7)
\end{aligned}$$

Note that:

$$1-\frac{\alpha_{n,0}}{1-\sum_{i=1}^{q_1}\alpha_{n,i}\delta^i}=\frac{1-[\sum_{i=1}^{q_1}\alpha_{n,i}\delta^i+\alpha_{n,0}]}{1-\sum_{i=1}^{q_1}\alpha_{n,i}\delta^i}\geq 1-[\sum_{i=1}^{q_1}\alpha_{n,i}\delta^i+\alpha_{n,0}]$$

hence, $\frac{\alpha_{n,0}}{1-\sum_{i=1}^{q_1}\alpha_{n,i}\delta^i}\leq\sum_{i=1}^{q_1}\alpha_{n,i}\delta^i+\alpha_{n,0}$ Let $\delta^i<\delta<1$, then

$$\sum_{i=1}^{q_1}\alpha_{n,i}\delta^i+\alpha_{n,0}\leq(1-\alpha_{n,0})\delta+\alpha_{n,0}$$

That is,

$$\frac{\alpha_{n,0}}{1-\sum_{i=1}^{q_1}\alpha_{n,i}\delta^i}\leq(1-\alpha_{n,0})\delta+\alpha_{n,0}. \qquad (3.8)$$

Therefore,

$$\begin{aligned} \|x_{n+1}(\omega)-p(\omega)\| &\leq [(1-\alpha_{n,0})\delta+\alpha_{n,0}][(1-\beta^1_{n,0})\delta+\beta^1_{n,0}][(1-\beta^{(2)}_{n,0})\delta+\beta^2_{n,0}]... \\ &\quad [(1-\beta^{k-2}_{n,0})\delta+\beta^{k-2}_{n,0}][(1-\beta^{k-1}_{n,0})\delta+\beta^{k-1}_{n,0}]\|x_n(\omega)-p(\omega)\| \\ &\leq [1-(1-\alpha_{n,0})(1-\delta)]\|x_n(\omega)-p(\omega)\| \\ &\leq \prod_i^n[1-(1-\alpha_{i,0})(1-\delta)]\|x_0(\omega)-p(\omega)\| \\ &\leq e^{-\sum_i^n[(1-\alpha_{i,0})(1-\delta)]}\|x_0(\omega)-p(\omega)\|. \end{aligned} \tag{3.9}$$

Hence, using the fact $\sum_i^n(1-\alpha_{i,0})=\infty$, then $\lim_{n\to\infty}\|x_{n+1}(\omega)-p(\omega)\|=0$. That is, the random implicit Kirk multistep iterative scheme (2.3) converges strongly to $p(\omega)$. This ends the proof. ∎

Theorem 3.1 leads to the following corollary:

Corollary 3.2. Let $(E,\|.\|)$ be a separable Banach space and $T:\Omega\times E\to E$ be a continuous generalized random φ-contractive-like operator with a random fixed point $p(\omega)\in F(T)$ which satisfies (2.7) for each $x,y\in E$, $0\leq\delta(\omega)<1$ and $\varphi:\mathbb{R}^+\to\mathbb{R}^+$ a continuous and nondecreasing function with $\varphi(t)>0\;\forall\;t\in(0,\infty)$ and $\varphi(0)=0$. Let $\{x_n\}_{n=0}^\infty$ be the random implicit Kirk-Noor, random implicit Kirk-Ishikawa, random implicit Kirk-Mann iterations defined respectively by (2.4), (2.5) and (2.6). Then
(a) T defined by (2.7) has a unique random fixed point $p(\omega)$.
(b)i) the random implicit Kirk-Noor iteration (2.4) converges strongly to $p(\omega)$ of T;
(b)ii) the implicit random Kirk-Ishikawa iteration (2.5) converges strongly to $p(\omega)$ of T;
(b)iii) the implicit random Kirk-Mann iteration (2.6) converges strongly to $p(\omega)$ of T.

3.2. **Stability Results in Normed Linear Spaces.**

Theorem 3.3. Let $(X,\|.\|)$ be a normed linear space and $T:\Omega\times X\to X$ be a continuous generalized random φ-contractive-like operator with a random fixed point $p(\omega)\in F(T)$ which satisfies (2.7) for each $x,y\in X$, $0\leq\delta(\omega)<1$ and $\varphi:\mathbb{R}^+\to\mathbb{R}^+$ a continuous and nondecreasing function with $\varphi(t)>0\;\forall\;t\in(0,\infty)$ and $\varphi(0)=0$. If the random implicit Kirk-multistep iterative scheme $\{x_n\}_{n=0}^\infty$ defined by (2.3) converges strongly to $p(\omega)$. Then the random iterative scheme (2.3) is $T-$stable.

Proof. Let $\{y_n(\omega)\}_{n=0}^\infty$ be any sequence of random variables in X and

let $\epsilon_n=\|y_{n+1}(\omega)-\alpha_{n,0}y_n^{(1)}(\omega)-\sum_{i=1}^{q_1}\alpha_{n,i}T^i(\omega,y_{n+1}(\omega))\|$

where
$y_n^{(1)}(\omega)=\beta^{(1)}_{n,0}y_n^{(2)}(\omega)+\sum_{i=1}^{q_1}\beta^{(1)}_{n,i}T^i(\omega,z_n^{(1)}(\omega))$, $\sum_{i=0}^{q_2}\beta^{(1)}_{n,i}=1$,

$y_n^{(l)}(w)=\beta^{(l)}_{n,0}y_n^{(l+1)}(\omega)+\sum_{i=1}^{q_{l+1}}\beta^{(l)}_{n,i}T^i(\omega,y_n^{(l)}(\omega))$,$\sum_{i=0}^{q_{l+1}}\beta^{(l)}_{n,i}=1$, $l=1(1)k-2$

$y_n^{(k-1)}(\omega)=\beta^{(k-l)}_{n,0}y_n(\omega)+\sum_{i=1}^{q_k}\beta^{(l)}_{n,i}T^i(\omega,y_n^{(k-1)}(\omega))$, $\sum_{i=0}^{q_k}\beta^{(k-1)}_{n,i}=1$.

Suppose $lim_{n\to\infty}\epsilon_n=0$, we show that $lim_{n\to\infty}y_n(\omega)=p(\omega)$ by using generalized random

φ-contractive-like condition (2.7).

$$\begin{aligned}
\|y_{n+1}(\omega)-p(\omega)\| &\leq \|y_{n+1}(\omega)-\alpha_{n,0}y_n^{(1)}(\omega)-\sum_{i=1}^{q_1}\alpha_{n,i}T^i(\omega,y_n(\omega))\| \\
&\quad +||\alpha_{n,0}y_n^{(1)}(\omega)+\sum_{i=1}^{q_1}\alpha_{n,i}T^i(\omega,y_{n+1}(\omega))-p(\omega)\| \\
&\leq \epsilon_n+\alpha_{n,0}\|y_n^{(1)}(\omega)-p(\omega)\|+\sum_{i=1}^{q_1}\alpha_{n,i}\|T^i(\omega,y_{n+1}(\omega))-p(\omega)\| \\
&\leq \epsilon_n+\alpha_{n,0}\|y_n^{(1)}(\omega)-p(\omega)\|+\sum_{i=1}^{q_1}\alpha_{n,i}\delta^i(\omega)\|y_{n+1}(\omega)-p(\omega)\| \\
&\quad -\sum_{i=1}^{q_1}\alpha_{n,i}\sum_{j=0}^{i}\left(\begin{array}{c} i \\ j \end{array}\right)\delta^{i-j}(\omega)\varphi^j(\|T^ip(\omega)-p(\omega)\|) \\
&= \epsilon_n+\alpha_{n,0}\|y_n^{(1)}(\omega)-p(\omega)\|+\sum_{i=1}^{q_1}\alpha_{n,i}\delta^i(\omega)\|y_{n+1}(\omega)-p(\omega)\|.
\end{aligned}$$

Thus,

$$\|y_{n+1}(\omega)-p(\omega)\| \leq \frac{\epsilon_n}{1-\sum_{i=1}^{q_1}\alpha_{n,i}\delta^i(\omega)}+\frac{\alpha_{n,0}}{1-\sum_{i=1}^{q_1}\alpha_{n,i}\delta^i(\omega)}\|y_n^{(1)}(\omega)-p(\omega)\|. \tag{3.10}$$

Let $\lambda_n=\frac{\alpha_{n,0}}{1-\sum_{i=1}^{q_1}\alpha_{n,i}\delta^i(\omega)}$ then

$$\begin{aligned}
1-\lambda_n &= 1-\frac{\alpha_{n,0}}{1-\sum_{i=1}^{q_1}\alpha_{n,i}\delta^i(\omega)}=\frac{1-\sum_{i=1}^{q_1}\alpha_{n,i}\delta^i(\omega)-\alpha_{n,0}}{1-\sum_{i=1}^{q_1}\alpha_{n,i}\delta^i(\omega)} \\
&\geq 1-\left(\sum_{i=1}^{q_1}\alpha_{n,i}\delta^i(\omega)+\alpha_{n,0}\right)
\end{aligned}$$

Therefore,

$$\lambda_n \leq \sum_{i=1}^{q_1}\alpha_{n,i}\delta^i(\omega)+\alpha_{n,0}=\sum_{i=1}^{q_1}\alpha_{n,i}\delta^i(\omega)<\sum_{i=1}^{q_1}\alpha_{n,i}=1. \tag{3.11}$$

Similarly,

$$\begin{aligned}
\frac{\beta_{n,0}}{1-\sum_{i=1}^{q_2}\beta_{n,i}^{(1)}\delta^i(\omega)} &\leq \sum_{i=1}^{q_2}\beta_{n,i}^{(1)}\delta^i(\omega)<\sum_{i=1}^{q_2}\beta_{n,i}^{(1)}=1; \\
\frac{\beta_{n,0}}{1-\sum_{i=1}^{q_3}\beta_{n,i}^{(2)}\delta^i(\omega)} &\leq \sum_{i=1}^{q_3}\beta_{n,i}^{(2)}\delta^i(\omega)<\sum_{i=1}^{q_3}\beta_{n,i}^{(2)}=1; \\
\frac{\beta_{n,0}}{1-\sum_{i=1}^{q_2}\beta_{n,i}^{(1)}\delta^i(\omega)} &\leq \sum_{i=1}^{q_4}\beta_{n,i}^{(3)}\delta^i(\omega)<\sum_{i=1}^{q_4}\beta_{n,i}^{(3)}=1; \\
&\vdots
\end{aligned}$$

$$\frac{\beta_{n,0}}{1-\sum_{i=1}^{q_k}\beta_{n,i}^{(k-1)}\delta^i(\omega)} \leq \sum_{i=1}^{q_k}\beta_{n,i}^{(k-1)}\delta^i(\omega)<\sum_{i=1}^{q_k}\beta_{n,i}^{(k-1)}=1. \tag{3.12}$$

We note that $\beta^l_{n,i}$ are measurable sequences in [0,1] for each l and q_1, q_l are fixed integer (for each l) for $n = 1, 2, 3, \ldots$ and $1 \leq l \leq k-2$. Similarly, following procedure in (3.10), we obtain

$$\|y_n^{(1)}(\omega) - p(\omega)\| \leq \frac{\beta^{(1)}_{n,0}}{1 - \sum_{i=1}^{q_2} \beta^{(1)}_{n,i}\delta^i(\omega)} \|y_n^{(2)}(\omega) - p(\omega)\|; \tag{3.13}$$

$$\|y_n^{(2)}(\omega) - p(\omega)\| \leq \frac{\beta^{(2)}_{n,0}}{1 - \sum_{i=1}^{q_3} \beta^{(2)}_{n,i}\delta^i(\omega)} \|y_n^{(3)}(\omega) - p(\omega)\|; \tag{3.14}$$

$$\|y_n^{(3)}(\omega) - p(\omega)\| \leq \frac{\beta^{(3)}_{n,0}}{1 - \sum_{i=1}^{q_4} \beta^{(3)}_{n,i}\delta^i(\omega)} \|y_n^{(4)}(\omega) - p(\omega)\|. \tag{3.15}$$

Continuing this process for $k-1$, we obtain

$$\|y_n^{(k-1)}(\omega) - p(\omega)\| \leq \frac{\beta^{(k-1)}_{n,0}}{1 - \sum_{i=1}^{q_k} \beta^{(k-1)}_{n,i}\delta^i(\omega)} \|y_n(\omega) - p(\omega)\|. \tag{3.16}$$

Observe that

$$\begin{aligned} \|y_n^{(1)}(\omega) - p(\omega)\| &\leq \|y_n^{(2)}(\omega) - p(\omega)\| \leq \|y_n^{(3)}(\omega) - p(\omega)\| \leq \cdots \\ &\leq \|y_n^{(k-1)}(\omega) - p(\omega)\| \leq \|y_n(\omega) - p(\omega)\|. \end{aligned} \tag{3.17}$$

Substituting (3.8) into (3.1), we obtain

$$\|y_{n+1}(\omega) - p(\omega)\| \leq \frac{\epsilon_n}{1 - \sum_{i=1}^{q_1} \alpha_{n,i}\delta^i(\omega)} + \frac{\alpha_{n,0}}{1 - \sum_{i=1}^{q_1} \alpha_{n,i}\delta^i(\omega)} \|y_n(\omega) - p(\omega)\|. \tag{3.18}$$

Let $\kappa = \frac{\alpha_{n,0}}{1-\sum_{i=1}^{q_1} \alpha_{n,i}\delta^i(\omega)}$ and $\eta_n = \frac{\epsilon_n}{1-\sum_{i=1}^{q_1} \alpha_{n,i}\delta^i(\omega)}$
Then, (3.18) becomes

$$\|y_{n+1}(\omega) - p(\omega)\| \leq \eta_n + \kappa\|y_n(\omega) - p(\omega)\|. \tag{3.19}$$

Since, $\eta_n \to 0$ and $\kappa < 1$, and from Lemma 2.4, we can conclude that $lim_{n\to\infty}\|y_n(\omega) - p(\omega)\| = 0$ or $lim_{n\to\infty} y_n(\omega) = p(\omega)$.

Conversely, suppose $lim_{n\to\infty}||y_n(w) = p(w)$ for $F(T) \neq \phi$, then we show that $lim_{n\to\infty}\epsilon_n = 0$.

$$\begin{aligned} \epsilon_n &= \|y_{n+1}(\omega) - \alpha_{n,0}y_n^{(1)}(\omega) - \sum_{i=1}^{q_1} \alpha_{n,i}T^i(\omega, y_{n+1}(\omega))\| \\ &\leq \|y_{n+1}(\omega) - p(\omega)\| + \|p(\omega) - \alpha_{n,0}y_n^{(1)}(\omega) + \sum_{i=1}^{q_1} \alpha_{n,i}T^i(\omega, y_{n+1}(\omega)) \\ &\leq \|y_{n+1}(\omega) - p(\omega)\| + \alpha_{n,0}\|y_n^{(1)}(\omega) - p(\omega)\| + \sum_{i=1}^{q_1} \alpha_{n,i}\|T^i(\omega, y_{n+1}(\omega)) - T^i(\omega, p(\omega))\| \\ &\leq \|y_{n+1}(\omega) - p(\omega)\| + \alpha_{n,0}\|y_n(\omega) - p(\omega)\| \\ &\leq (1 + \sum_{i=1}^{q_1} \alpha_{n,i}\delta^i(\omega))\|y_n(\omega) - p(\omega)\| \end{aligned}$$

Since $\|y_n(\omega) - p(\omega)\| \to 0$ as $n \to \infty$, then $lim_{n\to\infty}\epsilon_n = 0$.
Therefore, the random implicit Kirk multistep iteration (2.3) is $T-$ stable. This ends the proof. ∎

Theorem 3.3 leads to the following corollary:

Corollary 3.4. *Let $(X, \|.\|)$ be a normed linear space and $T : \Omega \times X \to X$ be a continuous generalized random φ-contractive-like operator with a random fixed point $p(\omega) \in F(T)$ which satisfies (2.7) for each $x, y \in X$, $0 \leq \delta(\omega) < 1$ and $\varphi : \mathbb{R}^+ \to \mathbb{R}^+$ a continuous and nondecreasing function with $\varphi(t) > 0 \; \forall \; t \in (0, \infty)$ and $\varphi(0) = 0$. If the random implicit Kirk-Noor, random implicit Kirk-Ishikawa, random implicit Kirk-Mann iterations $\{x_n\}_{n=0}^{\infty}$ defined respectively by (2.4), (2.5) and (2.6) converges strongly to the random fixed point $p(\omega)$. Then*
(i) the random iteration (2.4) is $T-$ stable;
(ii) the random iteration (2.5) is $T-$stable;
(iii) the random iteration (2.6) is $T-$stable.

3.3. **Rate of Convergence of the Various Random-Implicit-Kirk-Type Iterative Schemes.** We compare our new implicit random Kirk multistep iterative scheme (2.3) with others defined by (2.4), (2.5) and (2.6) using the following example.

Example 3.1. Let $E = [0, 1]$ and $T : [0, 1] \to [0, 1]$, $T(\omega, x(\omega)) = \frac{x}{8}$, with $x_0 \neq 0$, $\alpha_{n,0} = \beta_{n,0} = 1 - \frac{8}{\sqrt{n}}$,
$\alpha_{n,i} = \beta^l_{n,i} = \frac{8}{\sqrt{n}}$, $n \geq 25$, for each l and for $n = 1, 2, 3..., 24$, $\alpha_{n,i} = \beta^l_{n,i} = 0$ for each l. Let $p(\omega) = 0$.

From the random implicit Kirk Mann iteration (RIKM) defined by (2.6):
$x_{n+1}(\omega) = \alpha_{n,0}x_n(\omega) + \sum_{i=1}^{q1} \alpha_{n,1}T^i(\omega, x_{n+1}(\omega))$,

We have,

$$\begin{aligned} x_{n+1}(\omega) &= (1 - \frac{8}{\sqrt{n}})x_n(\omega) + \sum_{i=1}^{2} \alpha_{n,i}T^i(\omega, x_{n+1}(\omega)), \\ &= (1 - \frac{8}{\sqrt{n}})x_n(\omega) + \frac{8}{\sqrt{n}} \times \frac{x_{n+1}(\omega)}{8} + \frac{8}{\sqrt{n}} \times \frac{x_{n+1}(\omega)}{64} \end{aligned}$$

$\Rightarrow$

$$\begin{aligned} x_{n+1}(\omega) &= (1 - \frac{8}{\sqrt{n}})x_n(\omega) + \frac{1}{\sqrt{n}}x_{n+1}(\omega) + \frac{1}{8\sqrt{n}}x_{n+1}(\omega) \\ (1 - \frac{1}{\sqrt{n}} - \frac{1}{8\sqrt{n}})x_{n+1}(\omega) &= (1 - \frac{8}{\sqrt{n}})x_n(\omega) \\ (1 - \frac{9}{8\sqrt{n}})x_{n+1}(\omega) &= (1 - \frac{8}{\sqrt{n}})x_n(\omega) \\ \frac{8\sqrt{n} - 9}{8\sqrt{n}}x_{n+1}(\omega) &= \frac{\sqrt{n} - 8}{\sqrt{n}}x_n(\omega) \end{aligned}$$

$$x_{n+1}(\omega) = \frac{8(\sqrt{n} - 8)}{8\sqrt{n} - 9}x_n(\omega) = \frac{8\sqrt{n} - 64}{8\sqrt{n} - 9}x_n(\omega) = \prod_{j=25}^{n} \frac{8\sqrt{n} - 64}{8\sqrt{n} - 9}x_0(\omega). \tag{3.20}$$

Similarly, the random implicit Kirk-Ishikawa (RIKI) iteration gives

$$x_{n+1}(\omega) = (\frac{8\sqrt{n}-64}{8\sqrt{n}-9})^2 x_n(\omega) = \prod_{j=25}^{n} (\frac{8\sqrt{n}-64}{8\sqrt{n}-9})^2 x_0(\omega). \tag{3.21}$$

Also, the random implicit Kirk-Noor (RIKN) iteration gives

$$x_{n+1}(\omega) = (\frac{8\sqrt{n}-64}{8\sqrt{n}-9})^3 x_n(\omega) = \prod_{j=25}^{n} (\frac{8\sqrt{n}-64}{8\sqrt{n}-9})^3 x_0(\omega). \tag{3.22}$$

Finally, the random implicit Kirk-Multistep (RIKMulti) iteration gives

$$x_{n+1}(\omega) = (\frac{8\sqrt{n}-64}{8\sqrt{n}-9})^k x_n(\omega) = \prod_{j=25}^{n} (\frac{8\sqrt{n}-64}{8\sqrt{n}-9})^k x_0(\omega). \tag{3.23}$$

Next, we use definition 2.3 to compare the random implicit Kirk-type iterative schemes as follows:

Case 1: Comparison of RIKMulti and RIKN gives:
Let $J = lim_{n\to\infty} \frac{|x_{n+1}(RIKMulti)-p(\omega)|}{|x_{n+1}(RIKN)-p(\omega)|} = lim_{n\to\infty} \prod_{j=25}^{n} (\frac{8\sqrt{j}-64}{8\sqrt{j}-9})^{(k-3)}$
$= lim_{n\to\infty} \prod_{j=25}^{n} (1 - \frac{55}{8\sqrt{j}-9})^{(k-3)} \leq lim_{n\to\infty} \prod_{j=25}^{n} (1 - \frac{1}{j})^{(k-3)}$
$= lim_{n\to\infty} (\frac{24}{25} \times \frac{25}{26} \times \frac{26}{27} \dots \frac{n-1}{n})^{(k-3)} = 0.$

Case 2: Comparison of RIKN and RIKI gives:
Let $J = lim_{n\to\infty} \frac{|x_{n+1}(RIKN)-p(\omega)|}{|x_{n+1}(RIKI)-p(\omega)|} = lim_{n\to\infty} \prod_{j=25}^{n} (\frac{8\sqrt{j}-64}{8\sqrt{j}-9})^{(3-2)}$
$= lim_{n\to\infty} \prod_{j=25}^{n} (1 - \frac{55}{8\sqrt{j}-9}) \leq lim_{n\to\infty} \prod_{j=25}^{n} (1 - \frac{1}{j})$
$= lim_{n\to\infty} (\frac{24}{25} \times \frac{25}{26} \times \frac{26}{27} \dots \frac{n-1}{n}) = 0.$

Case 3: Comparison of RIKI and RIKM gives:
Let $J = lim_{n\to\infty} \frac{|x_{n+1}(RIKI)-p(\omega)|}{|x_{n+1}(RIKM)-p(\omega)|} = lim_{n\to\infty} \prod_{j=25}^{n} (\frac{8\sqrt{j}-64}{8\sqrt{j}-9})^{(2-1)}$
$= lim_{n\to\infty} \prod_{j=25}^{n} (1 - \frac{55}{8\sqrt{j}-9}) \leq lim_{n\to\infty} \prod_{j=25}^{n} (1 - \frac{1}{j})$
$= lim_{n\to\infty} (\frac{24}{25} \times \frac{25}{26} \times \frac{26}{27} \dots \frac{n-1}{n}) = 0.$

3.4. **Summary.** In case 1, it is shown that the random implicit Kirk-multistep (RIKMulti) iteration converges faster than the random implicit Kirk-Noor (RIKN) iteration to the random fixed point $p(\omega) = 0$. Case 2 shows that the random implicit Kirk-Noor (RIKN) iteration converges faster than the random implicit Kirk-Ishikawa (RIKI) iteration to the random fixed point $p(\omega) = 0$, while in case 3, the random implicit Kirk-Ishikawa (RIKI) iteration converges faster than the random implicit Kirk-Mann (RIKM) iteration to the random fixed point $p(\omega) = 0$.

4. CONCLUSION.

We have shown the convergence of random implicit kirk-multistep iteration to the unique random fixed point for generalized contractive-like random operator defined in a separable Banach space. We also proved stability results of this scheme in a normed linear space. It was also demonstrated through example, that our new scheme perform better than other random implicit iterations considered in this work in terms of convergence. Thus, the schemes have good potentials for further applications in equilibrium problems in computer science, physics and economics.

5. ABBREVIATIONS.

RIKM represents random implicit Kirk Mann iteration, RIKI represents random implicit Kirk-Ishikawa iteration, RIKN represents random implicit Kirk-Noor iteration, RIKMulti represents random implicit Kirk-multistep iteration.

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